Self-Organizing Maps of Spiking Neurons Using Temporal Coding (Summary)

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The self-organizing map (SOM) by Kohonen (see e.g. [1]) is among the most successful artificial neural network types. The basic principle of SOM's, namely to map similar input patterns to contiguous locations in the output space, also occurs in biological systems, e.g. in the visual cortex. However, the standard formulation of the SOM and the corresponding learning rule are not suitable for biological systems: Either the winner neuron has to be determined by a non-local mechanism like exhaustive search or one uses for this a recurrent network of analogue neurons that has to settle down into an equilibrium state. Here we show how networks of spiking neurons can be used to implement a variation of the SOM in temporal coding, which has the same characteristic behavior. In contrast to the standard formulation of the SOM our construction has the additional advantage that the winner among the competing neurons can be determined fast and locally by using lateral excitation and inhibition.

Recently Maass has shown in [2] how leaky integrate-and-fire neurons can compute weighted sums in temporal coding, where the firing time of a neuron encodes a value in the sense that an early firing of the neuron represents a large value. More precisely, one simply considers a neuron v which receives excitatory input from m neurons $u_1, \ldots, u_m$, where the corresponding weights are denoted by $w_1, \ldots, w_m$. Each u_i fires exactly once within a sufficiently small time intervall at a time t_i with $t_i = T_1 - s_i$, where s_i is the ith input to v and T_1 some constant. Under some weak assumptions (basically the initial rising segments of the excitatory postsynaptic potentials have to be linear) one can guarantee that v fires at a time determined by $T_2 - \sum w_i s_i$ with T_2 being some constant.

We used this construction to implement a variation of Kohonen's learning rule for SOM's in the context of spiking neuron networks as follows: Given a set S of m-dimensional input vectors $\mathbf{s}^l = (s_1^l, \ldots, s_m^l)$ and a spiking neuron network with m input neurons and n output neurons, where each output neuron v_j receives synaptic "feedforward" input from each input neuron u_i with weight w_{ij} and "lateral" synaptic input from each output neuron $v_k, k \neq j$ with weight $\tilde{w}_{kj}$. At every epoch of the learning procedure one $\mathbf{s}^l \in S$ is randomly chosen and the input neurons are made fire such that they temporally encode $\mathbf{s}^l$. Each

v_j then starts to compute $\sum_i w_{ij} s_i^l$. If we assume that the input vector and the weight vector for each neuron are normalized, then this weighted sum represents the similarity between the two vectors. Hence the earlier v_j fires, the more similar is its weight vector to the input vector and the output neurons form a competitive layer, where the winner is the neuron which fires first. Note that the firing time of the winner is not influenced by the firing of the other output neurons.

The neighborhood-function, which is an important part of the SOM, is implemented here by the lateral connections among the output neurons. We assume that initially neurons which are topologically close together have strong excitatory lateral connections whereas remote neurons have strong inhibitory connections. This means that the firing of the winner neuron, say v_k, at time t_k drives the firing times of neurons in the neighborhood of v_k towards t_k, thus increasing the values they encode. The firing of remote neurons is postponed by the lateral inhibition. We suggest the following learning rule:

$$\Delta w_{ij} = \eta \frac{T_{out} - t_j}{T_{out}} (c_i^l - w_{ij})$$

where the current pattern is $\mathbf{c}^l$, η is the learning rate and t_j the firing time of the jth output neuron. The learning rule applies only to neurons that have fired before a certain time T_{out} (which has to be chosen sufficiently large). The factor $(T_{out} - t_j)/T_{out}$ realizes the neighborhood function, which is largest for the winner neuron and decreases for neurons which fire at later times.

During the learning process the lateral weights $\tilde{w}_{kj}$ are decreased, thus reducing the size of the neighborhoods.

We performed computer simulations with this approach and achieved extremely promising results. The typical self-organization of topology-preserving behavior could be observed for a wide range of parameters. Furthermore it turned out that a normalization of the weight vectors after each epoch is not necessary, since the weight vectors converge "automatically" to normalized patterns.

To the best of our knowledge, this is the first implementation of Kohonen's learning algorithm for spiking neurons using temporal coding. This work is a first step towards showing that biological neurons can indeed achieve a topology-preserving behavior using a similar learning procedure like the one suggested by Kohonen.

References

[1] Kohonen, T.: Self-organizing maps. Springer Verlag, Berlin. (1995).

[2] Maass, W.: Fast sigmoidal networks via spiking neurons. Neural Computation **9**, 279–304, 1997.