Journal of Philosophical Logic (2025) 54:651–689
https://doi.org/10.1007/s10992-025-09798-3

Mīmāṃsā on 'better-not' Permissions

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Received: 22 March 2024 / Accepted: 22 April 2025 / Published online: 19 May 2025

Abstract

The traditional definition of permission as the dual of obligation oversimplifies its many applications, often yielding undesirable consequences. Recent literature recognizes the need to distinguish various types of permissions but often overlooks potential preferences associated with them. In contrast, the Sanskrit philosophical school of Mīmāṃsā refutes the interdefinability of deontic concepts, and asserts that permissions always refer to less desirable actions ('better-not' permissions), and are exceptions to prohibitions or negative obligations. This article analyzes the concept of Mīmāṃsā permission, compares it with contemporary theories and formalizes it while carefully preserving its essential characteristics. We transform Mīmāṃsā's reasoning principles for permission into Hilbert axioms and introduce neighbourhood semantics, incorporating ceteris-paribus preferences. The resulting logic is evaluated against various paradoxes from contemporary deontic logic and applied to a scenario from Sanskrit jurisprudence.

Keywords Deontic logic · Permission · Mīmāṃsā · Sanskrit philosophy · Ceteris-paribus preferences · Deontic paradoxes

1 Introduction

The concept of permission elaborated by the Mīmāṃsā school of Sanskrit philosophy offers a novel approach, resolving some of the ambiguities found in deontic literature.

Permission is of crucial importance in several settings, from law to ethics, and from theology to artificial intelligence. Despite its significance, the notion has received relatively less attention, especially when compared to obligation and prohibition. The

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concept of permission is inherently ambiguous as it becomes evident when one looks at the several manners it can be expressed in, such as “you are allowed to”, “it is open for you to”, “you are authorized to”, and “you have the right to”. Ever since von Wright’s introduction of the first formalized deontic logic system, now referred to as Standard Deontic Logic (SDL) [62], permission has been often viewed simply as the dual of obligation, similar to how possibility serves as the dual of necessity in modal logic. In addition, in SDL all obligatory actions are permitted. However, due to the unintuitive consequences (aka deontic paradoxes) mainly arising from these assumptions, many different deontic logics and types of permissions have been introduced; these include weak and strong permissions (e.g. [2, 5, 6, 63, 65]), bilateral and unilateral permissions (e.g. [13, 34, 38, 39]), positive and negative permissions (e.g. [47, 53]), explicit, tacit and implicit permissions (e.g. [38]), as well as defeasible permissions (e.g. [20, 31, 32]). Additionally, one can distinguish levels of preferences associated with them (see, e.g. [49, 51]): supererogatory permissions, which refer to morally superior actions, or suboptimal permissions, indicating actions that are allowed but represent the least desirable choices among a given set of options.

Here, we contribute to the debate by introducing and formalizing the concept of permission in Mīmāṃsā, one of the main Sanskrit philosophical schools. Philosophers of the Mīmāṃsā school adopt the standard Sanskrit terms for permission, but focus on only one aspect among the ones mentioned above (employing different terms for the others), thus offering a way out of the polysemy of the term ‘permission’. More in general, the Mīmāṃsā school is a largely unexplored source for deontic investigations. Mainly active between the last centuries BCE and the 20th c. CE, Mīmāṃsā centred around the analysis of the prescriptive portions of the Vedas - the sacred texts of (what is now called) Hinduism. Mīmāṃsā authors interpreted the Vedas independently of the will of any speaker, as a consistent and self-sufficient corpus of commands. The Vedas are assumed to be not contradictory, and Mīmāṃsā authors have thoroughly discussed and analyzed normative statements in order to explain what has to be done in the presence of seemingly conflicting norms. They invested all their efforts in creating a consistent deontic system.[1]

To translate this system into suitable deontic logics, we have used the reasoning principles of Mīmāṃsā known as *nyāya*s. The *nyāya*s are interpretative rules, intended to direct a reader through a prescriptive text, facilitating its comprehension without appealing to the author’s intention. *Nyāya*s can be divided into three categories, namely hermeneutic, linguistic, and deontic, see [24]. Certain deontic *nyāya*s describe the properties of deontic operators as discussed by Mīmāṃsā scholars and can be converted into Hilbert axioms, while others (e.g. the specificity principle discussed in Kumārila’s *Tantravārttika*) are metarules to resolve seeming contradictions [43]. The formalization of Mīmāṃsā deontic theories has been involving an interdisciplinary team effort that began with the discovery of the relevant *nyāya*s in Sanskrit texts, followed by their translation into English, interpretation, and formalization. It is important to remark that the resulting logics are solely based on principles extracted from Mīmāṃsā texts. The aim is indeed to faithfully formalize the deontic theories

[1] Different Mīmāṃsā authors interpret commands differently (see [9]), but most of them looked at the Veda as a text having only deontic, i.e., normative authority.

of the Mīmāṃsā authors, to shed light on the controversies they engage with and also offer new insights into contemporary deontic logic.

A distinctive feature of Mīmāṃsā deontics is the non-interdefinability of the deontic operators, i.e., obligation, prohibition, and permission. Mīmāṃsā deontics has been progressively formalized through a series of works [9, 15, 16, 43], each introducing new deontic operators and properties found in the original texts. The initial paper [15] presented the base logic, which considered only obligation, whose properties were extracted by analyzing around 40 *nyāya*s. Subsequently, prohibition was added in [43], and [9] included a weaker form of obligation, corresponding to elective duties, which tally with Vedic sacrifices to be performed only if one desires their specific outcome. The concept of permission within Mīmāṃsā has been investigated in the preliminary paper [16]. Its core attributes are: 1) permissions are always exceptions to general prohibitions or negative obligations, and 2) they inherently imply actions that are 'better-not' pursued. The paper introduced a permission operator formalizing the former characteristic, alongside the analysis of a *nyāya* corresponding to a version of the 'ought implies can' principle, see e.g. [8].

In this paper, we present a comprehensive analysis and formalization of the concept of permission in Mīmāṃsā. Building on previous investigations of the other deontic operators, our formalization is grounded in the *nyāya*s found in Mīmāṃsā texts. From the perspective of Mīmāṃsā studies, we improve on [25], by incorporating a broader range of Sanskrit sources and examining the interaction between permission and supererogation. In terms of Logic, we expand upon the logic in [16] by integrating the property that all permissions are better-not. To achieve this, we leverage a specific instance of the 'ceteris-paribus' modality from [7] allowing us to compare worlds that are identical in all respects except for the truth of the permitted action. We adopt ceteris-paribus preferences (as described in [37]) instead of the conventional notion of preference to avoid scenarios in which any world abstaining from the better-not permitted action is automatically deemed superior to any world in which that action occurs. For instance, abstaining from a better-not permitted action while simultaneously violating another norm should not be considered better than executing the permitted action without any norm violation. By taking this approach, we ensure that our analysis focuses solely on the impact of better-not permitted actions.

The semantics of the resulting logic – we call it $LM_P^{\preceq}$ (Mīmāṃsā Logic with permission and preferences) – combines Kripke semantics, where the relation between worlds is a preference relation, and neighbourhood semantics. To analyze the behaviour of $LM_P^{\preceq}$, we confront it with the best known deontic paradoxes concerning permission: free choice [64], Ross' paradox [55] and the paradox of the privacy act [30]. Additionally, we consider a scenario identified in Mīmāṃsā-influenced jurisprudence, namely the poor *brāhmaṇa*. In contrast to SDL, $LM_P^{\preceq}$ behaves well with respect to these paradoxes. The well-behaved nature of our logic can be attributed to the millennia-old philosophical and juridical foundation upon which it is built.

The paper is organized as follows: Section 2 gives a general overview of Mīmāṃsā deontics. Section 3 presents the new discoveries on Mīmāṃsā permission. In particular, Subsection 3.1 discusses the fundamental aspects of permissions in Mīmāṃsā, together with some basic metarules (*nyāya*s), whereas Subsection 3.2 examines the Mīmāṃsā sources we used in our reconstruction. Last, Subsection 3.3 discusses the 'ought implies can' principle and the failure of 'obligation implies permission' (aka axiom D [62]). Section 4 compares the Mīmāṃsā notion of permission to the literature on contemporary deontic logic. Mīmāṃsā permission is formalized in Section 5. In Section 6, the resulting logic is examined in light of the main paradoxes for permission in the deontic logic literature and one scenario from Mīmāṃsā-influenced Sanskrit jurisprudence (called Dharmaśāstra). Section 7 concludes the paper.

Sanskrit sources Throughout this paper, we refer to Jaimini's *Mīmāṃsā Sūtra* (or *Pūrva Mīmāṃsā Sūtra*, henceforth PMS, approximately 250 BCE) and Śabara's *Bhāṣya* 'commentary' thereon (henceforth ŚBh, approx. 5th c. CE), whose authority has been recognized by all Mīmāṃsā authors. We also refer to the following Mīmāṃsā texts: Kumārila's *Tantravārttika* (7th c., a key sub-commentary on PMS and ŚBh), and Rāmānujācārya's *Tantrarahasya* (possibly 14th c. or later), as well as to three key texts of Sanskrit jurisprudence, namely the *Mānavadharmaśāstra* (possibly 1st c. BCE or later), Medhātithi's commentary (early 9th c.) thereon, and Vijñāneśvara's *Mitākṣarā* (early 12th c.), a commentary on Yajñavalkya's code of norms. Whenever referring to a specific passage of these texts, we will indicate the book's number (if applicable), followed by the chapter's number and the verse (or aphorism)'s number. For instance, PMS 1.3.4 will indicate the fourth aphorism in the third chapter of the first book of the PMS. *Mānavadharmaśāstra* 10.91 will indicate the 91st verse of the tenth chapter of the *Mānavadharmaśāstra*.

2 Preliminaries on Mīmāṃsā Deontics

The Mīmāṃsā school focused on the rational interpretation and systematization of the prescriptive portions of the Vedas. These include *commands* of various kinds, such as prescriptions concerning the performance of sacrifices, and prohibitions applying either to the context of a sacrifice or to the entire life of a person (e.g. "One should not harm any living being"). Permissions are not easily recognised by their linguistic form, because linguistic forms can be misleading in Sanskrit, as they can be in English (for instance, within a series of instructions, one can encounter statements in the indicative that are, however, to be analysed as commands, e.g. "One takes two cups of flour", or "One threshes grains"). Therefore, Mīmāṃsā authors identify commands through a semantic and contextual analysis.

Sometimes commands may seem to be contradictory, like in the case of the Śyena sacrifice[2], that should be performed if one wants to kill an enemy, notwithstanding the prohibition to perform any violence. Mīmāṃsā thinkers introduced and applied metarules (*nyāya*s) in order to rigorously analyze the Vedic commands and solve seeming contradictions among them. The *nyāya*s are not listed explicitly and have to be carefully distilled from their concrete applications within the texts.

Example 1 An example of a *nyāya* is "if a certain action is obligatory but it implies other activities, then these other activities are also obligatory" (Rāmānujācārya's *Tantrarahasya* 4).

2 See [9] for the formalizations of the solutions to the Śyena controversy provided by the main Mīmāṃsā authors, and [33] for a formalization by Navya Nyāya thinkers.

The use of logic to analyze Mīmāṃsā reasoning is justified by the rigorous theory of inference implemented by the school, that implicitly refers to logical principles and methods, see [15, 27] and fn. 11 for more details.

Mīmāṃsā authors distinguish between obligations (*vidhi*) and prohibitions (*niṣedha*) and consider them to be ultimately and fundamentally distinct.[3] The former are identified by the potential to achieve a desired outcome upon fulfillment, whereas the latter by the liability to sanction, if not fulfilled. Hence, obligations and prohibitions are not one the dual of the other. If something is obligatory and one does it, one gets a reward, whereas if something is prohibited and one refrains from doing it, one only remains safe from sanctions, but does not receive any reward. Accordingly, negative obligations and prohibitions are distinct concepts, not mutually definable.

Example 2 Mīmāṃsā scholars discuss how to interpret the seeming prohibition to lie. If one interprets it as "it is forbidden to lie", this means that one will be liable to a sanction if one lies and that no reward will follow if one tells the truth. If, by contrast, it is interpreted as "it is obligatory not to lie", this means that one will receive a reward if one avoids any lie.

Commands are always uttered with regard to a specific person, called 'eligible' or 'responsible' (*adhikārin*), or to a specific situation in which an *adhikārin* might be in. In terms of deontic logic, this means that commands are always dyadic. For instance, the obligation to recite the Vedas is incumbent only on male members of the highest three classes who have undergone initiation, which can be rendered as $\mathcal{O}(reciteVedas/initiated)$.

A further salient characteristic of Mīmāṃsā deontics is that commands always have one goal, hence they do not have conjunctions or disjunctions within them. A seemingly unitary command like "You should offer clarified butter and pour milk" would be interpreted as two separate commands, namely "You should offer clarified butter" and "You should pour milk". Thus, all commands have only one action as their argument. see Remark 2 in Section 5.1.

Last, a metarule prescribes that commands should always convey something new (*apūrva*). A command that seems to prescribe an action one is already inclined to do should, therefore, be interpreted otherwise.

Example 3 The command "One should eat the five five-nailed animals" cannot be interpreted as enjoining the eating of certain animals, because one is naturally inclined to eat the meat of each animal. The command is instead interpreted as a prohibition of eating the meat of any other animal.

A connected *nyāya* prescribes that the Vedas are always purposeful and do not enjoin anything without purpose. Although the scope of these two *nyāya*s may overlap, they are different as it is possible to imagine a sentence being purposeful but not novel. As a consequence of these *nyāya*s, for instance, prohibitions need to refer to actions one would be naturally inclined to undertake (*rāgaprāpta*) or that have already been enjoined (*śāstraprāpta*). Prohibiting something one would never undertake, e.g. eating disgusting food (admitting that no one derives any pleasure from it) would be

[3] Obligations and prohibitions have been discussed in [26], and formalized in [9, 43].

purposeless because eating disgusting food is neither enjoined by another command nor obtained through one's natural inclinations. Purposefulness also dictates that the Veda does not command anything that is irreconcilably contradicted by another portion of the Veda —should this seem to be the case, one would need to revise one's understanding since the Veda is a priori assumed to be consistent.

3 Mīmāṃsā Permissions

The next two sections discuss the essentials of Mīmāṃsā on permissions, together with some basic metarules about permissions (Section 3.1), as well as the Mīmāṃsā sources from which we could extract such essentials (Section 3.2). Our conclusions are based on the main analyses of permissions in the oldest available sources. Section 3.3 discusses general consequences for Mīmāṃsā deontics that we have discovered while analyzing permissions.

3.1 Basic Characteristics

One of the most striking features of Mīmāṃsā deontics is the lack of interdefinability among deontic concepts, a feature that extends to the concept of permission. Therefore, Mīmāṃsā rejects the commonly accepted notion that permission is the absence of a prohibition and the dual of an obligation. The main characteristics of Mīmāṃsā permissions, identified and discussed in [16, 25], are: 1) permissions are always exceptions to general prohibitions or negative obligations, *and* 2) permissions are always better-not permissions.[4] Roughly speaking, Mīmāṃsā permissions correspond to the "strong permissions" discussed in the deontic literature with the additional characteristic of being better-not permissions, see Section 4.

Ad 1), in Mīmāṃsā, saying "it is permitted to do X given Y", always entails that X is negatively obligatory or forbidden given a condition Z that is more general than Y. This can be illustrated by the following applications of the underlying (but unspoken) *nyāya* "A permission is always an exception to a pre-existing prohibition or negative obligation":

(a) The permission to take a second wife can only occur as an exception to a general prohibition to remarry or an obligation not to remarry (see Example 4 in Section 3.2).
(b) The permission to take up the occupation of another class in times of distress depends on the underlying prohibition to take up any occupation other than the ones admitted for one's own class (see Example 8 in Section 3.2.)

[4] In this paper whenever we speak of "Mīmāṃsā permissions" we mean *permissions found in the Veda as interpreted by Mīmāṃsā authors*. We do not consider the Mīmāṃsā take on worldly permissions, about which, see Sect. 5.1 in [25]. The main reason for this choice is that metarules can only be applied in the context of a closed corpus of rules. By contrast, a conversation in which new information is continuously shared can make the common ground of what is known and shared by all speakers shift, and reinterpretations are always possible (for more on this, see [56]). Therefore, applying metarules to an ongoing conversation is hardly possible and permissions exchanged in such contexts cannot be formalized in the same way as Vedic permissions.

(c) The permission to eat at the place of a person who has undergone the ritual of initiation (a preliminary part of every sacrifice) after they bought Soma implies the prohibition to eat (or the obligation not to eat) before it (see Example 6 in Section 3.2.)
(d) The permission to sell while being a *brāhmaṇa* in distress implies the prohibition to sell while being a *brāhmaṇa* in general (see Example 8 in Section 3.2.)

Thus, these permissions are interpreted as presupposing an underlying prohibition or negative obligation, and not as stand-alone permissions. Hence, permissions only make sense for Mīmāṃsā authors with respect to acts that were previously prohibited or the abstention from which was obligatory.

To define the realm of "whatever is not prohibited is permitted", Mīmāṃsā authors introduce the concept of "normatively indifferent actions". These are actions that are possible, but neither prohibited *nor* enjoined (nor permitted in the Mīmāṃsā sense) and that constitute most of our everyday life. Normatively indifferent actions are the ones on which normative texts make an intervention. That is, prior to the intervention of an obligation or prohibition on a given action, the action was just not normed. The obligation or prohibition make an action which previously was normatively indifferent obligatory or prohibited. For instance, offering a ritual substance is not permitted in a Mīmāṃsā sense, because it is enjoined. In the following, we will call whatever is neither prohibited nor permitted nor enjoined "extra-normative". In sum, for Mīmāṃsā there are either normed actions (enjoined, prohibited or permitted) or extra-normative ones. The latter notion covers what deontic logicians refer to as "unconditional permissions".

Ad 2), if X is permitted given Y, doing X is not on the same level as not doing it, or as doing X while X is an extra-normative action. Rather, permissions allow an option that is less desirable than its counterpart. One of the main consequences of this approach is that performing a permitted X exposes one to the risk of restrictions, insofar as the permitted action is actually an action one should have 'better-not' performed. This is extracted from the *nyāya*, "Vedic permissions are always better-not permissions", whose application can be found below:

(e) If one still refrains from eating meat, even though eating meat in particular circumstances is permitted, it is a meritorious act which leads one to the accumulation of good *karman* (see Example 5 in Section 3.2).
(f) Eating meat, drinking wine, and making love are actions one is naturally inclined towards. Performing them leads to no penalty, but abstaining from them leads to merit (see Example 7 in Section 3.2).
(g) Eating at the place of a person who has undergone the ritual of initiation (a preliminary part of every sacrifice) before they bought Soma is prohibited, whereas the same is permitted after they bought Soma. However, once they have bought it, the decision not to eat is preferred to eating (see Example 6 in Section 3.2).

We can conclude that permissions are to be interpreted as better-not permissions: The permission of X means that $\neg X$ is generally obligatory or that X is generally forbidden, and that it would be preferable if one were to keep $\neg X$, but that one can do X if there is no way out.

