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Homework for

Komplexitätstheorie

A. Y. 16/17

Assignment 10

Exercise 10.1

Suppose that $T(n)$ is a monotonically increasing and time-constructible function. Define a function $f : \mathbb{N} \rightarrow \mathbb{N}$ inductively by setting $f(1) = 2$ and $f(i+1) = 2^{T(f(i))}$ (as in the proof of Theorem 15.5 of the lecture notes). In the lecture, we claimed that, given n, the smallest index i such that $f(i) < n \leq f(i+1)$ can be determined within time bound $O(T(n))$. Argue why this is true.

Exercise 10.2

Show that 2-SAT is $\mathcal{NL}$-hard.

Hint: We recall that the REACHABILITY problem is defined as follows. Given a digraph $G = (V, E)$ and two nodes $s, t \in V$, is there a path from s to t? UNREACHABILITY is the complement of REACHABILITY, and an $\mathcal{NL}$-complete problem by virtue of $\mathcal{NL} = co\text{-}\mathcal{NL}$. UNREACHABILITY is $\mathcal{NL}$-complete even for acyclic digraphs.

Exercise 10.3

Let us introduce the following definition. A language $L \in NL^{\star}$ if there exists a polynomial $p(n)$ and a DTM M with space bound $O(\log n)$ such that the following holds:

a) Besides the read-only input tape for input x and the working tape, M is equipped with another read-only input tape for a certificate y.

b) For all $x \in \Sigma^* : x \in L$ if and only if $\exists y \in \{0,1\}^{p(|x|)} : M(x, y) = 1$.

Show that $NL^{\star} = NP$.

Hint: You could show the following:

(i) $NL^\star \subseteq NP$.

(ii) 3-SAT $\in NL^\star$. Using the fact that Cook's theorem can be proved using log-space reductions, this yields $NP \subseteq NL^\star$.

Exercise 10.4

Let LINEAR SPACE ACCEPTANCE be the problem of deciding whether a given DTM M accepts an input word x in $O(n)$ space, where $n = |x|$. Both (an encoding of) M and x are part of the input. Show that LINEAR SPACE ACCEPTANCE is $PSpace$-hard.

Hint: It is easy to see that every language that can be recognized by a DTM with space bound $O(n)$ can be polynomially reduced to LINEAR SPACE ACCEPTANCE. Use a "padding argument" (padding = making an input string longer by adding redundant symbols) to show that this generalizes to languages that can be recognized within polynomial space.