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Homeworks for

Komplexitätstheorie

A. Y. 13/14

Sheet 8

Exercise 8.1 Suppose that $T_2(n)$ is a monotonically increasing and time-constructible function. Define function $f : \mathbb{N} \to \mathbb{N}$ inductively by setting $f(1) = 2$ and $f(i+1) = 2^{T_2(f(i))}$ (as in the proof of Satz 15.5 in the Lecture Notes). In the lecture, we claimed that, given n, the smallest index i such that $f(i) < n \leq f(i+1)$ can be determined within time bound $O(T_2(n))$. Argue why this is true.

Exercise 8.2 Let M be an NTM with a logarithmic space bound. Let $G_M(w)$ denote the configuration digraph of M with input string w. As usual, $K_0(w)$ denotes the initial configuration and K_+ the unique accepting configuration. In the lecture we claimed that the mapping $w \mapsto (G_m(w), K_0(w), K_+)$ is computable in logspace. Argue why this is true.

Exercise 8.3 Prove Satz 16.7 of the Lecture Notes (the characterization of the languages in $\mathcal{NL}$ in terms of read-once certificates).

Exercise 8.4 Let us introduce the following definition. A language $L \in \widetilde{NL}$ if there exists a polynomial $p(n)$ and a DTM M with space bound $O(\log n)$ such that the following holds:

a) Besides the read-only input tape for input x and besides the working tape, M is equipped with another read-only input tape for a certificate y.

b) For all $x \in \Sigma^* : x \in L \Longleftrightarrow \exists y \in \{0,1\}^{p(|x|)} : M(x,y) = 1$.

Show that $\widetilde{NL} = NP$. (This result shows why we used read-once certificates within Satz 16.7 of the Lecture Notes).

Hint: You could do the following:

1) Show that $\widetilde{NL} \subseteq NP$.

2) Show that $3SAT \in \widetilde{NL}$. Using the fact that Cook's theorem can be proved using log-space reductions, this yields $NP \subseteq \widetilde{NL}$.