3.2 Discussions of Permissions in Śabara, Kumārila, Medhātithi and Vijñāneśvara

The metarules mentioned in the previous sections have been extracted from their application in the discussions of permissions by Śabara and Kumārila within Mīmāṃsā, and by Medhātithi and Vijñāneśvara within Mīmāṃsā-influenced jurisprudence (Dharmaśāstra). In this section, we add more context to each discussion and show how we came to the conclusions discussed above based on their texts.

The first Mīmāṃsā author to explicitly mention permissions is **Śabara**. He discusses the following command (see (a) in Section 3.1):

Example 4 "One should take a second wife if the first one is not virtuous or fertile". This command, in theory, could be interpreted as an obligation to remarry given those circumstances. However, Śabara explains that it cannot be interpreted as an obligation, because one is naturally inclined to take as many wives as possible, and commands need to communicate something new. In contrast, he explains that the above command should be interpreted as the permission to take a second wife if the first one is unvirtuous and/or not fertile, which implies the prohibition to take a second wife otherwise (ŚBh ad PMS 6.8.17–18).

Śabara's analysis leads to the conclusion that this is a better-not permission, that better-not permissions only emerge as exceptions to previously stated prohibitions or negative obligations, and that better-not permissions enable one to perform actions one would be naturally inclined to do, such as re-marrying in this example. In fact, the permission to remarry limits a natural desire to specific cases (when one's wife is not virtuous or not fertile), only if its complete elimination is impossible. Several other passages by Śabara confirm that he interprets permissions as better-not permissions. For instance, ŚBh ad PMS 5.4.2 explains that there should be no more interruptions during a ritual than the one explicitly permitted, thus showing that interruptions are only permitted as suboptimal options and that the officiant would naturally be inclined to interrupt the ritual, were it not for the command to avoid interruptions.

Kumārila's analysis of permissions postdates Śabara's by about two centuries and is much more systematic. Accordingly, it is the basis of most later Mīmāṃsā and Dharmaśāstra discussions on permissions. Therefore, we mainly rely on it while discussing permissions in Mīmāṃsā. In *Tantravārttika* ad ŚBh 1.3.4, Kumārila discusses two kinds of better-not permissions: the first involves a small sanction upon performing the permitted action, while the second permits the action without any penalty but instead a reward if one, notwithstanding the permission, decides to follow the preceding negative obligation and refrains from performing the act.

For the former case, Kumārila discusses the following example (see application (e) from Section 3.1):

Example 5 If one risks starvation, one may eat meat.[5] While there is no explicit command in the Vedas that allows eating meat when starving, Kumārila evokes here the unspecific permission to follow different courses of action in times of hardship, mentioned in the *Mānavadharmaśāstra*. In the case of this unspecific permission, one is still sanctionable when eating meat but the sanction is minor and can easily be

[5] The background assumption, for most of Kumārila's audience, is that eating meat should be avoided.

cancelled out with an expiation (*prāyaścitta*) ritual. By contrast, in the presence of a specific permission, there is no need for any expiation, because no sanction at all applies (*Tantravārttika* ad 1.3.4, [59], vol. 1b, p.191).

Given a negative obligation and a specific permission, following the specific permission leads to no sanction, but a reward is obtained if one nonetheless follows the negative obligation and does not perform the permitted action, as explained in the following example (see applications (c) and (g) in Section 3.1):

Example 6 There is an explicit permission to eat at the house of someone who has purchased Soma (a plant to be offered during sacrifices) and is in the process of performing a ritual. Even though it is prohibited to eat during sacrifices, the Vedas provide an explicit permission to eat at the house of someone who has just purchased Soma (*Tantravārttika* ad 1.3.4). Kumārila offers two explanations: Firstly, this permission applies only when there is truly no alternative. Secondly, while it is permitted to eat at their place without sanction, refraining from doing so is a supererogatory action. Kumārila calls the act of not eating meat a 'mental act', i.e. something that counts as an action even though nothing seems to happen in the outside world.[6]

In summary, according to Kumārila, when faced with a prohibition or negative obligation, four potential scenarios may arise: 1) a sanction for violating a prohibition, 2) a minor sanction for violating a prohibition in times of hardship, 3) no sanction, but lack of reward, for following a specific permission, and 4) a reward for following a negative obligation even though a specific permission was available (see Ex. 6 above for an instance of the use of 3 and 4). It is worth underlying that case 4) covers what some Euro-American philosophers and deontic logicians call "supererogation". The term *supererogare* is already found in the Latin version of the Gospel of Luke (10.35), but supererogation was introduced as a distinct category distinguished from the other three being obligatory, forbidden and permissible in [60].[7] For Kumārila, not eating although there is a specific permission to eat and although there is no specific obligation about that specific act (e.g. "$\mathcal{O}(\neg eating/risk\ of\ starving)$"), means doing something meritorious which will be rewarded, thus covering the field of supererogation.

Lastly, the two main Sanskrit jurists, **Medhātithi and Vijñāneśvara**, largely follow the view of Kumārila. Medhātithi was the author of a commentary on the most well-known jurisprudential treatise, the *Mānavadharmaśāstra*. The date of *Mānavadharmaśāstra* is still debated, but it surely predates Śabara and Kumārila. It is therefore historically significant that it already contains, *in nuce*, the idea that there are actions one is naturally inclined to do, and that refraining from performing them could lead to rewards:

Example 7 "There is no flaw attached to eating meat, drinking wine, or making love, [apart from the specific cases in which these behaviours are prohibited]. Undertaking an action with regard to them is natural for living beings. Abstaining from

[6] The same example has been analyzed already in [26], section 6.2.2, but without the awareness that all permissions are better-not permissions. The current conclusions supersede the ones in [26].

[7] For more details on the later debates on supererogation within moral philosophy and ethics, see [40].

action, however, leads to great results”. (*Mānavadharmaśāstra* 5.56, [42] vol.1:444). This means that eating (all sorts of foods, including) meat, drinking (all sorts of drinks, including) wine and making love are actions one is naturally inclined to perform and that are not normed, that is, they belong to the extra-normative space (the *Mānavadharmaśāstra* precedes the systematization by Kumārila, but still it implements the Mīmāṃsā approach to permissions and does not say that these actions are “permissible”). Deciding to refrain from them, however, is supererogatory and leads to rewards. Kumārila used this quote in the context of Example 5.

The following example (to be elaborated in full in Section 6.4), from Vijñāneśvara, shows how permissions are considered exceptions to previous prohibitions and how they are about behaviours that are suboptimal and should better-not be performed (see applications (b) and (d) in Section 3.1).

Example 8 The permission to sell while being a *brāhmaṇa* in distress, implies that a *brāhmaṇa* not in distress should not be selling anything. Similarly, the permission to take up the occupation of another[8] class in times of distress depends on the underlying prohibition to take up any occupation other than the ones admitted for one’s own class (see Vijñāneśvara’s *Mitākṣarā* commentary on Yājñavalkya 3.35).

Medhātithi and Vijñāneśvara appear to propose systems with 1) absolutely forbidden acts, 2) permitted acts that lead to bad karman but can be expiated, and 3) permitted acts that lead to no sanction at all. However, they presuppose a slightly different scheme than Kumārila when it comes to specific and unspecific permissions, since for them, it is possible to obtain a minor sanction even in the case of a specific permission.

Summing up, Mīmāṃsā and Mīmāṃsā-influenced Dharmaśāstra authors agree in considering permissions within a closed corpus of law as better-not permissions and as exceptions to previous prohibitions or negative obligations. Within this article, we streamlined Kumārila’s scenario and followed the Dharmaśāstra approach in just connecting all permissions to a better-not output.

3.3 Ought Entails can and Obligation does not Imply Permission

The article [16] newly (identified and) formalized a characteristic of Mīmāṃsā deontics, that is a version of the ‘ought implies can’ principle. This is usually attributed to Immanuel Kant (see [57]), and in Mīmāṃsā’s case it can be formulated as “each command must be actionable”, thus including the claim that also forbidden entails can. This metarule is extracted from the *nyāya*s “Prescriptions can only prescribe actions that can be performed” and “Prohibitions can only prohibit actions that can be performed”, whose application is found below:

(h) Commands prescribing complicated sacrifices in order to get *svarga* (that is, heaven, to be understood as happiness) are addressed only to men who are able to perform them (see *Tantravārttika* on 1.3.4).

[8] For context, taking up the occupation of another class is frowned upon or prohibited in Dharmaśāstra literature.

(i) The seeming prohibition "The fire is not to be kindled on the earth, nor in the sky, nor in heaven" cannot be taken as a prohibition, because fire cannot be kindled in the sky nor in heaven (see ŚBh on 1.2.5 and 1.2.18).

The metarule regarding novelty (*apūrva*, see Section 2) also implies that each deontic operator needs to make a novel intervention and is therefore applied to an extra-normative situation, or, in the case of permissions, to a pre-existing negative obligation or prohibition. This also means that the same action cannot[9] be at the same time obligatory and permitted given the same circumstances (pace SDL [62]), since the operator for permission would not add anything novel if applied to a situation already normed by the deontic operator for obligation. For instance, if one already knows that male married *brāhmaṇa*s ought to perform a certain ritual at dawn, receiving the information that it is permitted to perform the same ritual at the same time and given the same circumstances would be redundant and purposeless, and no command in the Veda can be purposeless.

4 Mīmāṃsā Permission vs Deontic Logic Permission

We shall now analyze the deontic literature regarding permission, with the specific purpose of drawing comparisons and identifying parallels and distinctions with the concept of permission in Mīmāṃsā.

The interdefinability between obligation and permission is an old problem in Deontic Logic, dating back to von Wright's observation in [62] about its resemblance to the relationship between necessity and possibility. The deontic axiom D, as introduced in SDL, asserts that obligation implies permission. A main problem with this interdefinability is that the resulting system does not allow for gaps [2]. If everything that is not permitted is prohibited and everything that is not prohibited is permitted, then any normative system would regulate all possible states of affairs. This is counterintuitive since not all situations are subject to regulation, as also acknowledged by the Mīmāṃsā school and its recognition of extra-normative actions.

Mīmāṃsā's concept of extra-normativity aligns with the idea of indifference as defined in [50] in relation to supererogation. In McNamara's definition, an indifferent action is neither obligatory nor forbidden. Moreover, the author links an operator for indifference to one for "moral significance" and uses it to differentiate between indifference and supererogation. Both indifferent and supererogatory actions are neither obligatory nor forbidden, but supererogatory actions hold moral significance. In contrast to Mīmāṃsā permissions, where permission for A implies a preference for not doing A, supererogatory actions suggest the opposite: doing A is preferred over not doing A. Moral indifference, on the other hand, indicates no moral preference between A and not A.

In [63], von Wright treats the notion of permission more carefully than in his previous writings and introduces a distinction between weak permission and strong permission. Weak permission is permission as the absence of prohibition, whereas strong permission is a modality by itself. The latter is defined as follows: (i) "an

[9] This corresponds to (the refusal of) Axiom D, which is brought up in Section 4.

act will be said to be permitted in the strong sense if it is not forbidden but subject to norm", and (ii) "an act is permitted in the strong sense if the authority has considered its normative status and decided to permit it". Many authors have sought to formalize von Wright's definition of strong permission, mainly to obtain a consistent formalization of the so-called 'free choice inference', introduced in [64]. Notable attempts to develop a formalization free from undesirable consequences are [3, 5, 6, 19, 22]. These systems tend to be complex, and use, e.g., substructural logics, connectives other than those of classical logic, or semantical elements added to the language. We discuss in detail the free choice inference in Section 6.1.

Hansson's paper [38] argues that, next to strong and weak permission, there is a third permission to distinguish: implicit permission, which is implied by an obligation. For instance, the obligation to testify in court implies the permission to enter the courtroom. In contrast, for Mīmāṃsā an act cannot be both obligatory and permitted under the same circumstances and the obligation to perform X extends to the obligation to perform whatever is necessarily entailed by X. Thus, entering the courtroom is not the content of an implicit permission but of an obligation.

In the next subsections, we debate the alternative formalizations of strong permission that also take into account properties discussed by Mīmāṃsā authors.

4.1 Permissions as Exceptions

A Mīmāṃsā permission is always an exception to a more general prohibition or negative obligation. While permissions-as-exceptions have been considered in the literature, Mīmāṃsā stands out for its distinctive approach, focusing exclusively on permissions that serve as exceptions. In their framework, permissions are only considered meaningful when they alter the normative status of actions.

As we have seen in the previous section, this is not the only way of approaching permissions. For instance, Alchourrón famously recounts a story (originally from [23]) about a hunting tribe and its new chief, who emits a norm permitting hunting on certain days, but without prohibiting it on the others. The tribe is utterly dissatisfied, because one expected from the chief an intervention in the status quo ("The moral of this story is valuable. It shows that purely permissory norms are of little if any practical interest" [1]). Alchourrón's conclusion, is different from the Mīmāṃsā one, as he highlights the importance of permissions in the case of more than one source of norms, see [1]. In contrast, the tribe reasoned according to Mīmāṃsā principles, based on which each command needs to change something which was previously the case (see the novelty requirement discussed in Section 2, Example 3).

Viewing permissions as exceptions reflects a common practice in normative texts, such as legal codes in European jurisprudence, where permissions are typically stated only when there is an expectation of the opposite due to a general prohibition. Norms granting permissions usually derogate from what is stated in other norms, as Bouvier notes in the definition of permission in his legal dictionary [12]. He distinguishes between express permissions that "derogate from something which before was forbidden," and implied permissions, "which arise from the fact that the law has not forbidden the act to be done". The latter are therefore different from Hansson's "implicit permission" and rather correspond to what Hansson calls "tacit permissions" in [38], and to what von Wright calls "weak permissions" in [63]. Similarly, the idea that permissions grant one a different degree of freedom if compared to the non-normed space of indifferent actions is neatly reflected by the comparison of cases like "You are permitted to run 2km per day" (said by a physician to her patient, who is recovering from a heart attack), as opposed to the same person's freedom to run prior to the heart attack. The permission rules the realm of running by introducing a space of possibility that is, however, not as absolute as the space of extra-normative actions. Accordingly, permitted actions are actions one would be naturally inclined to do, prior to the intervention of a normative text prohibiting them (or obliging one to refrain from them). In Mīmāṃsā deontics, it would not make sense to have a permission that regards impossible actions like flying or undesirable actions like harming oneself (assuming that harming oneself is not desirable for anyone). The Mīmāṃsā position is also neatly distinguished from the one of, e.g., [38], who thinks that introducing permissions even in the absence of general prohibitions is useful to define rights.

Treating permissions as exceptions is also not uncommon in the deontic logic literature, as evidenced, e.g., by [11] and [58]. Both systems are based on Input/Output logic, a rule-based approach to deontic logic [48]. The work [11] examines permissions from the perspective of the legislator who has the ability to change the normative system by adding permissions and obligations, and the focus is on the hierarchy of authorities. For example, the permission to attend a party only if one brings a bottle of wine is an exception to the underlying prohibition to attend if one comes empty-handed. Unlike in the Mīmāṃsā perspective, the permissions discussed in this work are not associated with less desirable outcomes; for instance, the permission to attend a party only if one brings a bottle of wine does not lead to the conclusion that joining the party is an undesired outcome. Additionally to the permission-as-exception, the work [58] categorizes antithetic permissions, that is, permissions that cannot be prohibited by a code without creating a contradiction. Examples include actions protected by constitutional laws such as the freedom of expression, within the legal framework, or actions of compassion and kindness that cannot be prohibited without leading to a contradiction within the framework of moral principles. Both [11] and [58] refer to weak permissions as permissions whereas Mīmāṃsā treats these as describing extra-normative actions.

4.2 Permissions as Less Desirable Actions

A key trait of Mīmāṃsā permissions is that they always lead to less desirable options, and are therefore better-not permissions. This relates to common notions in ethical theory regarding preferences in permitted actions, see e.g. [40], distinguishing *supererogatory* actions — doing more than the obligatory — and *permissibly suboptimal* actions — doing only the bare minimum. The concepts of supererogation and permissible suboptimality are also discussed in the deontic logic literature. For example, they are explored in [29, 49, 51]. However, these works only provide a semantic characterization of these notions. Both [49] and [51] refer to permissibly suboptimal actions as actions that are the bare minimum but good enough, whereas Mīmāṃsā refers to actions that are less than acceptable. To illustrate these concepts, consider a situation where delicate information must be communicated in person or via email. In Mīmāṃsā terms we would say that the duty to communicate a given information in person is mitigated by the permission to communicate it via email. However, providing the information by e-mail is still better than not providing any information at all, and this distinguishes suboptimal permitted actions from the Mīmāṃsā better-not permission, where the permission to do X means that not doing X is always better.

The paper [41] classifies normative concepts, drawing from Alexius Meinong's work [52], which divides modalities into four classes: meritorious, required, excusable, and inexcusable. In the case of actions deemed meritorious or required doing the action is superior to not doing it. If an action is excusable or inexcusable, not doing this action is always better. The 'excusable' modality aligns with the Mīmāṃsā better-not permission, but, so far, it lacks a sound and complete formalization. Unlike the Mīmāṃsā permission, the permissibly suboptimal [49, 51] and the excusable actions [29] are not exceptions to general prohibition or negative obligations. Furthermore, Mīmāṃsā permissions are always dyadic, and thus only hold in a specific context, whereas this is not a specific property considered in [49, 51] and [29].

Within the formalization of deontic logic, preferences have been well explored, e.g., [4, 17, 29, 35, 36, 44, 61]. However, notions of suboptimal permissions have been only characterized semantically.

To formalize Mīmāṃsā permission, incorporating ceteris-paribus preferences appears to be the most suitable approach. Ceteris-paribus isolates the behaviour of a single action while holding all other factors constant, enabling us to express preferences such as 'I prefer a round table over a squared table'. This preference does not mean favouring all round tables over all square ones, but rather prioritizing a round table over a square one under the condition that all other factors (size, height, colour, etc.) remain constant. The formalization of ceteris-paribus preferences within modal logic has been addressed in the work of [7]. Leveraging this framework, in the present paper we use this established machinery to formalize Mīmāṃsā permission. Notions of ceteris-paribus have been introduced within deontic logic, e.g. [21, 45]. Specifically, in [45], obligatory formulas are true in all 'best' worlds, while permitted formulas are true in at least one 'best' world. By contrast, [21] defines obligation as in SDL, and permission through ceteris-paribus preferences. An action A is permitted if all the 'best' worlds where A is true, are considered normatively fine, meaning that all obligations are adhered to. Both in [21] and [45], a permission is completely defined through preferences, whereas in the Mīmāṃsā approach a permission *implies* a preference.

Conceiving permissions as better-not permissions also offers a solution to seeming problems like the "Interrupted promise", discussed by Zylberman [66]. In his scenario, one commits to participating in a conference, but then their daughter has an accident and the previous duty is overruled by the duty to stand by the daughter during surgery. Zylberman notes that despite having permission to withdraw, there is still an obligation to apologize or make reparations to the conference organizers. This sentiment contradicts the standard account of permissions, which does not mandate such actions. For instance, if it is permitted to drive at 18, no 18-year-old is expected to apologise because they are, in fact, driving. By contrast, the "interrupted promise" problem is instantly solved if the permission [66] is referring to a Mīmāṃsā permission (better-not) and hence requires some expiation (e.g., offering an apology).

