CELLULAR PROPERTIES OF FUSION SYSTEMS AND LIE GROUPS

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(with Ramón Flores and Alberto Gavira)

Let G be a finite group. In the homotopy theory of p-completed classifying spaces of finite groups (in the sens of Bousfield-Kan), the transfer map in stable homotopy theory provides an splitting describing $\Sigma^\infty_+ BG^\wedge_p$ as a retract of $\Sigma^\infty_+ BS$, where S is a p-Sylow subgroup of G. That is $\Sigma^\infty_+ BG^\wedge_p$ can be constructed from $\Sigma^\infty_+ BS$ as a telescope of a selfmap. In unstable homotopy theory this is not true, but we could ask if $BG^\wedge_p$ can be built out of BS in the sense of cellular approximations described by Dror-Farjoun.

Given connected pointed spaces A and X, X is *A-cellular* if it can be built from A by means of pointed homotopy colimits, possibly iterated. Many authors have contributed to the development of A-homotopy when the spaces involved are classifying spaces (see work of R. Flores, R. Flores-Foote, R- Flores-J. Scherer). Recently, importat results have been obtained by W. Chachólski, E. Dror-Farjoun, R. Flores and J. Scherer in describing cellular covers of nilpotent Postnikov stages.

In the more general case of classifying spaces of saturated fusion systems $(S, \mathcal{F})$ (defined by the p-completion of a centric linking system associated to it), with A. Gavira, we show that cellular properties with respect to $B\mathbb{Z}/p^r$ are controlled by strongly $\mathcal{F}$-closed subgroups.

In a joint work with R. Flores and A. Gavira we apply these techniques to the class of classifying spaces of compact Lie groups in order to understand their cellular properties with respect to $B\mathbb{Z}/p^\infty \times B\mathbb{Z}/p^r$.