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Homework for

Komplexitätstheorie

A. Y. 16/17

Assignment 2

Exercise 2.1

a) In the lecture we have described the codeword $\langle w_1, \ldots, w_k \rangle \in \{0,1\}^*$ of a k-tuple $(w_1, \ldots, w_k)$ such that $w_i \in \{0,1\}^*$. Let $n := \sum_{i=1}^{k} |w_i|$ denote the total length of $w_1, \ldots, w_k$. Express $|\langle w_1, \ldots, w_k \rangle|$ as a function of n and k.

b) Define a new codeword $C_k(w_1, \ldots, w_k) \in \{0,1\}^*$ of $(w_1, \ldots, w_k)$ inductively as follows:

$$C_1(w) = w$$
$$C_{i+1}(w_1, \ldots, w_{i+1}) = \langle C_i(w_1, \ldots, w_i), w_{i+1} \rangle$$

How does $|C_k(w_1, \ldots, w_k)|$ depend on n and k (in O-notation)? How is the dependence on n if k is considered constant?

Exercise 2.2

Let M be a 2-*state busy beaver*, i.e., $M = (Q, \Sigma, \Gamma, B, q_0, \delta, F)$ is a DTM where $Q = \{q_0, q_1, q_h\}$, $\Sigma = \{1\}$, $\Gamma = \{B, 1\}$, $F = \{q_h\}$ and δ is defined by the following table:

	B	1
q_0	$(q_1, 1, R)$	$(q_1, 1, L)$
q_1	$(q_0, 1, L)$	$(q_h, 1, R)$

Write the configurations of the machine M when it is initialized with the empty input.

Exercise 2.3

Sketch the proof of the "Linear Speedup" theorem for a 1-tape DTM (resp. NTM), that is, for each 1-tape DTM (resp. NTM) M which, on input of length n, makes at most $T(n)$ steps, there exists a DTM (resp. NTM) M' that simulates M in at most $\varepsilon T(n)+n+1$ steps, where $\varepsilon > 0$ is an arbitrarily small constant.

Hint: M' can have two tapes.

Exercise 2.4

Let PARTITION be the following problem: Given a set of natural numbers $A = \{a_1, \ldots, a_n\}$, is there $M \subseteq A$ such that $\sum_{a \in M} a = \sum_{a \in (A \setminus M)} a$?
Let PARTITION–INTO–3–SUMS be the following problem: Given a set of natural numbers $A = \{a_1, \ldots, a_n\}$ such that $\sum_{i=1}^{n} a_i = 3S$, are there pairwise disjoint $M, N, P \subseteq A$ such that $\sum_{a \in M} a = \sum_{a \in N} a = \sum_{a \in P} a$?

Show PARTITION $\leq_{pol}$ PARTITION–INTO–3–SUMS.