5 Formalizing Mīmāṃsā Permission

Following a bottom-up approach of extracting deontic principles from the Mīmāṃsā texts, we transform the identified properties of the permission operator into suitable Hilbert axioms. The axioms are added to the logic LM_P of [16] (Mīmāṃsā Logic with permission), which extends Kumārila's logic[10] LKu^+ [9] with permission-as-exception. We name the resulting logic $LM_P^{\preceq}$ (Mīmāṃsā Logic with permission and preferences). Here we present and justify its Hilbert axiomatization, introduce a neighbourhood semantics, and demonstrate soundness, completeness, and consistency.

5.1 Syntax

We start by defining the language $\mathcal{L}_{LM_P^{\preceq}}$. Recall that the language of Kumārila's logic extends that of the modal logic S5 with the modalities $\mathcal{O}(\phi/\psi)$ and $\mathcal{F}(\phi/\psi)$ for obligation and prohibition (read as "ϕ is obligatory/prohibited given ψ"). As explained in Section 2, all deontic operators in Mīmāṃsā are necessarily dyadic. The language $\mathcal{L}_{LM_P}$ of LM_P extends that of LKu^+ with the permission operator $\mathcal{P}(\phi/\psi)$, to be read as "ϕ is permitted given ψ". This operator is treated as a primitive modality, that is, $\mathcal{P}(\phi/\psi)$ is not defined as $\neg\mathcal{F}(\phi/\psi)$ or $\neg\mathcal{O}(\neg\phi/\psi)$.

To formalize the better-not permission, the language should include a notion of preference. To achieve this, we incorporate the ceteris-paribus preference modality of [7], which allows comparing two scenarios that agree on the truth of a given set of formulas Γ. We denote this modality as $[\preceq]^\Gamma\phi$, meaning that "ϕ is true in all worlds that are better and agree on the truth of all formulas in Γ". Its dual $\langle\preceq\rangle^\Gamma\phi = \neg[\preceq]^\Gamma\neg\phi$ reads as "ϕ is true in at least one better world that agrees on the truth of all formulas in Γ".

Remark 1 In the definition of the ceteris-paribus modality, $[\preceq]^\Gamma$, we only consider LM_P-formulas as nesting of preferences is not taken into account.

Hence, the language $\mathcal{L}_{LM_P^{\preceq}}$ is defined by:

$$\phi ::= p \in Atom \mid \neg\phi \mid (\phi \vee \phi) \mid [U]\phi \mid \mathcal{O}(\phi/\phi) \mid \mathcal{F}(\phi/\phi) \mid \mathcal{P}(\phi/\phi) \mid [\preceq]^\Gamma\phi$$

(where $Atom$ is the set of atomic propositions, and Γ a set of LM_P-formulas). We take the classical logic[11] connectives $\neg$ and $\vee$ as primitive, and define $\wedge, \rightarrow, \leftrightarrow$ in the usual way. The constants $\top$ and $\bot$ are abbreviations for $\neg\phi \vee \phi$ and $\neg\top$, respectively. $[U]$ is the universal S5 modality, read as 'in all scenarios, ϕ is true' and its dual $\langle U\rangle\phi = \neg[U]\neg\phi$ as 'there is at least one scenario where ϕ is true'. We write $[\preceq]\phi$ when $\Gamma = \emptyset$. To define

[10] The logic LKu^+ formalizes the deontic theories of two main Mīmāṃsā authors: Kumārila and Prabhākara (who probably both lived in the 7th c. CE). Their theories differ in the way elective duties are interpreted: as an obligation for Prabhākara, and as a recipe that guarantees to obtain a desired result, for Kumārila. The latter has been formalized in [9] with a modality $\mathcal{E}(\phi/\psi)$ having no deontic force. Since this modality does not interact with the deontic operators, we omit it from our logic for simplicity.

[11] The use of classical logic base is justified by the presence in Mīmāṃsā of *nyāya*s that express the reductio ad absurdum law, see [15].

Table 1 Axioms regarding obligation and prohibition from [9]

Ax1.	$(\boxed{u}(\phi \rightarrow \psi) \wedge \mathcal{O}(\phi/\theta) \wedge \neg\boxed{u}\psi) \rightarrow \mathcal{O}(\psi/\theta)$
Ax2.	$(\boxed{u}(\phi \rightarrow \psi) \wedge \mathcal{F}(\psi/\theta) \wedge \Diamond_u\phi) \rightarrow \mathcal{F}(\phi/\theta)$
Ax3.	$\neg(X(\phi/\theta) \wedge X(\neg\phi/\theta))$ for $X \in \{\mathcal{O}, \mathcal{F}\}$
Ax4.	$\neg(\mathcal{O}(\phi/\theta) \wedge \mathcal{F}(\phi/\theta))$
Ax5.	$(\boxed{u}(\psi \leftrightarrow \theta) \wedge X(\phi/\psi)) \rightarrow X(\phi/\theta)$ for $X \in \{\mathcal{O}, \mathcal{F}\}$
Ax6.	$(\Diamond_u(\phi \wedge \theta) \wedge \mathcal{O}(\phi/\top) \wedge \mathcal{O}(\theta/\top)) \rightarrow \mathcal{O}(\phi \wedge \theta/\top)$

the better-not permission, we use a specific instance of the ceteris-paribus modality $\boxed{\leq}^{\Gamma}$, comparing two scenarios that agree on *all* formulas of the language LM_P except for a single formula ψ. We denote this as $\boxed{\leq}^{\psi}\phi$, meaning that "ϕ holds in all better worlds that agree on the truth of all LM_P-formulas, except ψ".

The logic LKu^+ [9] consists of the axioms for obligation and prohibition in Table 1 as well as the axioms for the logic S5, and is closed under the rules of $\boxed{u}$-necessitation and modus ponens. The axioms for obligation and prohibition are based on the following principles extracted from suitable *nyāya*s:

1. If the accomplishment of an action presupposes the accomplishment of another action, the obligation to perform the first prescribes also the second. Conversely, if an action necessarily implies a prohibited action, this will also be prohibited. This corresponds to the *nyāya* given in Example 1, and is formalized by Ax1 and Ax2.
2. Two actions that exclude each other can neither be prescribed nor prohibited simultaneously to the same group of eligible people under the same conditions. This principle is the base for Ax3 and Ax4.
3. If two sets of conditions always identify the same group of eligible agents, then a command valid under the conditions in one of the sets is also enforceable under the conditions in the other set. This is formalized by Ax5.
4. If two fixed duties are prescribed and compatible, their conjunction is obligatory as well. This corresponds to Ax6.

Remark 2 Prima facie, Mīmāṃsā commands can have only one action as their argument. However, employing logical formulas and transitioning to an 'all-things-considered' stage, more than one action can be considered as the argument. The distinction between prima facie and derived norms is similar to the one made in [2], where the authors distinguish prescriptive permissions, which are explicit, and descriptive permissions, which are derived from a normative context. Our logic does not distinguish between prima facie norms and derived norms[12].

The logic LM_P from [16] extends LKu^+ with the axioms in Table 2. The principles that are extracted from applications of the *nyāya*s that led to the formalization of these axioms are discussed below.

(I) Permissions are always exceptions to more general prohibitions or negative obligations.

[12] A formal distinction might need ad hoc operators, along the line of [46].

This principle is extracted from the *nyāya* applied in (a)-(d) (see Section 3) and justifies two axioms. The first axiom represents the fact that a permission is always an exception to a general prohibition or negative obligation (cf. (a)-(c)), and is formalized as: $\mathcal{P}(\phi/\psi) \rightarrow (\mathcal{F}(\phi/\top) \vee \mathcal{O}(\neg\phi/\top))$. It follows from (d) that if something is allowed in one context and prohibited (or negatively obliged) in another, the context of the prohibition or negative obligation is more general. This is formalized by the axiom: $(\mathcal{P}(\phi/\psi) \wedge (\mathcal{F}(\phi/\theta) \vee \mathcal{O}(\neg\phi/\theta))) \rightarrow \boxdot(\psi \rightarrow \theta)$.

(II) No more than one deontic operator can be applied to the same action under the same circumstances.

In the domain of Mīmāṃsā deontics, this principle represents a foundational metarule (cf. the inner-consistency and the *apūrva*-metarules discussed in Section 2 and 3) and justifies the axioms stating that an action cannot be permitted and forbidden in the same context, $\neg(\mathcal{P}(\phi/\psi) \wedge \mathcal{F}(\phi/\psi))$, an action cannot be permitted and negatively obligatory in the same context, $\neg(\mathcal{P}(\phi/\psi) \wedge \mathcal{O}(\neg\phi/\psi))$, and an action cannot be permitted and obligatory in the same context, $\neg(\mathcal{P}(\phi/\psi) \wedge \mathcal{O}(\phi/\psi))$. The latter axiom is especially interesting since it contradicts the often-accepted inference in deontic logic that obligation implies permission (aka axiom D).

(III) Commands are about actions that are possible but not necessary, that is, actions that can be performed, but from which abstention is also possible.

This principle has been extracted from various contexts, summarized by the *nyāya*-applications (h), corresponding to 'ought implies can', and (i), corresponding to 'forbidden implies can' (cf. Section 3.3). Moreover, the axiom excludes obligations and prohibitions of tautologies and contradictions: it should be possible to follow as well as violate commands. The formalization of the principle is accomplished as follows: $(\mathcal{O}(\phi/\psi) \vee \mathcal{F}(\phi/\psi)) \rightarrow \diamondsuit(\phi \wedge \psi) \wedge \neg\boxdot\phi$, which we formalize as P3. This axiom results in the following formulas to be true: $\neg\mathcal{F}(\bot/\theta)$, $\neg\mathcal{O}(\bot/\theta)$, $\neg\mathcal{F}(\top/\theta)$, and $\neg\mathcal{O}(\top/\theta)$. Although we have not found an explicit statement that principle (III) applies to permissions, the fact that permitted actions are exceptions to prohibited or negatively obliged (possible) actions, is enough to conclude that this axiom should be present; as shown by Lemma 1.3 it is indeed derivable in LM_P.

Remark 3 The paper [16] uses a slightly different formulation of the axioms Ax1 and Ax2, w.r.t. [9], as their original version leads to contradictions in the presence of axiom P3. Ax1 was presented in [9] as $(\boxdot(\phi \rightarrow \psi) \wedge \mathcal{O}(\phi/\theta)) \rightarrow \mathcal{O}(\psi/\theta)$. Since $\boxdot(\phi \rightarrow \top)$ is true for any formula ϕ, we would derive $\mathcal{O}(\top/\theta)$ from $\mathcal{O}(\phi/\theta)$, for any ϕ and θ, from which $\diamondsuit\neg\top$ would follow. Ax2 was presented in [9] as $(\boxdot(\phi \rightarrow \psi) \wedge \mathcal{F}(\psi/\theta)) \rightarrow \mathcal{F}(\phi/\theta)$. The formula $\boxdot(\bot \rightarrow \psi)$ is true for any formula ψ, and therefore we derive $\mathcal{F}(\bot/\theta)$ from $\mathcal{F}(\psi/\theta)$ for any ψ and θ. Again, the formula $\mathcal{F}(\bot/\theta)$ would lead to a contradiction when applying P3, since it derives $\diamondsuit\bot$. The use of the versions of Ax1 and Ax2 in Table 1 permits to avoid these contradictions. At first glance, they still seem to derive undesired formulas. Regarding Ax1, the formula $\mathcal{O}(\bot/\theta)$ is derived whenever both $\boxdot(\phi \rightarrow \bot)$ and $\mathcal{O}(\phi/\theta)$ are true. However, since $\diamondsuit\phi$ follows directly from $\mathcal{O}(\phi/\theta)$ by axiom P3, $\boxdot(\phi \rightarrow \bot)$ and $\mathcal{O}(\phi/\theta)$ cannot both be true. Regarding Ax2, the formulas $\boxdot(\top \rightarrow \psi)$ and $\mathcal{F}(\psi/\theta)$ would derive the formula $\mathcal{F}(\top/\theta)$. However, as $\neg\Box_u\psi$ follows from $\mathcal{F}(\psi/\theta)$ by axiom P3, $\Box_u(\top \rightarrow \psi)$ and $\mathcal{F}(\psi/\theta)$ cannot both be true.

Lastly, LM_P contains the following substitution axioms: $(\Box_u(\psi \leftrightarrow \theta) \wedge \mathcal{P}(\phi/\psi)) \rightarrow \mathcal{P}(\phi/\theta)$ and $(\Box_u(\phi \leftrightarrow \psi) \wedge \mathcal{P}(\phi/\theta)) \rightarrow \mathcal{P}(\psi/\theta)$. They do not follow from any explicit discussions by Mīmāṃsā authors but are implicitly used in Dharmaśāstra discussions of permissions under extreme circumstances, as shown in the following example.

Example 9 Vijñāneśvara states that it is permitted to sell certain vegetables if one has assumed the occupation of the *vaiśya* class, and then refers to the permission to sell the same vegetables if one is working as a merchant, given that assuming the occupation of a *vaiśya* is equivalent to being a merchant (*Mitākṣarā* on Yājñavalkya 3.35). Furthermore, the permission to act as a *vaiśya* when being a *brāhmaṇa* in distress is equivalent to the permission to sell when being a *brāhmaṇa* in distress.

Remark 4 In contrast with obligation and prohibition, LM_P does not contain a monotonicity axiom for permission, i.e., $(\mathcal{P}(\phi/\theta) \wedge \Box_u(\phi \rightarrow \psi)) \rightarrow \mathcal{P}(\psi/\theta)$. The main reason is that we have not found it in Mīmāṃsā texts. It is also unlikely that we will find it since this axiom would lead to unwanted consequences. For instance, from "eating meat implies being alive" and $\mathcal{P}(eating\ meat/during\ extreme\ circumstances)$, would follow $\mathcal{P}(being\ alive/during\ extreme\ circumstances)$ which is not meaningful as we have no control over being alive. Additionally, as shown by the following derivation, the monotonicity of permissions would imply an unconditional prohibition or negative obligation for any other feasible action:

1. $\mathcal{P}(\phi/\theta) \rightarrow \mathcal{P}(\phi \vee \psi/\theta)$ (monotonicity for permissions)
2. $\mathcal{P}(\phi \vee \psi/\theta) \rightarrow (\mathcal{F}(\phi \vee \psi/\top) \vee \mathcal{O}(\neg(\phi \vee \psi)/\top))$ (P1)
3. $\Box_u(\psi \rightarrow (\phi \vee \psi)) \wedge \mathcal{F}(\phi \vee \psi/\top) \wedge \Diamond_u\psi \rightarrow \mathcal{F}(\psi/\top)$ (Ax2)
4. $\Box_u((\neg\phi \wedge \neg\psi) \rightarrow \neg\psi) \wedge \mathcal{O}(\neg(\phi \vee \psi)/\top) \wedge \Diamond_u\psi \rightarrow \mathcal{O}(\neg\psi/\top)$ (Ax1)
5. $\mathcal{P}(\phi/\theta) \wedge \Diamond_u\psi \rightarrow \mathcal{F}(\psi/\top) \vee \mathcal{O}(\neg\psi/\top)$ (from (1)-(4))

The logic $LM_P^{\preceq}$ extends the logic LM_P by including the formalization of the principle that led to the better-not permission:

(iv) Permissions are always better-not permissions.

Derived from the *nyāya* applications (e)-(g), it asserts that when a formula ϕ is permitted in the context ψ, $\neg\phi$ is preferred over ϕ (and ϕ is not preferred over $\neg\phi$).

To formalize it, we want to compare a scenario, or world, where $\neg\phi$ is true to one where ϕ is true. For this, we define a relation $Pref^{\theta}(\phi)$, that expresses that ϕ is preferred over its negation, $\neg\phi$, when the condition θ is true. However, not all scenarios warrant comparison. For instance, a scenario where someone refrains from drinking wine but commits a serious offense cannot be considered superior to a world where wine is consumed, but no harm is done. Therefore, for meaningful comparisons, we appeal to ceteris-paribus preferences [37]. This means that we compare two scenarios when they agree on the truth of a set of formulas. When comparing formulas $\neg\phi$ and ϕ when the condition θ is true, our objective is to ensure alignment on the truth of all formulas, except for ϕ (and all formulas equivalent to ϕ). We make this deliberate choice because our concern is not to compare any two arbitrary worlds; rather, we view it as a decision-making process: when a condition θ is true, a choice has to be made between doing ϕ or not, and therefore we only focus on the comparison between worlds where this is the only aspect in which they differ. This leads to the following definition.

Definition 1 The preference of a formula ϕ over its negation, when a formula θ is true, denoted by $Pref^{\theta}(\phi)$, is defined as follows:

$$Pref^{\theta}(\phi) := \boxed{\cup}((\theta \wedge \phi) \rightarrow \boxed{\preceq}^{\phi}(\theta \rightarrow \phi)) \wedge \Diamond_{\cup}(\theta \wedge \neg\phi \wedge \Diamond_{\preceq}^{\phi}(\theta \wedge \phi)).$$

This formula combines a $\forall\forall$ preference (in the first conjunct) and an $\exists\exists$ preference relation (in the second conjunct). The former says that if θ and ϕ are true in a world, then in all better worlds that agree on the truth of all LM_P-formulas except ϕ, ϕ remains true. The latter states that there is a world where θ and $\neg\phi$ are true and a better world, agreeing on the truth of all LM_P-formulas except the formula ϕ, where ϕ and θ are true. This ensures that $Pref^{\theta}(\phi) \rightarrow \neg Pref^{\theta}(\neg\phi)$ is true (see Lemma 1.4).

Drawing from [7], we take $\boxed{\preceq}^{\Gamma}$ to be an S4-modality, which includes the axioms for reflexivity and transitivity. This setup enables the formalization of property (iv), as axiom P6 (see Def. 2): $\mathcal{P}(\phi/\psi) \rightarrow Pref^{\psi}(\neg\phi)$ (if ϕ is permitted given ψ, then $\neg\phi$ is preferred over ϕ given ψ). Note that, permission is the only modality that implies a preference for a formula over its negation. Obligations and prohibitions lead to rewards and sanctions, respectively, which are unrelated to preferences.

Remark 5 From axioms P1, P2a, and P2c of Table 2, it follows that $\neg\mathcal{P}(\phi/\top)$ is a theorem, indicating that no action is permitted unconditionally. While unconditional permissions are, in general, valuable for expressing rights, Mīmāṃsā permissions are not rights. If they were, the "right to vote" would imply that it is better not to vote, which contradicts the fundamental intuition behind rights.

Lastly, the modality $\boxed{\preceq}^{\Gamma}$ requires the additional axioms P7, as well as P8-P12 from [7]. We added P7 to guarantee that if a formula holds in all worlds, it also holds in all better worlds that agree on all formulas in Γ, implying that the set of better worlds is a subset of the set of all worlds. P8 guarantees that if Γ is a subset of Γ', the set of worlds agreeing on Γ' is not larger than the set of worlds agreeing on Γ. Axiom P9 (resp. P10) ensures that if a formula is true (resp. false) in any world, then it remains true (resp. false) in all better worlds that agree on a set of formulas containing it. P11 and P12 serve to build the set of formulas Γ on which a world and a better world agree.

