Hans U. Simon
Francesco Aldà

Bochum, December 15th 2016
Deadline on December 22nd 2016

Homework for

Komplexitätstheorie

A. Y. 16/17

Assignment 9

Exercise 9.1

Show the following:

a) Every constant function $n \mapsto c$, for $c \in \mathbb{N}$, is time-constructible.

b) The functions $n \mapsto \lfloor \log n \rfloor$ and $n \mapsto \lceil \log n \rceil$ are space-constructible.

Exercise 9.2

Show the following:

a) If $f(n)$ is time-constructible, then $f(n)$ is space-constructible.

b) Time- and space-constructible functions are closed under addition and multiplication.

Exercise 9.3

Show that $\text{DSpace}(S(n)) = \text{co-DSpace}(S(n))$ provided that $S(n) \geq \log n$.

Hint: In the lecture, this was shown under the additional assumption that $S(n)$ is space-constructible.

Exercise 9.4

Consider the following definition of a space-constructible function. A function $S \colon \mathbb{N} \to \mathbb{N}$ is space-constructible if the function $1^n \mapsto \text{bin}(S(n))$ can be computed by a DTM using only $O(S(n))$ cells.

Show that this definition and the one you have seen in the lecture are equivalent.