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Homework for

Komplexitätstheorie

A. Y. 16/17

Assignment 12

Exercise 12.1

Show (without using Wrathall's Theorem) that, for each $k \geq 0$ and for each language $L \in \Sigma'_k$, the language $\{\langle u_1, \ldots, u_r \rangle | \ r \geq 1 \wedge u_1, \ldots, u_r \in L\}$ belongs to Σ'_k as well.

Exercise 12.2

In the lecture, we have shown that each language in P can be realized by a circuit family $C = (C_n)_{n \geq 0}$ of polynomial size (transformation of software into hardware). Show that $desc(C_n)$ can be constructed within space $O(\log n)$ (so that the family C is uniform).

Exercise 12.3

Let CIRCUIT-SAT (or CSAT) be the following language:

$$\text{CSAT} = \bigcup_{n \geq 0} \{\langle C_n \rangle \mid C_n \text{ is a } n\text{-input circuit s.t. } \exists a \in \{0,1\}^n \text{ s.t. } C_n(a) = 1\}.$$

Show that CSAT is NP-complete.

Exercise 12.4

A DTM M is said to be *oblivious* if the movements of the head depends only on the input length n.

a) Show that a DTM M with time bound $T(n)$ and space bound $S(n)$ can be simulated by an oblivious DTM M' with time bound $S(n)T(n)$ and space bound $S(n)$. It suffices to explain the main idea of the simulation and to provide a short argument concerning its time- and space-bound.

b) Let L be a language that is recognized by an oblivious DTM with time bound $T(n)$. Show that L can be realized by a circuit of size $O(T(n))$. It suffices to reconsider the transformation of software into hardware from the lecture and to indicate why and where this transformation becomes more hardware-efficient.