Definition 2 The logic $LM_P^{\preceq}$ extends the logic LM_P with the axioms and rules for the S4 modality for $\boxed{\preceq}^{\Gamma}$ and the following axioms:

P6. $\mathcal{P}(\phi/\psi) \rightarrow Pref^{\psi}(\neg\phi)$
P7. $\boxed{\cup}\phi \rightarrow \boxed{\preceq}^{\Gamma}\phi$
P8. $\boxed{\preceq}^{\Gamma}\phi \rightarrow \boxed{\preceq}^{\Gamma'}\phi$ for $\Gamma \subseteq \Gamma'$
P9. $\gamma \rightarrow \boxed{\preceq}^{\Gamma}\gamma$ for $\gamma \in \Gamma$

Table 2 Axioms regarding permission from [16]

P1.	$\mathcal{P}(\phi/\psi) \to (\mathcal{F}(\phi/\top) \vee \mathcal{O}(\neg\phi/\top)$
P2.	a) $\neg(\mathcal{P}(\phi/\psi) \wedge \mathcal{F}(\phi/\psi)$
	b) $\neg(\mathcal{P}(\phi/\psi) \wedge \mathcal{O}(\phi/\psi)$
	c) $\neg(\mathcal{P}(\phi/\psi) \wedge \mathcal{O}(\neg\phi/\psi)$
P3.	$(\mathcal{O}(\phi/\psi) \vee \mathcal{F}(\phi/\psi)) \to \Diamond_{u}(\phi \wedge \psi) \wedge \neg\Box_{u}\phi$
P4.	a) $(\Box_{u}(\psi \leftrightarrow \theta) \wedge \mathcal{P}(\phi/\psi)) \to \mathcal{P}(\phi/\theta)$
	b) $(\Box_{u}(\phi \leftrightarrow \psi) \wedge \mathcal{P}(\phi/\theta)) \to \mathcal{P}(\psi/\theta)$
P5.	$(\mathcal{P}(\phi/\psi) \wedge (\mathcal{F}(\phi/\theta) \vee \mathcal{O}(\neg\phi/\theta))) \to \Box_{u}(\psi \to \theta)$

P10. $\neg\gamma \to \Box_{\preceq}^{\Gamma}\neg\gamma$ for $\gamma \in \Gamma$
P11. $\gamma \wedge \Diamond_{\preceq}^{\Gamma}(\gamma \wedge \phi) \to \Diamond_{\preceq}^{\{\gamma\}\cup\Gamma}\phi$
P12. $\neg\gamma \wedge \Diamond_{\preceq}^{\Gamma}(\neg\gamma \wedge \phi) \to \Diamond_{\preceq}^{\{\gamma\}\cup\Gamma}\phi$

Remark 6 The notion of supererogation in Mīmāṃsā (see Section 3.2) can be simulated in $LM_P^{\preceq}$ without the need for an extra operator. We showcase this by formalizing the permission to have sex at specific times, mentioned in Example 7. From this permission, we can deduce an underlying obligation to refrain from having sex, and this interpretation is confirmed by the mention of a reward as well as by other passages, see e.g. [18]. Following a negative obligation always leads to a reward, and adhering to this negative obligation, particularly when there is a specific permission to act contrarily, constitutes a supererogatory act. Then, one can argue that $\mathcal{O}(\neg having\ sex/\top) \wedge \mathcal{P}(having\ sex/specific\ times)$[13]—from which follows $Pref^{\text{specific times}}(\neg\text{having sex})$—is a shorthand indicating that, at specific times, not having sex is a supererogatory act and leads to a reward.

The logic, $LM_P^{\preceq}$, enables us to derive consequences that align with various topics discussed by Mīmāṃsā authors. We present these consequences in Lemma 1 and describe their meaning afterward.

Lemma 1 *The following formulas are derivable in* $LM_P^{\preceq}$*:*

1. $\Box_{u}(\phi \to \psi) \wedge \neg\Box_{u}\psi \to \neg(\mathcal{O}(\phi/\theta) \wedge \mathcal{P}(\psi/\theta))$
2. $\Box_{u}(\phi \to \psi) \wedge \Diamond_{u}\phi \to \neg(\mathcal{F}(\psi/\theta) \wedge \mathcal{P}(\phi/\theta))$
3. $\mathcal{P}(\phi/\psi) \to \Diamond_{u}(\phi \wedge \psi) \wedge \neg\Box_{u}\phi$
4. $Pref^{\psi}(\phi) \to \neg Pref^{\psi}(\neg\phi)$
5. $\neg(\mathcal{P}(\phi/\theta) \wedge \mathcal{P}(\neg\phi/\theta))$
6. $Pref^{\psi}(\phi) \wedge \Box_{u}(\phi \leftrightarrow \theta) \to Pref^{\psi}(\theta)$

Proof Formula 1 follows by Ax1 and P2b, while Formula 2 by Ax2 and P2a. We derive both 3 and 4 from Def. 1. Formula 5 follows by P1, Ax3 and Ax4. Formula 6 follows from the general substitution of formulas and the definition of the modality $\Box_{\preceq}^{\phi}$. □

The first two formulas from Lemma 1 are generalizations of the D-axiom for permission (see Section 4). They occur in the following scenario: being a merchant implies selling karelas (among other vegetables), and it cannot be the case that it is obligatory to be a merchant and permitted to sell karelas under the same circumstances. Likewise,

[13] We reconstruct supererogation as an exception to a general negative obligation as opposed to a general prohibition as the performance of supererogatory actions leads to rewards.

it cannot be the case that it is prohibited to be a merchant and permitted to sell karelas. Formula 3, which will be utilized in the formalization of the free choice inference in Section 6, constitutes a variation of the 'commands entail possibility' principle for permissions. For something to be permitted, it cannot be impossible nor should it be necessary. Hence, kindling a fire in the sky cannot be permitted, as it is impossible, and desiring happiness while being alive cannot be permitted as it is automatically given for each living being. Formula 4 denotes a kind of strict preference: If a formula ϕ is preferred over its negation, in a context θ, then $\neg\phi$ is not preferred over its negation, ϕ, in a context θ. Indeed, as discussed in Example 7, if not drinking wine is preferred over drinking it, then drinking wine is not preferred over not drinking it. Although Formula 5 is not a property of permission in the English language, e.g. it is possible to permit both having coffee and not having coffee, in the context of Mīmāṃsā, permissions are treated as exceptions to general prohibitions or negative obligations and there cannot be a prohibition or negative obligation regarding both a particular action and its negation. This corresponds to Example 7 on how eating meat is permitted and not eating cannot be permitted (rather, it is encouraged). Formula 6 is a substitution formula for the defined preference relation and appears in the following example: If it is preferred to take up the occupation of a *vaiśya* over not doing so, then it is also preferred to work as a merchant, as being a *vaiśya* is equivalent to being a merchant. This formula will be used in the soundness proof.

5.2 Semantics

In line with [9] and [16], we use neighbourhood semantics to assign a meaning to formulas of $LM_P^{\preceq}$. The decision for neighbourhood semantics over the standard Kripke semantics is mainly driven by the metarule concerning novelty (*apūrva*), see Section 2, entailing the invalidity of trivial norms such as $\mathcal{O}(\top/\theta)$, $\mathcal{P}(\top/\theta)$ and $\mathcal{F}(\bot/\theta)$ for any θ. The (monadic variants) of these formulas are indeed true in all Kripke frames.

Neighbourhood semantics generalizes Kripke semantics. It consists of a set of worlds W and a valuation function V, and contains neighbourhood functions N_x that map a world to a set of ordered pairs of sets of worlds. Each of the three modalities, obligation, permission and prohibition, has its own neighbourhood function. For example, let $w \in W$, if (X, Y) is in w's obligation-neighbourhood, this means that X represents the worlds of compliance 'from the point of view' of Y. Then, if X is exactly the set of worlds where ϕ is true, and Y is exactly the set of worlds where ψ is true, then $\mathcal{O}(\phi/\psi)$ is true in w.

Definition 3 An $LM_P^{\preceq}$-frame $F = \langle W, N_{\mathcal{O}}, N_{\mathcal{P}}, N_{\mathcal{F}}, \leq\rangle$ is a tuple where $W \neq \emptyset$ is a set of worlds $w, v, u, \ldots$ and $N_\chi : W \to P(P(W) \times P(W))$ is a neighbourhood function for $\chi \in \{\mathcal{O}, \mathcal{P}, \mathcal{F}\}$, and $\leq \subseteq W \times W$ is a transitive, reflexive relation between worlds. Let $X, Y, Z \subseteq W$, then M satisfies the following conditions:

(i) If $(X, Z) \in N_{\mathcal{P}}(w)$ then $(X, W) \in N_{\mathcal{F}}(w)$ or $(\overline{X}, W) \in N_{\mathcal{O}}(w)$.
(ii) If $(X, Z) \in N_{\mathcal{P}}(w)$ then $(X, Z) \notin N_{\mathcal{F}}(w)$ and $(X, Z) \notin N_{\mathcal{O}}(w)$.
(iii) If $(X, Z) \in N_\chi(w)$ then $X \bigcap Z \neq \emptyset$ and $X \neq W$ for $(\chi \in \{\mathcal{O}, \mathcal{F}\})$.
(iv) If $(X, Y) \in N_{\mathcal{P}}(w)$ and $((X, Z) \in N_{\mathcal{F}}(w)$ or $(\overline{X}, Z) \in N_{\mathcal{O}}(w))$ then $Y \subset Z$.
(v) If $(X, Z) \in N_{\mathcal{P}}(w)$ then $(\overline{X}, Z) \notin N_{\mathcal{O}}(w)$.

(vi) If $(X, Z) \in \mathcal{N}_{\mathcal{O}}(w)$ and $X \subseteq Y$ and $Y \neq W$, then $(Y, Z) \in \mathcal{N}_{\mathcal{O}}(w)$.
(vii) If $(X, Z) \in \mathcal{N}_{\mathcal{F}}(w)$ and $Y \subseteq X$ and $Y \neq \emptyset$, then $(Y, Z) \in \mathcal{N}_{\mathcal{F}}(w)$.
(viii) If $(X, Y) \in \mathcal{N}_{\mathcal{X}}(w)$, then $(\overline{X}, Y) \notin \mathcal{N}_{\mathcal{X}}(w)$ for $\mathcal{X} \in \{\mathcal{O}, \mathcal{F}\}$.
(ix) If $(X, Z) \in \mathcal{N}_{\mathcal{O}}(w)$ then $(X, Z) \notin \mathcal{N}_{\mathcal{F}}(w)$.
(x) If $X \bigcap Y \neq \emptyset$ and $(X, W), (Y, W) \in \mathcal{N}_{\mathcal{O}}(w)$, then $(X \bigcap Y, W) \in \mathcal{N}_{\mathcal{O}}(w)$.

An $LM_P^{\preceq}$ model is a tuple $M = \langle F, V \rangle$ where F is an $LM_P^{\preceq}$−frame and V is a valuation function mapping atomic propositions to sets of worlds, $V : Atom \rightarrow \mathcal{P}(W)$.

(i) corresponds to axiom P1, (ii) and (v) to axioms P2a-c, (iii) to axiom P3[14] and (iv) to P5. Note that, when assuming $(X, W) \in N_{\mathcal{P}}(w)$ a contradiction arises due to conditions (i), (ii), and (v). Therefore, as a consequence of Theorems 2, 11 and 12 below, $(X, W) \notin N_{\mathcal{P}}(w)$. This aligns with the fact that there are no unconditional permissions according to Mīmāṃsā (Remark 5). Moreover, (vi) and (vii) correspond to axioms Ax1 and Ax2, expressing the property of monotonicity in the first argument of the deontic operators; these conditions are based on the ones in [9], adjusted to comply with the new version of the monotonicity axioms (see Remark 3). (viii) corresponds to Ax3, avoiding the accumulation of deontic operators, (ix) corresponds to Ax4, and (x) to Ax6. Axioms P4a, P4b, and Ax5 hold in any neighbourhood model, as shown by [14], and do not require explicit conditions. Axiom P6 lacks a neighbourhood condition. Ceteris-paribus preferences depend on the valuation of the atoms in a model, and therefore the condition corresponding to the better-not permission is added as a property of the model in Def. 4.

Following [7], to model the modality $\boxed{\preceq}^{\Gamma}\phi$, the semantics requires an equivalence relation denoted by $\equiv_{\Gamma}^{M} \subseteq W \times W$. $w \equiv_{\Gamma}^{M} v$ expresses that w and v agree on the truth of all formulas in Γ. Specifically, for the instance $\boxed{\preceq}^{\psi}\phi$, the corresponding equivalence relation is $\equiv_{\psi}^{M} \subseteq W \times W$ and expresses that w and v agree on the truth of all formulas in LM_P except for the formula ψ, and the formulas equivalent to ψ. Additionally, we denote by $f_{\not\equiv}^{M}(\psi)$ the set of all formulas not equivalent to a formula ψ in a model M, and by $\|\phi\|^{M}$ the set of all worlds where ϕ is true in M.

Definition 4 Let M be an $LM_P^{\preceq}$-model. We define the satisfaction relation of an $LM_P^{\preceq}$-formula ϕ at any $w \in W$ as follows:

$M, w \models p$	iff	$w \in V(p)$, for $p \in Atom$
$M, w \models \neg\phi$	iff	$M, w \not\models \phi$
$M, w \models \phi \vee \psi$	iff	$M, w \models \phi$ or $M, w \models \psi$
$M, w \models \boxed{\cup}\phi$	iff	for all $w_i \in W$ $M, w_i \models \phi$
$M, w \models \langle\cup\rangle\phi$	iff	there exists a $w_i \in W$ $M, w_i \models \phi$
$M, w \models \boxed{\preceq}^{\Gamma}\phi$	iff	$\forall w_i$ if $w \equiv_{\Gamma}^{M} w_i$ and $w \leq w_i$, then $M, w_i \models \phi$
$M, w \models \boxed{\preceq}^{\psi}\phi$	iff	$\forall w_i$ if $w \equiv_{\psi}^{M} w_i$ and $w \leq w_i$, then $M, w_i \models \phi$
$M, w \models \mathcal{X}(\phi/\psi)$	iff	$(\|\phi\|^{M}, \|\psi\|^{M}) \in \mathcal{N}_{\mathcal{X}}(w)$ for $\mathcal{X} \in \{\mathcal{O}, \mathcal{F}\}$
$M, w \models \mathcal{P}(\phi/\psi)$	iff	1. $(\|\phi\|^{M}, \|\psi\|^{M}) \in \mathcal{N}_{\mathcal{P}}(w)$; 2. $\forall w_i, w_j \in \|\psi\|^{M}$ if $w_i \in \|\neg\phi\|^{M}$, $w_i \leq w_j$ and $w_i \equiv_{\neg\phi}^{M} w_j$ then $w_j \in \|\neg\phi\|^{M}$; 3. there exists a $w_j \in \|\psi \wedge \phi\|^{M}$ and a $w_i \in \|\psi \wedge \neg\phi\|^{M}$ such that $w_j \leq w_i$ and $w_j \equiv_{\neg\phi}^{M} w_i$

[14] (iii) prevents the formulas $\mathcal{O}(\top/\phi)$, $\mathcal{F}(\top/\phi)$, $\mathcal{O}(\bot/\phi)$, $\mathcal{F}(\bot/\psi)$ from being true in any $LM_P^{\preceq}$-model.

Where $\|\phi\|^M = \{w \in W : M, w \vDash \phi\}$, $w \equiv^M_\Gamma v$ iff for all $\gamma \in \Gamma$ $M, w \vDash \gamma$ iff $M, v \vDash \gamma$, $w \equiv^M_\psi v$ iff $w \equiv^M_\Gamma v$ where $\Gamma := f^M_{\not\equiv}(\psi) = \{\chi : M \nvDash \chi \leftrightarrow \psi$ & χ is an LM_p-formula$\}$.

Using the strategy outlined in [10] and the corresponding definitions, we demonstrate that $LM^{\preceq}_P$, as it is given in Def. 2, is sound and complete for the given neighbourhood semantics.

A derivation in the Hilbert system for $LM^{\preceq}_P$ is defined as usual.

Definition 5 A formula ϕ is *valid* in $LM^{\preceq}_P$, if for all worlds w in all $LM^{\preceq}_P$-models M it is the case that $M, w \vDash \phi$. A formula ϕ is a *theorem* of $LM^{\preceq}_P$, if there is a derivation in the Hilbert system for $LM^{\preceq}_P$. A formula ϕ *holds* in a model M iff $M \vDash \phi$.

Theorem 2 (Soundness) *If a formula ϕ is a theorem of $LM^{\preceq}_P$, then ϕ is valid.*

Proof We show that the axioms of $LM^{\preceq}_P$ are true in all worlds of any $LM^{\preceq}_P$-model M. For each axiom, we assume that the antecedent holds in a world in a model and use the neighbourhood restrictions of Def. 3 and the truth conditions of Def. 4 to derive the intended consequent. Showing that modus ponens and the necessitation rules preserve validity is easy. We detail the case of axioms P1 and P6 – the main properties of Mīmāṃsā permission, Ax1 and Ax2 – the axioms that changed with respect to [9] (see Remark 3), P4b – one of the substitution axioms, for which we use Lemma 1.6, and P11 – one of the ceteris-paribus axioms. All other axioms are proven similarly.

P1) Consider the theorem $\mathcal{P}(\phi/\psi) \to (\mathcal{F}(\phi/\top) \vee \mathcal{O}(\neg\phi/\top))$. Assume a world w in M such that $M, w \vDash \mathcal{P}(\phi/\psi)$. Def. 4 gives us $(\|\phi\|^M, \|\psi\|^M) \in N_{\mathcal{P}}(w)$, while (i) of Def. 3 says that $(\|\phi\|^M, W) \in N_{\mathcal{F}}(w)$ or $(\|\neg\phi\|^M, W) \in N_{\mathcal{O}}(w)$. Since $W = \|\top\|^M$, it follows that $M, w \vDash \mathcal{F}(\phi/\top)$ or $M, w \vDash \mathcal{O}(\neg\phi/\top)$. Therefore, $M, w \vDash \mathcal{P}(\phi/\psi) \to (\mathcal{F}(\phi/\top) \vee \mathcal{O}(\neg\phi/\top))$.

P6) Consider the theorem $\mathcal{P}(\phi/\psi) \to \mathit{Pref}^{\psi}(\neg\phi)$. Assume a world w in a model M, such that $M, w \vDash \mathcal{P}(\phi/\psi)$. According to Def. 4 the following is true:

1. $(\|\phi\|^M, \|\psi\|^M) \in N_{\mathcal{P}}(w)$;
2. for all $u, v \in \|\psi\|^M$ if $u \in \|\neg\phi\|^M$, $u \leq v$ and $u \equiv^M_{\neg\phi} v$ then $v \in \|\neg\phi\|^M$;
3. there exists a $u \in \|\psi \wedge \phi\|^M$ and a $v \in \|\psi \wedge \neg\phi\|^M$ such that $u \leq v$ and $u \equiv^M_{\neg\phi} v$.

