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Homework for
Komplexitätstheorie
A. Y. 16/17
Assignment 13

Exercise 13.1

Show that there exists a PPTM that, given $N \in \mathbb{N}$, achieves the following:

- a) It returns as output an element of the set $\{0, \dots, N - 1\} \cup \{?\}$.
- b) The question mark is returned with probability at most $1/2$.
- c) Conditioned to the event of not returning “?”, the output is uniformly distributed over $\{0, \dots, N - 1\}$.

Note that the PPTM must be $\text{poly}(\log N)$ time-bounded.

Exercise 13.2

Show that a coin with $\Pr[\text{Head}] = \rho$ can be simulated by a PPTM in expected time $O(1)$ provided that the i -th bit of ρ is computable in $\text{poly}(i)$ time.

Exercise 13.3

Describe a real number $0 < \rho < 1$ such that a Turing machine equipped with a coin that returns “Head” with probability ρ can decide an undecidable language in polynomial time.

Exercise 13.4

Sketch the proof of the following statement: If $NP \subseteq BPP$ then $NP = RP$.