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Homeworks for

# Komplexitätstheorie

A. Y. 13/14

Sheet 5

**Exercise 5.1** Let EXACT-$k$-SAT be the version of SAT where every clause consists of exactly $k$ literals. Argue that a random assignment satisfies, on the average, a fraction $1 - 2^{-k}$ of all clauses. Can you design a *deterministic* polynomial time procedure which finds an assignment that satisfies a fraction of at least $1 - 2^{-k}$ of all clauses?
**Hint:** One could use the following fact. Let $S$ be the fraction of clauses which are satisfied by a random assignment, then

$$\mathbb{E}[S] = \frac{\mathbb{E}[S|x_i = 0] + \mathbb{E}[S|x_i = 1]}{2},$$

for $1 \leq i \leq m$, where $m$ is the number of variables.

**Exercise 5.2** Show that Sahni's algorithm $A_0$ for KNAPSACK has performance ratio $\infty$.

**Exercise 5.3** In the lecture, we have seen that Sahni's algorithm $A_k$ for KNAPSACK differs from optimum at most by a factor $1 + 1/k$. Show that the performance ratio of $A_k$ actually is $1 + 1/k$.

**Exercise 5.4** Read Theorem 8.13 in the Lecture Notes (Transformation of a FPTA-scheme into a pseudopolynomial algorithm). In the lecture, the theorem was proved for minimization problems. Prove it for maximization problems.