From 2, we obtain that $M, u \vDash \psi \wedge \neg\phi \to \Box_{\preceq}^{\neg\phi}(\psi \to \neg\phi)$ for all $u \in W$, and therefore we get that $M, w \vDash \Box_U((\psi \wedge \neg\phi) \to \Box_{\preceq}^{\neg\phi}(\psi \to \neg\phi))$. From 3, we obtain that $M, u \vDash \psi \wedge \phi \wedge \Diamond_{\preceq}^{\neg\phi}(\psi \wedge \neg\phi)$ for some $u \in W$, and therefore $M, w \vDash \Diamond_U(\psi \wedge \phi \wedge \Diamond_{\preceq}^{\neg\phi}(\psi \wedge \neg\phi))$. We conclude that $M, w \vDash \mathit{Pref}^{\psi}(\neg\phi)$.

Ax1) Consider the theorem $(\Box_U(\phi \to \psi) \wedge \mathcal{O}(\phi/\theta) \wedge \neg\Box_U\psi) \to \mathcal{O}(\psi/\theta)$. Assume a world w in a model M, such that $M, w \vDash \Box_U(\phi \to \psi) \wedge \mathcal{O}(\phi/\theta) \wedge \neg\Box_U\psi$. According to Def. 4, this entails that $\|\phi\|^M \subseteq \|\psi\|^M$, $(\|\phi\|^M, \|\theta\|^M) \in N_{\mathcal{O}}(w)$ and $\|\psi\|^M \neq W$. Then, from (vi) in Def. 3, we derive $(\|\psi\|^M, \|\theta\|^M) \in N_{\mathcal{O}}(w)$. Thus, we conclude that $M, w \vDash \mathcal{O}(\psi/\theta)$.

Ax2) Consider the theorem $(\Box_U(\phi \to \psi) \wedge \mathcal{F}(\psi/\theta) \wedge \Diamond_U\phi) \to \mathcal{F}(\phi/\theta)$. Consider a world w in a model M, such that $M, w \vDash \Box_U(\phi \to \psi) \wedge \mathcal{F}(\psi/\theta) \wedge \Diamond_U\phi$. Then, from Def. 4, $\|\phi\|^M \subseteq \|\psi\|^M$, $\|\phi\|^M \neq \emptyset$ and $(\|\psi\|^M, \|\theta\|^M) \in N_{\mathcal{F}}(w)$. From (vii) in Def. 3, we obtain that $(\|\phi\|^M, \|\theta\|^M) \in N_{\mathcal{F}}(w)$. It follows that $M, w \vDash \mathcal{F}(\phi/\theta)$.

P4b) Consider the theorem $(\boxed{\cup}(\phi \leftrightarrow \psi) \wedge \mathcal{P}(\phi/\theta)) \rightarrow \mathcal{P}(\psi/\theta)$. Assume a world w in a model M such that $M, w \vDash \boxed{\cup}(\phi \leftrightarrow \psi) \wedge \mathcal{P}(\phi/\theta)$. To show that $M, w \vDash \mathcal{P}(\psi/\theta)$, we need the following to be true, according to Def. 4:

1. $(\|\psi\|^M, \|\theta\|^M) \in N_{\mathcal{P}}(w)$;
2. for all $u, v \in \|\theta\|^M$ if $u \in \|\neg\psi\|^M$, $u \leq v$ and $u \equiv^M_{\neg\psi} v$ then $v \in \|\neg\psi\|^M$
3. there exists a $u \in \|\theta \wedge \psi\|^M$ and a $v \in \|\theta \wedge \neg\psi\|^M$ such that $u \leq v$ and $u \equiv^M_{\neg\psi} v$.

1. follows directly from $\|\phi\|^M = \|\psi\|^M$ and $(\|\phi\|^M, \|\theta\|^M) \in N_{\mathcal{P}}(w)$. Then 2. and 3. follow from Lemma 1.6: since $M, w \vDash \boxed{\cup}(\phi \leftrightarrow \psi)$ it follows that $M, w \vDash Pref^{\theta}(\neg\psi)$. This gives us both 2. and 3.

P11) Consider the theorem $\gamma \wedge \Diamond_{\preceq}^{\Gamma}(\gamma \wedge \phi) \rightarrow \Diamond_{\preceq}^{\{\gamma\}\cup\Gamma}\phi$. Assume a world w in a model M such that $M, w \vDash \gamma$ and $M, w \vDash \Diamond_{\preceq}^{\Gamma}(\gamma \wedge \phi)$. There is a world v such that $w \equiv_{\Gamma} v$, and $M, v \vDash \gamma$, from which we can conclude that $w \equiv_{\Gamma\cup\{\gamma\}} v$. Thus, since $M, v \vDash \phi$ and $w \leq v$, we obtain that $M, w \vDash \Diamond_{\preceq}^{\Gamma\cup\{\gamma\}}\phi$. □

5.3 Completeness

In this section, we prove completeness and consistency of $LM_P^{\preceq}$.

We start by illustrating the roadmap of the completeness proof, which uses the method of canonical models. We first define the canonical model $M^c = \langle W^c, N^c_{\mathcal{O}}, N^c_{\mathcal{P}}, N^c_{\mathcal{F}}, R^c_{\boxed{\cup}}, \leq^c, \equiv^c_{\Gamma}, \preceq^c_{\Gamma}, V^c\rangle$, which is a model that satisfies *every* consistent formula, in such a way that for each formula ϕ and world w, $M^c, w \vDash \phi$ if and only if $\phi \in w$. The Truth Lemma (Lemma 7) proves the property that $M^c, w \vDash \phi$ if and only if $\phi \in w$ and implies that the set of formulas that hold true in all worlds of M^c are precisely the theorems of $LM_P^{\preceq}$. We mostly follow the strategy outlined in [10], with two key differences. First, as shown in [7], we need $\preceq^c_{\Gamma}$ to be the intended relation. This requires that $w \preceq^c_{\Gamma} v$ if and only if $w \equiv^c_{\Gamma} v$ and $w \leq^c v$ (Lemma 5)[15]. Second, M^c may not be a $LM_P^{\preceq}$-model. The universal modality $\boxed{\cup}$ is an S5 modality, which is well known to be canonical for the equivalence relation, i.e. $R^c_{\boxed{\cup}} \subseteq W^c \times W^c$. For the universal modality, the required property is $R^c_{\boxed{\cup}} = W^c \times W^c$. Therefore, we use world $w \in W^c$ to generate a submodel $M^* = \langle W^*, N^*_{\mathcal{O}}, N^*_{\mathcal{P}}, N^*_{\mathcal{F}}, R^*_{\boxed{\cup}}, \leq^*, \equiv^*_{\Gamma}, \preceq^*_{\Gamma}, V^*\rangle$ of the canonical model M^c, such that $R^*_{\boxed{\cup}} = W^* \times W^*$. We then show that M^* is a $LM_P^{\preceq}$-model (Lemma 8), and use it to prove the Truth Lemma for M^* (Lemma 10). Lastly, completeness is proven by showing that if ϕ is not a theorem, then there is a model M and world w such that $M, w \nvDash \phi$, and M^* is that model.

We start by defining the canonical model M^c:

Definition 6 (Canonical model) $M^c = \langle W^c, N^c_{\mathcal{O}}, N^c_{\mathcal{P}}, N^c_{\mathcal{F}}, R^c_{\boxed{\cup}}, \leq^c, \equiv^c_{\Gamma}, \preceq^c_{\Gamma}, V^c\rangle$ for $LM_P^{\preceq}$, where:

- W^c be the set of all $LM_P^{\preceq}$-maximally consistent sets of formulas;
- $(Y, Z) \in N^c_{\mathcal{O}}(w)$ iff $Y \neq W^c$ and there is a formula $\mathcal{O}(\phi/\psi) \in w$ such that $\{z_j \in W^c : \phi \in z_j\} \subseteq Y$ and $\{z_j \in W^c : \psi \in z_j\} = Z$;

[15] The use the ceteris-paribus modality from [7], allows us to leverage keys aspects of their canonical model construction (Def. 6) and corresponding proofs (Lemmas 4 and 5).

- $(Y, Z) \in N^c_{\mathcal{P}}(w)$ iff there is a formula $\mathcal{P}(\phi/\psi) \in w$ such that $Y = \{z_j \in W^c : \phi \in z_j\}$ and $\{z_j \in W^c : \psi \in z_j\} = Z$;
- $(Y, Z) \in N^c_{\mathcal{F}}(w)$ iff $Y \neq \emptyset$ and there is a formula $\mathcal{F}(\phi/\psi) \in w$ such that $Y \subseteq \{z_j \in W^c : \phi \in z_j\}$ and $\{z_j \in W^c : \psi \in z_j\} = Z$;
- $(w, v) \in R^c_{\boxed{U}}$ iff for all $\phi \in v$, $\Diamond_{U}\phi \in w$;
- $(w, v) \in \leq^c$ iff $(w, v) \in R^c_{\boxed{U}}$ and for all $\phi \in v$, $\Diamond_{\leq}\phi$;
- $(w, v) \in \equiv^c_\Gamma$ iff $(w, v) \in R^c_{\boxed{U}}$ and $\forall\gamma \in \Gamma$ $\gamma \in w$ iff $\gamma \in v$, for any set of LM_P-formulas Γ;
- $(w, v) \in \leq^c_\Gamma$ iff $(w, v) \in R^c_{\boxed{U}}$ and for all $\phi \in v$, $\Diamond_{\leq}^{\Gamma}\phi \in w$, for any set of LM_P-formulas Γ;
- $w \in V^c(p)$ iff $p \in w$.

We will use the following shorthand throughout the proof $\|\phi\|^c = \{z_j \in W^c : \phi \in z_j\}$. We write $w \equiv^c_\psi v$ iff $w \equiv^c_\Gamma v$ for $\Gamma := f^c_{\not\equiv}(\psi) = \{\phi : \phi \leftrightarrow \psi \notin w$ for some $w \in W^c\}$.

Note that the relations $\leq^c_\Gamma$ and $\leq^c$ are defined slightly differently from the usual canonical Kripke relations, as they include the additional requirement that $(w, v) \in R^c_{\boxed{U}}$. This condition ensures that the relation holds only between worlds within the same equivalence class. We impose the same requirement on $\equiv^c_\Gamma$, as well. This implies that in the proof of Lemma 3, when we need to prove $(w, v) \in \leq^c_\Gamma$ or $(w, v) \in \leq^c$, we need to show that $(w, v) \in R^c_{\boxed{U}}$.

We demonstrate that the canonical relations—$R^c_{\boxed{U}}$, $\leq^c_\Gamma$, and $\leq^c$—adequately represent the desired relations among worlds. For example, for the relation $R^c_{\boxed{U}}$, this means that if $\boxed{U}\phi$ holds in a world w and $(w, v) \in R^c_{\boxed{U}}$, it follows that ϕ is true in v (Lemma 3). Additionally, we show that if $\Diamond_{U}\phi$ is true in w, there exists a world v such that ϕ is true in v (Lemma 4).

Lemma 3 *The following is true:*

1. $(w, v) \in R^c_{\boxed{U}}$ *iff for all* ψ, $\boxed{U}\psi \in w$ *implies* $\psi \in v$;
2. $(w, v) \in \leq^c_\Gamma$ *iff for all* ψ, $\boxed{\leq}^{\Gamma}\psi \in w$ *implies* $\psi \in v$;
3. $(w, v) \in \leq^c$ *iff for all* ψ, $\boxed{\leq}\psi \in w$ *implies* $\psi \in v$.

Proof We only show claim 2, as the others are similar. For the left-to-right direction, assume $(w, v) \in \leq^c_\Gamma$ and assume $\psi \notin v$. From $\psi \notin v$, follows $\neg\psi \in v$, and thus $\Diamond_{\leq}^{\Gamma}\neg\psi \in w$ (from Def. 6). Then $\neg\boxed{\leq}^{\Gamma}\psi \in w$ and therefore $\boxed{\leq}^{\Gamma}\psi \notin w$.

For the right-to-left direction, we assume that $\boxed{\leq}^{\Gamma}\psi \in w$ implies $\psi \in v$ and need to prove: (i) $wR^c_{\boxed{U}}v$ and (ii) $\phi \in v$ implies $\Diamond_{\leq}^{\Gamma}\phi \in w$. For (i), assume $\Diamond_{U}\phi \notin w$. This means $\neg\Diamond_{U}\phi \in w$ and thus $\boxed{U}\neg\phi \in w$. From axiom P7, we derive that $\boxed{\leq}^{\Gamma}\neg\phi \in w$, and from our assumption we obtain $\neg\phi \in v$ and thus $\phi \notin v$. For (ii), assume $\Diamond_{\leq}^{\Gamma}\phi \notin w$. Then $\neg\Diamond_{\leq}^{\Gamma}\phi \in w$ and thus $\boxed{\leq}^{\Gamma}\neg\phi \in w$. From our assumption, it follows that $\neg\phi \in v$, and $\phi \notin v$. □

Lemma 4 (Existence Lemma)

1. *For any state* $w \in W^c$, *if* $\Diamond_{U}\phi \in w$ *then there is a state* $v \in W^c$ *such that* $(w, v) \in R^c_{\boxed{U}}$ *and* $\phi \in v$.

2. *For any state* $w \in W^c$*, if* $\diamondsuit_{\leq}^{\Gamma}\phi \in w$ *then there is a state* $v \in W^c$ *such that* $(w, v) \in \leq_{\Gamma}^{c}$ *and* $\phi \in v$.
3. *For any state* $w \in W^c$*, if* $\diamondsuit_{\leq}\phi \in w$ *then there is a state* $v \in W^c$ *such that* $(w, v) \in \leq^c$ *and* $\phi \in v$.

Proof Claim 1 is proven using a standard method which can be found, for example, in [10]. Claims 2 and 3 have been proven in [7], and we provide a proof sketch here. For Claim 2, assume $\diamondsuit_{\leq}^{\Gamma}\phi \in w$, and define the set of formulas $v^- = \{\phi\} \cup \{\theta : \boxdot_{\leq}^{\Gamma}\theta \in w\} \cup \{\gamma : \gamma \in \Gamma, \gamma \in w\} \cup \{\neg\gamma : \gamma \in \Gamma, \neg\gamma \in w\}$. To show that v^- is consistent, we assume it is not, and use $\boxdot_{\leq}^{\Gamma}$-necessitation and the K-axiom to derive a contradiction. Then, we extend v^- to a maximally consistent set v by applying Lindenbaum's lemma, ensuring that v contains ϕ and, by construction, $(w, v) \in \leq_{\Gamma}^{c}$. The strategy for Claim 3 follows similarly. □

Next, the relation $\leq_{\Gamma}^{c}$ should be the intended one. If $w \leq_{\Gamma}^{c} v$ is true, then v is preferred over w, and w and v agree on all formulas in Γ (Lemma 5). For the sake of completeness, we reproduce the proof below as presented in [7].

Lemma 5 $w \leq_{\Gamma}^{c} v$ *iff* $w \leq^c v$ *and* $w \equiv_{\Gamma}^{c} v$.

Proof Assume $w \leq_{\Gamma}^{c} v$. Take $\psi \in v$, then it follows that $\diamondsuit_{\leq}^{\Gamma}\psi \in w$, and therefore, by axiom P8, $\diamondsuit_{\leq}\psi \in w$. To show that $w \equiv_{\Gamma}^{c} v$, take $\gamma \in \Gamma$ such that $\gamma \in w$. Then, from axiom P9 follows that $\boxdot_{\leq}^{\Gamma}\gamma \in w$. By $w \leq_{\Gamma}^{c} v$, and Lemma 3, it follows that $\gamma \in v$. Assume $\gamma \in \Gamma$ such that $\gamma \notin w$. Then, $\neg\gamma \in w$ and from axiom P10 follows that $\boxdot_{\leq}^{\Gamma}\neg\gamma \in w$. Thus, by $w \leq_{\Gamma}^{c} v$, and Lemma 3, it follows that $\neg\gamma \in v$. So $\gamma \in w$ iff $\gamma \in v$.

For the other direction assume $w \leq^c v$ and $w \equiv_{\Gamma}^{c} v$, and consider $\phi \in v$. For each $\gamma \in \Gamma$, either $\gamma \in w$ or $\neg\gamma \in w$. First, assume $\gamma \in w$. From this follows that $\gamma \in v$. Then we have that $\phi \wedge \gamma \in v$ and thus $\diamondsuit_{\leq}(\phi \wedge \gamma) \in w$ and $\gamma \wedge \diamondsuit_{\leq}(\phi \wedge \gamma) \in w$. From axiom P11, it follows that $\diamondsuit_{\leq}^{\{\gamma\}}\phi \in w$. If $\neg\gamma \in w$ then we can then apply axiom P12, to get that $\diamondsuit_{\leq}^{\{\gamma\}}\phi$ from $\neg\gamma \wedge \diamondsuit_{\leq}(\phi \wedge \neg\gamma) \in w$. We can do this iteratively for all $\gamma \in \Gamma$ until we obtain $\diamondsuit_{\leq}^{\Gamma}\phi \in w$, from which follows that $w \leq_{\Gamma}^{c} v$. □

Additionally, we have to show that the neighborhood functions–$N_{\mathcal{O}}^{c}$, $N_{\mathcal{P}}^{c}$, and $N_{\mathcal{F}}^{c}$–are the intended ones, and satisfy the restrictions defined in Def. 3.

Lemma 6 *The canonical model* M^c *satisfies the restrictions of Def. 3.*

Proof We highlight only cases (i) and (iv), as all other cases follow the same strategy.

(i) If $(X, Z) \in N_{\mathcal{P}}^{c}(w)$ then $(X, W) \in N_{\mathcal{F}}^{c}(w)$ or $(\overline{X}, W) \in N_{\mathcal{O}}^{c}(w)$. Assume $(X, Z) \in N_{\mathcal{P}}^{c}(w)$. Then there is a formula $\mathcal{P}(\phi/\psi) \in w$ such that $X = \|\phi\|^c$ and $Z = \|\psi\|^c$. Since w is consistent and complete, we have that $\mathcal{O}(\neg\phi/\top) \in w$ or $\mathcal{F}(\phi/\top) \in w$. Assume $\mathcal{F}(\phi/\top) \in w$. Then for all $Y \subseteq \|\phi\|^c$ and some $Z = W^c$, we have that $(Y, Z) \in N_{\mathcal{F}}^{c}(w)$, thus also $(X, W^c) \in N_{\mathcal{F}}(w)$.

