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Homework for
Komplexitätstheorie
A. Y. 16/17
Assignment 7

Exercise 7.1

In the lecture, we have seen that Sahni's algorithm A_k for KNAPSACK outputs a solution that differs from the optimum at most by a factor $1+1/k$. Show that the performance ratio of A_k actually is $1+1/k$ for $k \geq 1$.

Exercise 7.2

Read Theorem 8.14 in the lecture notes (transformation of a FPTA-scheme into a pseudopolynomial algorithm). In the lecture, the theorem was proved for minimization problems. Prove it for maximization problems.

Exercise 7.3

Given a maximization problem Π and two real numbers $0 \leq a \leq b$, the $[a,b]$-gap-Π problem consists of distinguishing between the following two cases for an instance I of problem Π:

- $OPT(I) \geq b$;
- $OPT(I) < a$.

The instances satisfying the former case are the YES instances and the latter are the NO instances. Show that if the $[a,b]$-gap version of a maximization problem Π is NP-hard, then it is not possible to find an approximate solution for Π within a factor of a/b, unless $P = NP$.

Exercise 7.4

A *kernelization algorithm* for a decision problem Π is a polynomial-time transformation that maps an instance (I,k) into an instance (I',k') such that

a) $(I,k) \in \Pi$ if and only if $(I',k') \in \Pi$;

b) $k' \leq k$; and

c) $|I'| \leq f(k)$ for some computable function f.

Show that Π is decidable and has a kernelization algorithm if and only if $\Pi \in \mathrm{FPT}$.