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Homeworks for

Komplexitätstheorie

A. Y. 13/14

Sheet 3

Recall that a Problem List had been distributed in the lecture. This list is also found in Appendix A of the Lecture Notes (Skript zur Vorlesung). Another problem list is found on the handout named "Handzettel zur Grenze zwischen P und NPC".[1] All problem definitions relevant for this Exercise Sheet are found either on the Problem List or on the "Handzettel".

Exercise 3.1 Design a Levin-reduction from the problem "Hamiltonian Circuit" to the problem "Longest Path Between Two Vertices".

Exercise 3.2

a) Argue why the problem "Transitive Reduction" belongs to P. **Hint:** Two nodes are said to be *strongly connected* if they are mutually connected by a path in the graph. This is an equivalence relation which leads to a decomposition of V into so-called *strongly connected components*. This decomposition helps in solving the exercise.

b) Argue why the NP-complete problem "Directed Hamiltonian Circuit (DHC) restricted to strongly connected digraphs (i.e., to digraphs for which all pairs of nodes are mutually connected by a path)" can be considered a special case of the problem "Minimum Equivalent Digraph".

Exercise 3.3 In the lecture, you have seen the following chain of reductions: "Colorability" $\leq_{POL}$ "3-Colorability" $\leq_{POL}$ "3-Colorability of Planar Graphs" $\leq_{POL}$ "3-Colorability of Planar Graphs of Maximum Node Degree 4". Each of the three polynomial reductions used a very special gadget with some

[1] All this material can furthermore be downloaded from this website.

very special properties.[2] Show that these gadgets indeed have the special properties as it was claimed in the lecture.

Exercise 3.4 Consider the following variant of the problem "Partition":

Instance: $a_1, \ldots, a_{2n} \in \mathbb{N}$, $S := \sum_{i=1}^{2n} a_i$

Question: Is there a subset $I \subseteq [2n]$ of size $|I| = n$ such that $\sum_{i \in I} a_i = S/2$?

Show that this variant of "Partition" can be solved in pseudo-polynomial time.

[2] As for the precise definition of the gadgets and their properties, see the Lecture Notes (Vorlesungsskript) and the "Handzettel".