(iv) If $(X, Y) \in N_{\mathcal{P}}^{c}(w)$ and $((X, Z) \in N_{\mathcal{F}}^{c}(w)$ or $(\overline{X}, Z) \in N_{\mathcal{O}}^{c}(w))$ then $Y \subset Z$. Wlog assume $(X, Y) \in N_{\mathcal{P}}^{c}(w)$ and $(X, Z) \in N_{\mathcal{F}}^{c}(w)$. Then $\mathcal{P}(\phi/\psi) \in w$ and $\mathcal{F}(\phi/\theta)$ such that $\|\phi\|^c = X$ and $\|\psi\|^c = Y$ and $\|\theta\|^c = Z$. From axiom P5 follows that $\boxdot_{\cup}(\psi \to \theta) \in w$, and thus for all u such that $wR_{\boxdot_{\cup}}^{c}v$ $\psi \to \theta \in v$, and thus

$\neg\psi \in v$ or $\theta \in v$, which implies that $\|\psi\|^c \subseteq \|\theta\|^c$. We have that $\|\psi\|^c \neq \|\theta\|^c$, since this implies that $\mathcal{P}(\phi/\psi) \in w$ and $\mathcal{F}(\phi/\psi) \in w$, contradicting axiom P2a. Therefore $\|\psi\|^c \subset \|\theta\|^c$, and thus $Y \subset Z$. □

Now we can show that the Truth Lemma holds for the model M^c.

Lemma 7 (Truth Lemma for M^c) $M^c, w \vDash \phi$ *iff* $\phi \in w$.

Proof We showcase the cases of $\mathcal{P}(\phi/\psi)$ and $\Diamond_{\preceq}^{\Gamma}\phi$, as the others are straightforward.

If $M^c, w \vDash \mathcal{P}(\phi/\psi)$, then $(\|\phi\|^c, \|\psi\|^c) \in N^c_{\mathcal{P}}(w)$. By the canonical model, there is a formula $\mathcal{P}(\theta_1/\theta_2) \in w$ such that $\|\phi\|^c = \|\theta_1\|^c$ and $\|\psi\|^c = \|\theta_2\|^c$. By axioms P4ab, we have that $\mathcal{P}(\phi/\psi) \in w$. For the other direction, assume $\mathcal{P}(\phi/\psi) \in w$. We need to show that $M^c, w \vDash \mathcal{P}(\phi/\psi)$ and thus that:

(1) $(\|\phi\|^c, \|\psi\|^c) \in N^c_{\mathcal{P}}(w)$;
(2) for all $u, v \in \|\psi\|^c$, if $u \in \|\neg\phi\|^c$ and $u \preceq^c_{\neg\phi} v$ then $v \in \|\neg\phi\|^c$;
(3) there exists a $u \in \|\phi \wedge \psi\|^c$ and a $v \in \|\neg\phi \wedge \psi\|^c$ such that $u \preceq^c_{\neg\phi} v$.

Since $\mathcal{P}(\phi/\psi) \in w$, it follows from the definition of the canonical model that $(X, Y) \in N_{\mathcal{P}}(w)$ for $X = \|\phi\|^c$ and $Y = \|\psi\|^c$, and thus (1) is true. We use the induction hypothesis to prove (2) and (3). According to axiom P6, $Pref^{\psi}(\neg\phi) \in w$, and thus $\Box_{\mathsf{U}}((\psi \wedge \neg\phi) \rightarrow \Box_{\preceq}^{\neg\phi}(\psi \rightarrow \neg\phi)) \in w$. It follows that for any world u such that $wR^c_{\boxed{\mathsf{U}}}u$ where $\psi \in u$ and $\neg\phi \in u$, and for all v such that $u \preceq^c_{\neg\phi} v$, $\neg\phi \in v$. By the induction hypothesis, we derive that for all $u \in \|\psi\|^c$ and $u \in \|\neg\phi\|^c$, and for all v such that $u \preceq^c_{\neg\phi} v$ that $v \in \|\neg\phi\|^c$, and (2) is proven. Furthermore, from $Pref^{\psi}(\neg\phi) \in w$, it follows that $\Diamond_{\mathsf{U}}(\psi \wedge \phi \wedge \Diamond_{\preceq}^{\neg\phi}(\psi \wedge \neg\phi)) \in w$. Thus, there is a u $wR^c_{\boxed{\mathsf{U}}}u$ such that $\psi \in u$, $\phi \in u$ and there is a v where $\psi \in v$ and $\neg\phi \in v$ such that $u \preceq^c_{\neg\phi} v$. By IH, there exists $u, v \in \|\psi\|^c$ such that $u \in \|\phi\|^c$ and $v \in \|\neg\phi\|^c$, and $u \preceq^c_{\neg\phi} v$. This proves (3), and we can conclude that $M^c, w \vDash \mathcal{P}(\phi/\psi)$ iff $\mathcal{P}(\phi/\psi) \in w$.

If $M^c, w \vDash \Diamond_{\preceq}^{\Gamma}\phi$ then there is a world v such that $w \equiv^c_{\Gamma} v$ and $w \preceq^c v$ $M^c, v \vDash \phi$ then (Lemma 5) there is a world v such that $w \preceq^c_{\Gamma} v$ $M^c, v \vDash \phi$, then (induction hypothesis) there is a world v such that $w \preceq^c_{\Gamma} v$ $\phi \in v$, then $\Diamond_{\preceq}^{\Gamma}\phi \in w$ (canonical model). If $\Diamond_{\preceq}^{\Gamma}\phi \in w$, then (Lemma 4) there is a world v such that $w \preceq^c_{\Gamma} v$ and $\phi \in v$, then there is a world v such that $w \preceq^c v$ and $w \equiv^c_{\Gamma} v$ (Lemma 5) and $M^c, v \vDash \phi$ (induction hypothesis), and then $M^c, v \vDash \Diamond_{\preceq}^{\Gamma}\phi$. □

However, as M^c is not necessarily a $LM^{\preceq}_{P}$-model, we define the submodel M^* of M^c, generated by a world $z \in W^c$, $M^* = \langle W^*, N^*_{\mathcal{O}}, N^*_{\mathcal{P}}, N^*_{\mathcal{F}}, R^*_{\boxed{\mathsf{U}}}, \equiv^*_{\Gamma}, \preceq^*, \preceq^*_{\Gamma}, V^* \rangle$, where: $W^* = \{v \in W^c : \text{for all } \Box_{\mathsf{U}}\phi \in z, \phi \in v\}$; $N^*_{\mathcal{O}}(v) = N^c_{\mathcal{O}}(v) \cap W^*$; $N^*_{\mathcal{P}}(v) = N^c_{\mathcal{P}}(v) \cap W^*$; $N^*_{\mathcal{F}}(v) = N^c_{\mathcal{F}}(v) \cap W^*$; $R^*_{\boxed{\mathsf{U}}}(v) = R^c_{\boxed{\mathsf{U}}}(v) \cap W^*$; $\equiv^*_{\Gamma}(v) = \equiv^c_{\Gamma}(v) \cap W^*$; $\preceq^*(v) = \preceq^c(v) \cap W^*$; $\preceq^*_{\Gamma}(v) = \preceq^c_{\Gamma}(v) \cap W^*$; and $V^*(p) = V^c(p) \cap W^*$.

Lemma 8 *Let M^* be a submodel of M^c generated by the world $z \in W^c$. M^* is a $LM^{\preceq}_{P}$-model.*

Proof First, we show that $R^*_{\boxed{\mathsf{U}}} = W^* \times W^*$. Assume $(u, v) \in R^*_{\boxed{\mathsf{U}}}$ then $(u, v) \in R^c_{\boxed{\mathsf{U}}} \cap W^* \times W^*$, and thus $(u, v) \in W^* \times W^*$. For the other direction, assume that $(u, v) \in W^* \times W^*$. Since $R^*_{\boxed{\mathsf{U}}} = R^c_{\boxed{\mathsf{U}}} \cap W^* \times W^*$, all is left, is to show that: $(u, v) \in R^c_{\boxed{\mathsf{U}}}$, and thus $\phi \in v$ implies $\langle u \rangle\phi \in u$. Assume $\phi \in v$. Then, $\langle u \rangle\phi \in w$, then $[u]\langle u \rangle\phi \in w$. Therefore $\langle u \rangle\phi \in u$, since $u \in W^*$.

From this, it directly follows that $\leq^c_\Gamma (w) = \leq^*_\Gamma (w)$, and thus $\leq^* (w) = \leq^c (w)$, $\equiv^*_\Gamma (w) = \equiv^c_\Gamma (w)$, and $N^*_\chi(w) = N^c_\chi(w)$ for $\chi \in \{\mathcal{O}, \mathcal{P}, \mathcal{F}\}$. This follows from Lemma 5 as well as the fact that $\leq^*_\Gamma$, $\leq^*$ and $\equiv^*_\Gamma$ are transitive and reflexive. Now is left to show that M^* satisfies the restrictions of Def. 3. This follows from Lemma 6. We showcase the case of (i). (i) If $(X, Y) \in N^*_{\mathcal{P}}(w)$ then $(X, W^*) \in N^*_{\mathcal{F}}(w)$ or $(\overline{X}, W^*) \in N^*_{\mathcal{O}}(w)$. To see why, consider $(X, Y) \in N^*_{\mathcal{P}}(w)$ for some $w \in W^*$. Then, by definition of the submodel M^*, it follows that $X = X' \cap W^*$ and $Y = Y' \cap W^*$ for some $(X', Y') \in N^c_{\mathcal{P}}(w)$. Since M^c is an $LM^{\preceq}_P$-model, we know that $(X', W^c) \in N^c_{\mathcal{F}}(w)$ or $(\overline{X'}, W^c) \in N^c_{\mathcal{O}}(w)$. Thus, $(X' \cap W^*, W^c \cap W^*) = (X, W^*) \in N^*_{\mathcal{F}}(w)$ or $(\overline{X'} \cap W^*, W^c \cap W^*) = (\overline{X}, W^*) \in N^*_{\mathcal{O}}(w)$. Thus, M^* is a $LM^{\preceq}_P$-model. □

Theorem 9 *For all $w \in W^*$, $M^*, w \vDash \phi$ iff $M^c, w \vDash \phi$.*

Proof By induction. In the proof of Lemma 8, we established that, for $w \in W^*$, $\leq^c_\Gamma (w) = \leq^*_\Gamma (w)$, and thus $\leq^* (w) = \leq^c (w)$, $\equiv^*_\Gamma (w) = \equiv^c_\Gamma (w)$, and $N^*_\chi(w) = N^c_\chi(w)$ for $\chi \in \{\mathcal{O}, \mathcal{P}, \mathcal{F}\}$. We use these facts in this proof.

We show the cases of $\langle\leq\rangle^\Gamma \phi$ and $\mathcal{P}(\phi/\psi)$. The left-to-right direction is trivially true, therefore we only show the right-to-left direction. $M^c, w \vDash \langle\leq\rangle^\Gamma \phi$ implies there exists a world $w \leq^c_\Gamma v$ $M^c, v \vDash \phi$. By IH, we obtain that there exists a world $w \leq^c_\Gamma v$ $M^*, v \vDash \phi$ (IH). Since $\leq^c_\Gamma (w) = \leq^*_\Gamma (w)$ for $w \in W^*$, we get that there exists a world $w \leq^*_\Gamma v$ $M^*, v \vDash \phi$, and thus $M^*, w \vDash \langle\leq\rangle^\Gamma \phi$.

Assume $M^c, w \vDash \mathcal{P}(\phi/\psi)$. Then, the following holds: (1) $(\|\phi\|^c, \|\psi\|^c) \in N^c_{\mathcal{P}}(w)$, (2) $\forall u, v$, $u \equiv^c_{\neg\phi} v$, $u, v \in \|\psi\|^c$, and $u \in \|\neg\phi\|^c$ imply $v \in \|\neg\phi\|^c$, (3) $\exists u, v$ $u \equiv^c_{\neg\phi} v$ $u, v \in \|\psi\|^c$, $u \in \|\phi\|^c$ and $v \in \|\neg\phi\|^c$. First, we need to show that (1') $(\|\phi\|^*, \|\psi\|^*) \in N^*_{\mathcal{P}}(w)$. This follows from (1): if $(\|\phi\|^c, \|\psi\|^c) \in N^c_{\mathcal{P}}(w)$, then $(\|\phi\|^c \cap W^*, \|\psi\|^c \cap W^*) \in N^*_{\mathcal{P}}(w)$. By IH, we know that $\|\phi\|^c \cap W^* = \|\phi\|^*$ and $\|\psi\|^c \cap W^* = \|\psi\|^*$, and thus $(\|\phi\|^*, \|\psi\|^*) \in N^*_{\mathcal{P}}(w)$.

Then we need to show, that (2') $\forall u, v$ s.t. $u \leq^*_{\neg\phi} v$, $u, v \in \|\psi\|^*$ and $u \in \|\neg\phi\|^*$ it holds that $v \in \|\neg\phi\|^*$, which follows from (2). Assume $u, v \in W^*$ such that a. $u \leq^*_{\neg\phi} v$, b. $u, v \in \|\psi\|^*$ and c. $u \in \|\neg\phi\|^*$. From a. and Lemma 5 follows that $u \equiv^c_{\neg\phi} v$, from b. and the IH, it follows that $u, v \in \|\psi\|^c$ and from c. and the IH follows that $u \in \|\neg\phi\|^c$. Therefore we can use (2) from which follows that $M^c, v \vDash \neg\phi$, and from IH follows then that $M^*, v \vDash \neg\phi$, and thus $v \in \|\neg\phi\|^*$.

Lastly, we can show that (3') $\exists u, v$ $u \equiv^c_{\neg\phi} v, u, v \in \|\psi\|^*, u \in \|\phi\|^*$ and $v \in \|\neg\phi\|^*$ using a similar strategy as for (2'). □

Lemma 10 (Truth Lemma for M^*) *$M^*, w \vDash \phi$ iff $\phi \in w$.*

Proof By induction. From Lemma 9 follows that for $w \in W^*$, $M^*, w \vDash \phi$ iff $M^c, w \vDash \phi$, and thus, from Lemma 7, $M^*, w \vDash \phi$ iff $\phi \in w$. □

Theorem 11 (Completeness) *If a formula ϕ is valid, then ϕ is a theorem of $LM^{\preceq}_P$.*

Proof By contraposition. Assume that ϕ is not a theorem of $LM^{\preceq}_P$, and thus $\nvdash \phi$. We need to show that there is a world w and a $LM^{\preceq}_P$-model M such that $M, w \nvDash \phi$. As

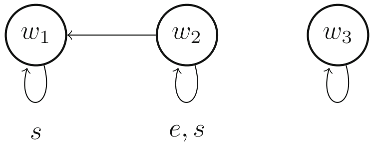


Fig. 1 An $LM_P^{\preceq}$-model on eating at someone’s house after they purchased Soma

$\nvdash \phi$ holds, we know that $\neg\phi \nvdash \bot$, and that (using Lindenbaum’s Lemma) we can construct a maximally consistent set w such that $\neg\phi \in w$. Since w is a maximally consistent set, we know that $w \in W^c$ and thus that $M^c, w \models \neg\phi$, by Lemma 7. To obtain a $LM_P^{\preceq}$-model, we create a submodel M^* generated by w (Lemma 8). Since $M^c, w \models \neg\phi$, we know from Lemma 10 that $M^*, w \models \neg\phi$, and therefore $M^*, w \not\models \phi$. □

Lemma 12 (Consistency) *The logic $LM_P^{\preceq}$ is consistent.*

Proof To demonstrate the consistency of $LM_P^{\preceq}$, we present a model M satisfying all axioms, yet containing at least one formula that fails to hold. The latter is straightforward due to the presence of the classical negation (if M models a formula it automatically does not model its negation). M is based on Kumārila’s statement of eating at the home of someone who has just purchased Soma (see Ex. 6). There is a general negative obligation to eat at someone’s house during a ritual, but there is an explicit permission to do so when someone has just purchased Soma.

To model this, we take e to mean “eating during a ritual”, and s for “having purchased Soma”, and use the formulas $\mathcal{O}(\neg e/\top)$, $\mathcal{P}(e/s)$, and $Pref^s(\neg e)$. Let $M = \langle W, N_{\mathcal{O}}, N_{\mathcal{F}}, N_{\mathcal{P}}, \leq, V\rangle$, where $W = \{w_1, w_2, w_3\}$, $V(e) = \{w_2\}$, $V(s) = \{w_1, w_2\}$, $N_{\mathcal{F}}(w_i) = \emptyset$[16], $N_{\mathcal{P}}(w_i) = \{(V(e), \{w_1, w_2\})\}$ and $N_{\mathcal{O}}(w_i) = \{(X, Y) : \{w_1, w_3\} \subseteq X, X \neq W, Y = W\}$ for $i \in \{1, 2, 3\}$, and $\leq = \{(w_1, w_1), (w_2, w_2), (w_2, w_1), (w_3, w_3)\}$. The model is depicted in Fig. 1, where the arrows indicate the preference relation $\leq$.

Axiom P1 holds because condition (i) of Def. 3 is met: $(X, Y) \in N_{\mathcal{P}}(w_i)$ implies $(\overline{X}, W) \in N_{\mathcal{O}}(w_i)$ for $i \in \{1, 2, 3\}$. Since $Y \subset W$, we also see that P5 holds. To show that axiom P6 is valid, since $M, w_i \models \mathcal{P}(e/s)$ for $i \in \{1, 2, 3\}$, we need to show that (a) $M, w_i \models \boxed{\mathsf{U}}((s \wedge \neg e) \to \boxed{\leq}^{\neg e}(s \to \neg e))$ and (b) $M, w_i \models \langle\mathsf{U}\rangle(s \wedge e \wedge \langle\leq\rangle^{\neg e}(s \wedge \neg e))$.

For (a), $M, w_i \models (s \wedge \neg e) \to \boxed{\leq}^{\neg e}(s \to \neg e)$ needs to be true for $i \in \{1, 2, 3\}$. Since $M, w_2 \models e$, the formula is trivially true in this world. For world w_1, we just need to show that for all worlds $v \in W$ such that $w_1 \leq v$ and $w_1 \equiv^M_{\neg e} v$, it holds that $M, v \models s \to \neg e$. Since only $w_1 \leq w_1$ holds and $M, w_1 \models s \wedge \neg e$, this statement is true.

The statement (b) is true, because $M, w_2 \models s \wedge e \wedge \langle\leq\rangle^{\neg e}(s \wedge \neg e)$. We see that $M, w_2 \models s \wedge e$, and we have that $w_2 \leq w_1$ and $w_1 \equiv^M_{\neg e} w_2$, and $M, w_1 \models s \wedge \neg e$. Therefore $M, w_i \models \langle\mathsf{U}\rangle(s \wedge e \wedge \langle\leq\rangle^{\neg e}(s \wedge \neg e))$ for $i \in \{1, 2, 3\}$.

The axioms P2a, P2b, P2c, Ax3, Ax4 hold, since there is no element $(x, y) \in N_\eta(w)$ such that $(x, y) \in N_\zeta(w)$ for $\eta, \zeta \in \{\mathcal{O}, \mathcal{P}, \mathcal{F}\}$ and $\eta \neq \zeta$. The model trivially satisfies

[16] An empty neighbourhood function for, e.g., prohibition implies that there is no formula in the model that is prohibited.

the axioms Ax5, P4a, P4b, P7, and P8. Ax1 holds since the neighbourhood function $N_{\mathcal{O}}(w_i)$ for $i \in \{1, 2, 3\}$ is closed under monotonicity. P3 holds since $(W, Y) \notin N_{\mathcal{O}}(w_i)$ for all $Y \subseteq W$ and $X \cap Y \neq \emptyset$ for all $(X, Y) \in N_{\mathcal{O}}(w_i)$, for $i \in \{1, 2, 3\}$.

