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Homeworks for
Komplexitätstheorie
A. Y. 13/14

Sheet 10

The solutions can be submitted within January 14th 2014

Exercise 10.1 Show (without using Wrathall's Theorem) that, for each $k \geq 0$ and for each language $L \in \Sigma'_k$, the language $\{\langle u_1, \ldots, u_r\rangle |\ r \geq 1 \wedge u_1, \ldots, u_r \in L\}$ belongs to Σ'_k as well. (A statement of this kind was used in the lecture within the proof of Wrathall's Theorem.)

Exercise 10.2 In the lecture, we have shown that, for each $k \geq 1$, the problem $\mathcal{B}_k$ is Σ_k-hard. Show that this even holds when we consider the restriction of $\mathcal{B}_k$ to input instances whose Boolean formula is in conjunctive normal form (CNF) for odd k and in disjunctive normal form (DNF) for even k.

Exercise 10.3 In the lecture, we have shown that each language in P can be realized by a circuit family $C = (C_n)_{n\geq 0}$ of polynomial size (transformation of software into hardware). Show that the circuits C_n used for this purpose can be constructed within space $O(\log n)$ (so that the family C is uniform).

Exercise 10.4 A DTM M is said to be *oblivious* if the movements of the head depend on the input string x only weakly over $n = |x|$. In other words, computations on input strings of the same length result in the same series of head movements.

a) Show that a DTM M with time bound $T(n)$ and space bound $S(n)$ can be simulated by an oblivious DTM M' with time bound $S(n)T(n)$ and space bound $S(n)$. It suffices to verbally explain the main idea of the simulation and to provide a short argument concerning its time- and space-bound.

b) Let L be a language that is recognized by an oblivious DTM with time bound $T(n)$. Show that L can be realized by a circuit of size $O(T(n))$. It suffices to reconsider the transformation of software into hardware from the lecture and to indicate why and where this transformation becomes more hardware-efficient.