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Homeworks for

Komplexitätstheorie

A. Y. 13/14

Sheet 12

Exercise 12.1 Let BPL be the class of all languages L that can be recognized by a logspace-bounded PTM that accepts inputs from L and rejects words from $\bar{L}$ with probability at least 2/3, respectively. Show that $BPL \subseteq P$.

Exercise 12.2 Show that if there is a polynomial-time algorithm that approximates the number of cycles in a given digraph up to a factor 1/2, then $P = NP$.

Exercise 12.3

a) Show that the problem of counting the number of satisfying assignments $a \in \{0,1\}^n$ of a CNF-Formula F such that $a_1 = 1$ is computationally equivalent to #SAT (by presenting mutual Cook-reductions between these two problems).

b) Show that #3-SAT is $\#P$-complete.

Exercise 12.4 Show that the XOR gadget in the proof of the $\#P$-completeness of the permanent has the required properties. Specifically, let G be any digraph with integer edge-weights containing a pair of edges $(u, u'), (v, v')$. Let G' be the graph obtained by replacing these edges by the XOR gadget. The following holds:

a) Every cycle cover of G of weight 1 that uses exactly one of the edges $(u, u'), (v, v')$ is mapped to a set of cycle covers in G' whose total weight is 4.

b) All the other cycle covers of G' have total weight 0.