Axioms Ax2 and Ax6 hold since these are implications with false antecedent. The remaining axioms –P7-P12– are true in any ceteris-paribus preference model, see [7]. □

6 Deontic Paradoxes According to Mīmāṃsā

To analyze the behaviour of $LM_P^{\preceq}$ we use as benchmarks the main paradoxes in the deontic literature involving permission: the free choice inference [64], Ross' paradox [55] and the paradox of the privacy act [30]. We also examine an additional scenario found in Mīmāṃsā-influenced Dharmaśāstra: the poor *brāhmaṇa*. Although we refer to them as paradoxes, they are intended here in a broad sense as conclusions derivable in SDL which are counter-intuitive from a common-sense reading.

6.1 The Free Choice Inference

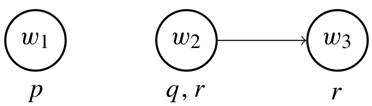
It is plausible to say that "you may have coffee or tea" implies that you may have a coffee and you may have a tea (though possibly not both at once). This very intuitive principle, first mentioned in [64], is known as the free choice inference (FCI) and is formalized in SDL as $\mathcal{P}(\phi \vee \psi) \rightarrow \mathcal{P}(\phi)$.

The paradoxical consequences of accepting FCI have been widely discussed in deontic logic, see, e.g. [5, 13, 19, 28]. Among them, as demonstrated in [28], SDL with (FCI) derives (i) $\mathcal{O}(\phi) \rightarrow \mathcal{O}(\phi \wedge \psi)$, (ii) $\mathcal{O}(\phi) \rightarrow \mathcal{P}(\psi)$, (iii) $\mathcal{P}(\phi) \rightarrow \mathcal{P}(\psi)$ and (iv) $\mathcal{P}(\phi) \rightarrow \mathcal{P}(\phi \wedge \psi)$. As a special instance of (iii), we get (v) $\mathcal{P}(\phi) \rightarrow \mathcal{P}(\perp)$, which is a particularly undesirable consequence in Mīmāṃsā, where permitted actions should be possible, see Lemma 1. As a result, the free choice inference was modified in [16] for LM_P to ensure that every inferred permission corresponds to a feasible action:

$$\mathcal{P}(\phi \vee \psi/\theta) \wedge \Diamond\phi \rightarrow \mathcal{P}(\phi/\theta). \qquad (\mathrm{FCI}\Diamond)$$

We demonstrate that the (dyadic variant of) (i)-(v) cannot be derived in $LM_P^{\preceq}$ in the presence of FCI$\Diamond$ by providing a model such that FCI$\Diamond$ is true in it, but (i)-(v) are not. This is demonstrated in the example below.

Example 10 Let $M = \langle W, N_{\mathcal{O}}, N_{\mathcal{P}}, N_{\mathcal{F}}, \leq, V\rangle$ be the $LM_P^{\preceq}$-model such that $W = \{w_1, w_2, w_3\}$, $V(p) = \{w_1\}$, $V(q) = \{w_2\}$, $V(r) = \{w_2, w_3\}$, $N_{\mathcal{P}}(w_i) = \{(V(q), V(r))\}$, $N_{\mathcal{O}}(w_i) = \{(X, Y) : V(p) \subseteq X, X \neq W, Y = V(r)\}$, $N_{\mathcal{F}}(w_i) = \{(V(q), W)\}$ for $i \in \{1, 2, 3\}$. The preference relation between the three worlds is given in the following figure (with the reflexive arrows omitted):



We see that for all $(X, Y) \in N_{\mathcal{P}}(w_i)$, $X = \{w_2\}$, for each $i \in \{1, 2, 3\}$. This means that if $M, w_i \models \mathcal{P}(\phi \vee \psi/\theta)$ and $M, w_i \models \Diamond\phi$, then $M, w_i \models \Box(\phi \leftrightarrow \phi \vee \psi)$, since $\|\phi\|^M = \|\phi \vee \psi\|^M = \{w_2\}$. From Axiom P4b, it follows that $M, w_i \models \mathcal{P}(\phi/\theta)$. Consequently, it follows that FCI$\Diamond$ holds for all $w_i \in W$. We show that M does not satisfy (i)-(v). For (i), we see that $M, w_i \models \mathcal{O}(p/r)$ and $M, w_i \not\models \mathcal{O}(p \wedge q/r)$. For (ii), $M, w_i \models \mathcal{O}(p/r)$ and $M, w_i \not\models \mathcal{P}(r/r)$. For (iii), we have $M, w_i \models \mathcal{P}(q/r)$ and $M, w_i \not\models \mathcal{P}(p/r)$. For (iv), we have that $M, w_i \models \mathcal{P}(q/r)$ and $M, w_i \not\models \mathcal{P}(p \wedge q/r)$. Lastly, for (v), we have $M, w_i \models \mathcal{P}(q/r)$ and $M, w_i \not\models \mathcal{P}(\bot/r)$.

Remark 7 The undesirable consequences (i)-(v) can be derived in SDL using instances of obligation implies permission (aka axiom D), interdefinability of the deontic operators, and monotonicity of permission. Due to the lack of these principles, the undesirable inferences regarding obligation (i.e., (i) and (ii)), and prima facie permission[17] (i.e., (iii) and (v)) are blocked in $LM_P^{\preceq}$ even when (an unrelated action) ψ is possible. Nonetheless, $LM_P^{\preceq}$ cannot get rid of *all* unwanted results at the 'all-things-considered' level (see Remark 2). Indeed, the formula $\mathcal{P}(\phi/\theta) \wedge \Diamond(\phi \wedge \psi) \rightarrow \mathcal{P}(\phi \wedge \psi/\theta)$ holds in $LM_P^{\preceq}$ due to axiom P4b. Although weaker than (iv), this formula is undesirable as it states that if ϕ is better not permitted under the condition θ, then for all ψ that are possible alongside ϕ, $\psi \wedge \phi$ is better not permitted under the condition θ.

6.2 Ross' Paradox

Ross' paradox [55] is a frequently debated issue. Introduced as a paradox for obligation, it states that the obligation to mail a letter implies the obligation to mail or burn the letter. Here we consider its version for permission formalized as the valid formula in SDL: $\mathcal{P}(\phi) \rightarrow \mathcal{P}(\phi \vee \psi)$. The prima facie version of this paradox does not apply to permissions in Mīmāṃsā, because all commands in Mīmāṃsā have only one action as their argument. Even if we consider the all-things-considered deontic situation, the consequences of the paradox can be avoided. In fact, as discussed in Section 3, unconditional permissions do not exist in Mīmāṃsā, and all permissions are better-not permissions. Thus, instead of the mailing letter example, which is not a better-not permission, we consider the paradox to be the derivation of formulas like 'you may eat meat or murder someone when starving' from the existing better-not permission 'you may eat meat when starving' (see *nyāya* (e) in Section 3). The dyadic version of the paradox is therefore

$$\mathcal{P}(\phi/\theta) \rightarrow \mathcal{P}(\phi \vee \psi/\theta).$$

This formula is not derivable in $LM_P^{\preceq}$, as shown by the following countermodel: Let $M = \langle W, N_{\mathcal{O}}, N_{\mathcal{P}}, N_{\mathcal{F}}, V\rangle$ be a $LM_P^{\preceq}$-model, such that $W = \{w_1, w_2, w_3\}$, $V(p) = \{w_1\}$, $V(q) = \{w_3\}$, $V(r) = \{w_1, w_2\}$, $N_{\mathcal{P}}(w_i) = \{(V(p), V(r))\}$, $\leq=$

[17] In Mīmāṃsā deontic logic, prima facie commands have only one action as an argument.

$\{(w_1, w_1), (w_2, w_2), (w_3, w_3), (w_1, w_2)\}$, $N_{\mathcal{F}}(w_i) = \{(V(p), W)\}$ and $N_{\mathcal{O}}(w_i) = \emptyset$ for $i \in \{1, 2, 3\}$. Note that the neighbourhood function of prohibition is not empty in order to satisfy condition (i) stated in Def. 3. We see that $(V(p), V(r)) \in N_{\mathcal{P}}(w_i)$, but $(V(p) \cup V(q), V(r)) \notin N_{\mathcal{P}}(w_i)$. Thus $M, w_i \models \mathcal{P}(p/r)$ while $M, w_i \not\models \mathcal{P}(p \vee q/r)$.

Remark 8 Ross' paradox does not appear in either $LM_P^{\preceq}$ or LM_P as Mīmāṃsā permission is not monotonic in the first argument, see Remark 4. For example, if we were to derive $\mathcal{P}(eatMeat \vee sing/starving)$ from $\mathcal{P}(eatMeat/starving)$, then we would need to have a pre-existing command $\mathcal{F}(sing/\top)$ or $\mathcal{O}(\neg sing/\top)$, saying that it is generally forbidden or negatively obligatory to sing a song. This is impossible if such a pre-existing prohibition or negative obligation is not available.

6.3 The Paradox of the Privacy Act

Introduced in [30], this paradox consists of a privacy act containing the norms:

(i) The collection of personal information is forbidden unless acting on a court order authorising it.
(ii) The destruction of illegally collected personal information before accessing it is a defence against the illegal collection of personal data.
(iii) The collection of medical information is forbidden unless the entity collecting the medical information is permitted to collect personal information.[18]

To properly assess this act, we need to consider five distinct scenarios as all other possible scenarios are variations of these. These scenarios are denoted as Scenarios 1–5 for reference. Scenario 1 involves a court order that authorizes the collection of personal data. Regardless of whether the data is ultimately collected or not, this scenario is compliant with the privacy act. Scenario 2, where a court has not authorized the collection of data and neither personal nor medical data is collected, is compliant as well. Scenario 3, where personal data is collected illegally but is compensated by its destruction, is called 'weakly compliant'. Lastly, there are two non-compliant situations: Scenario 4, involving the unauthorized collection of personal data, and Scenario 5, involving the unauthorized collection of medical data.

While SDL can formalize the norms (i)-(iii) in a consistent way, it derives a contradiction when considering the compliant Scenarios 1 and 2. For, by formalizing (i) as $\mathcal{F}(collPersInf)$ and $auth \to \mathcal{P}(collPersInf)$, when $auth$ is true (as in Scenario 1), we derive $\mathcal{P}(collPersInf)$, contradicting $\mathcal{F}(collPersInf)$.

This contradiction is prevented in $LM_P^{\preceq}$. We formalize the norms (i)-(iii) in the following way: (i) is $\mathcal{F}(collPersInf/\top)$ and $\mathcal{P}(collPersInf/auth)$; (ii) represents a contrary-to-duty obligation (see e.g. [54]) since the violation of collecting personal data must be compensated by its destruction, and is formalized as $\mathcal{O}(destrPersInf/collPersInf)$. Lastly, (iii) is formalized as $\mathcal{F}(collMedInf/\top)$ and $\mathcal{P}(collPersInf/X) \to \mathcal{P}(collMedInf/X)$ for any X, since the permission to

[18] In Mīmāṃsā terms, the permission to collect medical information in specific cases would not lead to any sanction, because it has been specifically authorised (see Kumārila's discussion of a similar case, Section 3.2). However, it would still be better if one could avoid collecting it and arrive at the same result without violating the patients' privacy.

collect medical data depends on the condition X of the permission to collect personal data.

We show that $LM_P^{\preceq}$ can adequately model the privacy act, by giving a model where all norms (i)-(iii) hold. Each world represents one of the scenarios, showing that we do not derive any contradiction:

$M = \langle W, N_{\mathcal{O}}, N_{\mathcal{P}}, N_{\mathcal{F}}, \leq, V\rangle$, where: $W = \{w_i : 1 \leq i \leq 7\}$, $V(collPersInf) = \{w_3, w_4, w_6\}$, $V(destrPersInf) = \{w_3\}$, $V(auth) = \{w_1, w_6, w_7\}$, $V(collMedInf) = \{w_5, w_7\}$, $N_{\mathcal{F}}(w_i) = \{(X, Y) : X \neq \emptyset, X \subseteq V(collPersInf), Y = W\} \cup \{(U, Z) : U \neq \emptyset, U \subseteq V(collMedInf), Z = W\}$, $N_{\mathcal{O}}(w_i) = \{(X, Y) : V(destrPersInf) \subseteq X, Y = V(collPersInf)\}$, and $N_{\mathcal{P}}(w_i) = \{(V(collPersInf), V(auth)), (V(collMedInf), V(auth))\}$. We denote the relation $\leq$ in Fig. 2, which illustrates the model. The reflexive arrows are omitted.

Note that, specifically, the exception-based definition of permission in $LM_P^{\preceq}$ is well-suited for the formalization of the privacy act, which also considers permissions as exceptions to prohibitions.

Remark 9 The paradox arising from the privacy act is resolved in both $LM_P^{\preceq}$ and LM_P by the use of dyadic deontic operators. In contrast to SDL, both logics enable the derivation of context-dependent prohibitions, permissions, and obligations, accommodating changing situations, and thus allowing, e.g., the formulas $\mathcal{F}(collPersInf/\top)$ and $\mathcal{P}(collPersInf/auth)$ to be true simultaneously.

6.4 The Poor *Brāhmaṇa*

The following case study arises from a discussion found in Mīmāṃsā-influenced jurisprudence (Dharmaśāstra), and more specifically in a commentary by the jurist Vijñāneśvara. To explain the scenario: According to Dharmaśāstra, the Indian society is divided into four classes and each class is only allowed to perform a certain number of occupations. The classes are: *brāhmaṇa* (the priestly class); *kṣatriya* (the warrior

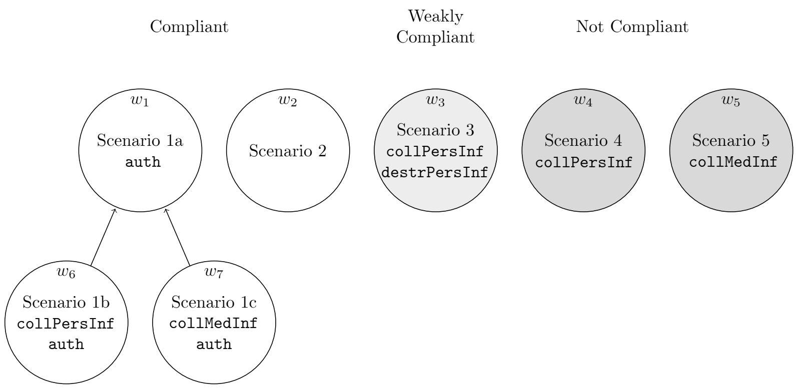


Fig. 2 An $LM_P^{\preceq}$ model of the scenarios 1-5

class); *vaiśya* (the merchant class) and *śūdra* (the servant class). For Dharmaśāstra authors, assuming the occupation of another class is a suboptimal option. The following norms apply to the *brāhmaṇa*:

1. It is prohibited for a *brāhmaṇa* to take up the occupation of a *kṣatriya* (warrior), *vaiśya* (merchant) or *śūdra* (servant).[19]
2. When a *brāhmaṇa* is in distress, it is permitted for them to take up the occupation of a *kṣatriya* or *vaiśya*. This permission is understood as an exception to the prohibition in 1.
3. While it is permitted for a *brāhmaṇa* in distress to sell (and thus take up the occupation of a *vaiśya*), it is prohibited to trade sesame. [20] This prohibition is again an exception to the permission in 2.
4. It is obligatory for a *brāhmaṇa* to perform rituals, and performing a ritual implies using grains.
5. When a *brāhmaṇa* in distress does not have grains, it is permitted for them to trade sesame in exchange for grains.[21] This is again a counter-exception to the prohibition in 3.

Although norms 1–5 can consistently be formalized in SDL, contradictions and undesired results arise due to its inability to handle exceptions. A problem, for instance, occurs by applying norm 2: when a *brāhmaṇa* is in distress, one derives a permission to take up the occupation of a *vaiśya* contradicting norm 1. Similarly, for norm 5, when a *brāhmaṇa* is in distress and does not have grains, one derives the permission to trade sesame contradicting norm 3.

We formalize the above norms in $LM_P^{\preceq}$ and show that these problems do not occur. Consider the following atoms: $occK$, $occV$, and $occS$ stand for 'taking up the occupation of a *kṣatriya* (warrior), a *vaiśya* (merchant), and a *śūdra* (servant)', respectively, br for 'being a *brāhmaṇa*', dis for 'being in distress', tr, trS, $trSG$ stand for 'trading', 'trading sesame', and 'trading sesame for grains', respectively, rit for 'performing rituals', and $grain$ for 'having grains'. Furthermore, the following relations hold: $trS \to tr$, $occV \leftrightarrow tr$, $trSG \to trS$. The norms 1-5 are formalized as follows:

1a. $\mathcal{F}(occK/br) \wedge \mathcal{F}(occV/br) \wedge \mathcal{F}(occS/br)$
2a. $\mathcal{P}(occK/br \wedge dis) \wedge \mathcal{P}(occV/br \wedge dis)$
3a. $\mathcal{F}(trS/br \wedge dis \wedge occV)$
4a. $\mathcal{O}(rit/br)$ and $rit \to grain$
5a. $\mathcal{P}(trSG/br \wedge dis \wedge occV \wedge \neg grain)$

We show that $LM_P^{\preceq}$ models the above norms, by providing a model in which 1a-5a hold: $M = \langle W, N_{\mathcal{O}}, N_{\mathcal{P}}, N_{\mathcal{F}}, \leq, V \rangle$, where $W = \{w_1, w_2, w_3, w_4, w_5, w_6\}$, $V(br) = W$, $V(dis) = \{w_1, w_2, w_3, w_4, w_5\}$, $V(occV) = V(tr) = \{w_1, w_2, w_3\}$, $V(trSG) = V(trS) = \{w_1\}$, $V(grain) = \{w_3, w_4, w_5, w_6\}$, $N_{\mathcal{F}}(w_i) = \{(X, Y) : X \subseteq V(occV), X \neq \emptyset, Y = V(br)\} \cup \{(X, Y) : X \subseteq V(trS), X \neq \emptyset, Y = V(br) \cap V(dis) \cap V(occV)\} \cup \{(X, Y) : X \subseteq V(occK), X \neq$

[19] From Vijñāneśvara's *Mitākṣarā* commentary on Yājñvalkya 3 (on expiations), verse 35.

[20] From Vijñāneśvara's *Mitākṣarā* commentary on Yājñvalkya 3 (on expiations), verses 37–39.

[21] The *Mitākṣarā* quotes here *Mānavadharmaśāstra* 10.91.

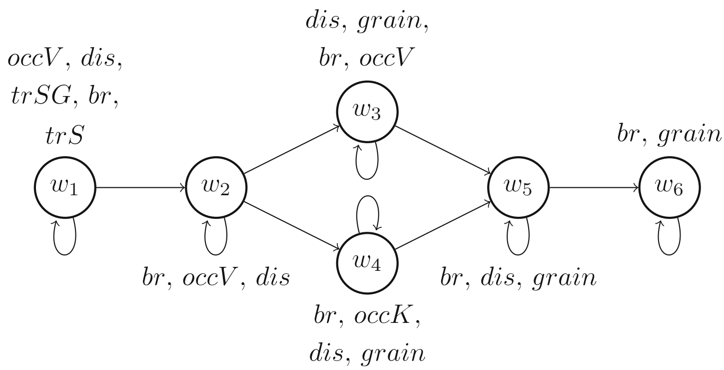


Fig. 3 An $LM^{\preceq}_{\mathcal{P}}$-model of the *brāhmaṇa* in distress

$\emptyset, Y = V(br)\}$, $N_{\mathcal{P}}(w_i) = \{(V(occV), V(br) \cap V(dis)), (V(occK), V(br) \cap V(dis))(V(trSG), V(br) \cap V(dis) \cap V(occV) \setminus V(grain)\}$. The relation $\leq$ is depicted in Fig. 3. The transitive arrows are omitted. In contrast to SDL, no contradiction occurs when a *brāhmaṇa* is in distress (see world w_3), and no contradition occurs when the distressed *brāhmaṇa* lacks grains (see world w_1).

7 Conclusions

The Mīmāṃsā school of Sanskrit philosophy is a very important yet largely unexplored source for deontic investigations. It is a treasure trove that still awaits systematic studies, but has already proven its value in rethinking basic deontic concepts.

Previous formalization of the deontic theories of Mīmāṃsā thinkers are in [9, 15, 43], each introducing new deontic operators and properties found in the original texts (obligations, prohibitions, and elective duties, respectively). These studies shed new light on centuries-old controversies within Mīmāṃsā and discussed the potential of Mīmāṃsā contributions to deontic paradoxes, and to deontic logic in general.[22]

The present paper focuses on permissions in Mīmāṃsā. These are always exceptions to general prohibitions or negative obligations, and refer to less desirable actions. While [16] formalized only the former characteristic, our contribution here lies in offering a comprehensive analysis of Mīmāṃsā permissions encompassing both aspects. We use Sanskrit sources to formalize permissions as exceptions and how they refer to less desirable actions (that is, how they are better-not permissions). This result is achieved by translating and interpreting relevant *nyāya*s from Mīmāṃsā and Dharmaśāstra into Hilbert axioms, and introducing a suitable semantics, which includes ceteris-paribus preferences. This approach has been also utilized to make sense of supererogation as the dual of permission. The choice to use ceteris-paribus preferences enables us to isolate the effect of one variable (action) while assuming that all other relevant factors remain constant. For example, a scenario where one

[22] For more details on the project that led to these articles, see http://mimamsa.logic.at.

refrains from eating meat is considered to be better than a scenario where one consumes meat, under the assumption that all other variables maintain the same truth value. The resulting logic, $LM_P^{\preceq}$, was tested against the major paradoxes of permission in contemporary deontic literature (the free choice paradox, Ross's paradox, and the paradox of the privacy act), as well as a scenario drawn from Mīmāṃsā itself (the poor *brāhmaṇa*). By rejecting assumptions such as the monotonicity of deontic operators, obligation implies permission, and the interdefinability of deontic operators, and by adopting the better-not interpretation of permissions, $LM_P^{\preceq}$ performs well with respect to (almost) all of these paradoxes. However, an undesired formula related to the free choice paradox, albeit weaker than its counterpart in SDL, remains provable.

On the Mīmāṃsā side, this interdisciplinary study is currently the most advanced one on Mīmāṃsā permissions (improving on [16, 25]). The level of detail achieved through the preference formalization has contributed to clarifying the essential characteristics of Mīmāṃsā permissions as well as their link to supererogatory actions.

With regard to logic, we are confident that our formalization of the Mīmāṃsā better-not permission contributes to the discussions on permissions and to the development of deontic logic. In particular, we want to stress the disambiguation features of the Mīmāṃsā better-not permissions, which offer a solution to seeming problems like the "interrupted promise" [66] (see Section 4.2) and more generally, a way out of the ambiguous use of the term 'permission'.

Acknowledgements We thank the anonymous reviewers for their detailed and constructive feedback. Work partially supported by the FWF project I 6372 (Grant-DOI 10.55776/I6372), by the European Union's Horizon 2020 research and innovation programme under grant agreement No 101034440 and by the SSHRC project "New voices in deontic philosophy" (application number 435-2022-0324).

Author Contributions The authors contributed equally to this work.

Funding Open access funding provided by TU Wien (TUW). Work partially supported by the FWF project I 6372 (Grant-DOI 10.55776/I6372), by the European Union's Horizon 2020 research and innovation programme under grant agreement No 101034440, and by the SSHRC project "New voices in deontic philosophy" (application number 435-2022-0324).

Declarations

Competing Interests The authors have no competing interests to declare that are relevant to the content of this article.

References

1. Alchourrón, C. E. (2015). Permissory Norms and Normative Systems (1984/86/2012). In: C. Bernal, C. Huerta, T. Mazzarese, et al. (Eds.), *Essays in Legal Philosophy*. Oxford University Press, (pp. 324–336). https://doi.org/10.1093/acprof:oso/9780198729365.003.0023
2. Alchourrón, C. E., & Bulygin, E. (1984). Permission and permissive norms. In: H. Rüßmann (Ed.), *Theorie der Normen*. Festgabe für Ota Weinberger zum 65. Geburtstag. Duncker & Humblot Berlin, (pp. 349–371).
3. Anglberger, A. J. J., Dong, H., & Roy, O. (2014). Open reading without free choice. In: F. Cariani, D. Grossi, J. Meheus, et al. (Eds.), *Deontic Logic and Normative Systems*. Springer International Publishing, Cham, (pp. 19–32). https://doi.org/10.1007/978-3-319-08615-6_3
4. Åqvist, L. (1984). Deontic logic. In: D. Gabbay, & F. Guenthner (Eds.) *Handbook of Philosophical Logic: Volume II*. Springer, Dordrecht, (pp. 605–714). https://doi.org/10.1017/s0022481200041256
5. Asher, N., & Bonevac, D. (2005). Free choice permission is strong permission. *Synth, 145*(3), 303–323. https://doi.org/10.1007/s11229-005-6196-z
6. Barker, C. (2010). Free choice permission as resource-sensitive reasoning. *Semantics and Pragmatics, 3*(10), 1–38. https://doi.org/10.3765/sp.3.10
7. van Benthem, J., Girard, P., & Roy, O. (2009). Everything else being equal: A modal logic for Ceteris Paribus preferences. *Journal of Philosophical Logic, 38*(1), 83–125. https://doi.org/10.1007/S10992-008-9085-3
8. van Berkel, K., & Lyon, T. (2021). The varieties of ought-implies-can and deontic STIT logic. In: F. Liu, A. Marra, P. Portner, et al. (Eds.), *Deontic Logic and Normative Systems - 15th International Conference, DEON 2020/21, Munich, Germany [virtual], July 21-24, 2021*. College Publications, (pp. 56–76).
9. van Berkel, K., Ciabattoni, A., Freschi, E., et al. (2023). Deontic paradoxes in Mīmāṃsā logics: There and back again. *Journal of Logic, Language and Information, 32*(1), 19–62. https://doi.org/10.1007/S10849-022-09375-W
10. Blackburn, P., Rijke, M., & d., & Venema, Y. (2001). Modal Logic. *Cambridge Tracts in Theoretical Computer Science, Cambridge University Press.* https://doi.org/10.1017/CBO9781107050884
11. Boella, G., & van der Torre, L. (2003). Permissions and obligations in hierarchical normative systems. In: J. Zeleznikow, G. Sartor (Eds.), *Proceedings of the 9th International Conference on Artificial Intelligence and Law*. Association for Computing Machinery, New York, NY, USA, ICAIL '03, (p 109–118). https://doi.org/10.1145/1047788.1047818
12. Bouvier, J. (1856). *A Law Dictionary*. Philadelphia: Childs and Peterson.
13. Broersen, J., & van der Torre, L. (2012). *Ten problems of deontic logic and normative reasoning in computer science*. Lectures on Logic and Computation: ESSLLI 2010, Selected Lecture Notes (pp. 55–88). https://doi.org/10.1007/978-3-642-31485-8_2
14. Chellas, B. F. (1980). Modal Logic. *Cambridge University Press, Cambridge.* https://doi.org/10.1017/CBO9780511621192
15. Ciabattoni, A., Freschi, E., Genco, F. A., et al. (2015). Mīmāṃsā deontic logic: Proof theory and applications. In: H. De Nivelle (Ed.), *Automated Reasoning with Analytic Tableaux and Related Methods*. Springer International Publishing, Cham, (pp. 323–338). https://doi.org/10.1007/978-3-319-24312-2_22
16. Ciabattoni, A., Dik, J., & Freschi, E. (2023). Disambiguating Permissions: A Contribution from Mīmāṃsā. In: J. Maranhão , C. Peterson, & C. Straßer, et al. (Eds.), *Deontic Logic and Normative Systems - 16th International Conference, DEON 2023*, Trois-Rivières, QC, Canada, July 5-7, 2023. College Publications, (pp. 99–118).
17. Danielsson, S. (1968). *Preference and Obligation*. Uppsala: Filosofiska Färeningen.
18. Davis, D. R. Jr. (2023). Sex with Purpose. Mīmāṃsā Rules and the Dharma of Marital Intercourse. *Religions of South Asia, 17*(1). https://doi.org/10.1558/rosa.25454
19. Dignum, F., Meyer, J. J. C., & Wieringa, R. (1994). Contextual permission: A solution to the free choice paradox. In: A. Jones, & M. Sergot (Eds.), *Second International Workshop on Deontic Logic in Computer Science*, (pp. 107–135).
20. Dong, H. (2021). Logic of defeasible permission and its dynamics. *Journal of Logic and Computation, 31*(4), 1158–1193. https://doi.org/10.1093/logcom/exab022

21. Dong, H. (2022). Permission and obligation in ceteris paribus. In: *Computability Theory and Foundations of Mathematics: Proceedings of the 9th International Conference on Computability Theory and Foundations of Mathematics*, (pp. 157–165). https://doi.org/10.1142/9789811259296_0007
22. Dong, H., Gratzl, N., & Roy, O. (2018). Open reading and free choice permission: a perspective in substructural logics. In: *International Conference on Logic and Argumentation*, Springer, (pp. 81–115).
23. Echave, D. T., Urquijo, M. E., & Guibourg, R. (1980). *Lógica, proposición y norma*. Buenos Aires: Astrea.
24. Freschi, E. (2018). The role of paribhāṣās in Mīmāṃsā: Rational rules of textual exegesis. *Asiatische Studien/Études Asiatiques, 72*(2), 567–595.
25. Freschi, E. (2024). Mīmāṃsā and dharmaśāstra sources on permissions. In: P. A., Maas & A. Cerulli (Eds.), *Suhṛdayasaṃhitā. A Compendium of Studies on South Asian Culture, Philosophy, and Religion*. Dedicated to Dominik Wujastyk, Studia Indologica Universitatis Halensis, (vol. 28). Universitätsverlag Halle-Wittenberg, Halle-Wittenberg.
26. Freschi, E., & Pascucci, M. (2021). Deontic concepts and their clash in Mīmāṃsā: Towards an interpretation. *Theoria, 87*(3), 659–703. https://doi.org/10.1111/theo.12307
27. Freschi, E., Ciabattoni, A., Genco, F. A., et al. (2017). Understanding prescriptive texts: Rules and logic as elaborated by the Mīmāṃsā school. *Journal of World Philosophies, 2*(1), 47–66. https://doi.org/10.2979/jourworlphil.2.1.05
28. Gabbay, D., Gammaitoni, L., & Sun, X. (2014). The paradoxes of permission an action based solution. *Journal of Applied Logic, 12*(2), 179–191. https://doi.org/10.1016/j.jal.2014.01.003
29. Gabbay, D., Horty, J., Parent, X., et al. (2021). *2021*. Handbook of deontic logic and normative systems: College Publications.
30. Governatori, G. (2015). *Thou shalt is not you will*. Association for Computing Machinery, New York, NY, USA, ICAIL '15, (pp. 63–68). https://doi.org/10.1145/2746090.2746105
31. Governatori, G., Olivieri, F., Rotolo, A., et al. (2011). Three concepts of defeasible permission. In: *Legal Knowledge and Information Systems*. IOS Press, (pp. 63–72). https://doi.org/10.3233/978-1-60750-981-3-63
32. Governatori, G., Olivieri, F., Rotolo, A., et al. (2013). Computing strong and weak permissions in defeasible logic. *Journal of Philosophical Logic, 42*, 799–829. https://doi.org/10.1007/s10992-013-9295-1
33. Guhe, E. (2021). Killing gently by means of the Śyena: The Navya-Nyāya analysis of vedic and secular injunctions (vidhi) and prohibitions (niṣedha) from the perspective of dynamic deontic logic. *Journal of Indian Philosophy, 49*(3), 421–449. https://doi.org/10.1007/s10781-021-09465-2
34. Gustafsson, J. E. (2020). Permissibility is the only feasible deontic primitive. *Philosophical Perspectives, 34*(1), 117–133. https://doi.org/10.1111/phpe.12137
35. Hansson, B. (1969). An analysis of some deontic logics. *Noûs*, *3*(4), 373–398. https://doi.org/10.2307/2214372
36. Hansson, S. O. (1990). Preference-based deontic logic (pdl). *Journal of Philosophical Logic,* 93–122. https://doi.org/10.1007/BF00211186
37. Hansson, S. O. (1996). What is ceteris paribus preference? *Journal of Philosophical Logic, 25*(3), 307–332. https://doi.org/10.1007/BF00248152
38. Hansson, S. O., et al. (2013). The varieties of permission. In D. M. Gabbay, J. Horty, & X. Parent (Eds.), *Handbook of deontic logic and normative systems* (pp. 195–240). London: College Publications.
39. Hansson, S. O. (2015). In defence of deontic diversity. *Journal of Logic and Computation, 29*(3), 349–367. https://doi.org/10.1093/logcom/exv057
40. Heyd, D. (Ed.) (2023). Handbook of Supererogation. Springer Nature Singapore.
41. Hilpinen, R., & McNamara, P. (2013). Deontic logic: A historical survey and introduction. In: D. Gabbay, J. Horty, X. Parent, et al. (Eds.), *Handbook of Deontic Logic and Normative Systems*. College Publications, (pp. 3–136).
42. Jhā, G. (Ed.). (1999). *Manu-smṛti. With the 'Manubhāṣya' of Medhātithi. Edited with the help of several manuscripts* (2nd ed.). Delhi: Motilal Banarsidass.
43. Lellmann, B., Gulisano, F., & Ciabattoni, A. (2021). Mīmāṃsā deontic reasoning using specificity: a proof theoretic approach. *Artificial Intelligence and Law, 29*(3), 351–394. https://doi.org/10.1007/s10506-020-09278-w
44. Lewis, D. (1973). *Counterfactuals*. Oxford: Blackwell.
45. Loreggia, A., Lorini, E., & Sartor, G. (2022). Modelling ceteris paribus preferences with deontic logic. *Journal of Logic and Computation, 32*(2), 347–368. https://doi.org/10.1093/logcom/exab088

46. Lorini, E. (2020). Rethinking epistemic logic with belief bases. *Artificial Intelligence, 282*, 103233. https://doi.org/10.1016/j.artint.2020.103233 https://www.sciencedirect.com/science/article/pii/S0004370219300293
47. Makinson, D., & van der Torre, L. (2003). Permission from an input/output perspective. *Journal of philosophical logic, 32*(4), 391–416. https://doi.org/10.1023/A:1024806529939
48. Makinson, D., & van der Torre, L. W. N. (2000). *Input/output logics. Journal of philosophical logic, 29*(4), 383–408. https://doi.org/10.1023/A:1004748624537
49. McNamara, P. (1996). Doing well enough: Toward a logic for common-sense morality. *Studia logica, 57*, 167–192. https://doi.org/10.1007/bf00370674
50. McNamara, P. (2006). Deontic logic. In: D. M. Gabbay, & J. Woods (Eds.), *Logic and the Modalities in the Twentieth Century, Handbook of the History of Logic*, (vol. 7, p 197–288). North-Holland. https://doi.org/10.1016/S1874-5857(06)80029-4
51. McNamara, P. (2023). Extensions and variations of the DWE framework. In: J. Maranhão, C. Peterson, C. Straßer, et al (Eds.), *Deontic Logic and Normative Systems - 16th International Conference, DEON 2023*, Trois-Rivières, QC, Canada, July 5-7, 2023. College Publications, (pp. 177–197)
52. Meinong, A. (1894). *Psychologisch-ethische Untersuchungen zur Werththeorie*. Leuschner & Lubensky.
53. Olszewski, M., Parent, X., & van der Torre, L. (2021). Input/output logic with a consistency check-the case of permission. In: F. Liu, A. Marra, P. Portner, et al. (Eds.), *Proceedings of Deontic Logic and Normative Systems - 15th International Conference, DEON 2020/21*, Munich, Germany [virtual], July 21-24, 2021, (pp. 358–375).
54. Prakken, H., & Sergot, M. (1996). Contrary-to-duty obligations. *Studia Logica, 57*, 91–115. https://doi.org/10.1007/BF00370671
55. Ross, A. (1944). Imperatives and logic. *Philosophy of Science, 11*(1), 30–46. https://doi.org/10.2307/2268596
56. Stalnaker, R. (2014). *Context*. Oxford: Oxford University Press.
57. Stern, R. (2004). Does ‘ought’ imply ‘can’? And did Kant think it does? *Utilitas, 16*(1), 42–61. https://doi.org/10.1017/s0953820803001055
58. Stolpe, A. (2010). A theory of permission based on the notion of derogation. *Journal of applied logic, 8*(1), 97–113. https://doi.org/10.1016/j.jal.2010.01.001
59. Subbāśāstrī (Ed.) (1929-1934) Śrīmajjaiminipraṇitaṃ Mīmāṃsādarśanam. Ānandāśramamudrāṇālaya, Poona.
60. Urmson, J. O. (1958). Saints and heroes. In: A. I. Melden (Ed.), *Essays in Moral Philosophy*. University of Washington Press. https://doi.org/10.1017/S0031819100050762
61. Van Fraassen, B. C. (1973). The logic of conditional obligation. *Exact Philosophy*, D Reidel, Dordrecht, Holland (pp. 151–172). https://doi.org/10.1007/bf00255570
62. von Wright, G. H. (1951). Deontic logic. *Mind, 60*(237), 1–15. https://doi.org/10.1093/mind/LX.237.1
63. von Wright, G. H. (1963). Norm and Action. *Routledge and Kegan Paul, London,*. https://doi.org/10.2307/2218217
64. von Wright, G. H. (1968). *An Essay in Deontic Logic and the General Theory of Action: With a Bibliography of Deontic and Imperative Logic*. Amsterdam: North-Holland Pub. Co.
65. von Wright, G. H. (1970). Deontic logic and the theory of conditions. In: R. Hilpinen (Ed.), *Deontic Logic: Introductory and Systematic Readings*. Springer Netherlands, (pp. 159–177). https://doi.org/10.1007/978-94-010-3146-2_7
66. Zylberman, A. (2022). Moral rights without balancing. *Philosophical Studies, 179*, 549–569. https://doi.org/10.1007/s11098-021-01670-9