Proof. To compute the matrix product

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix},$$

first compute the following products.

$$\begin{aligned}
m_1 &= (a_{12} - a_{22})(b_{21} + b_{22}), \\
m_2 &= (a_{11} + a_{22})(b_{11} + b_{22}), \\
m_3 &= (a_{11} - a_{21})(b_{11} + b_{12}), \\
m_4 &= (a_{11} + a_{12})b_{22}, \\
m_5 &= a_{11}(b_{12} - b_{22}), \\
m_6 &= a_{22}(b_{21} - b_{11}), \\
m_7 &= (a_{21} + a_{22})b_{11}.
\end{aligned}$$

Then compute the c_{ij}'s, using the formulas

$$\begin{aligned}
c_{11} &= m_1 + m_2 - m_4 + m_6, \\
c_{12} &= m_4 + m_5, \\
c_{21} &= m_6 + m_7, \\
c_{22} &= m_2 - m_3 + m_5 - m_7.
\end{aligned}$$

```
    begin
1.      for i ← 1 until ⌈n/m⌉ do
            begin
                comment We compute the sums of the rows b_1^(i), . . . , b_m^(i)
                  of B_i;
2.              ROWSUM[0] ← [0, 0, . . . , 0];   (n zeros)
3.              for j ← 1 until 2^m − 1 do
                    begin
4.                      let k be such that 2^k ≤ j < 2^(k+1);
5.                      ROWSUM[j] ← ROWSUM[j − 2^k] + b_(k+1)^(i)†
                    end;
6.              let C_i be the matrix whose jth row, 1 ≤ j ≤ n, is
                  ROWSUM[NUM(a_j)], where a_j is the jth row of A_i
            end;
7.      let C be Σ_(i=1)^⌈n/m⌉ C_i
    end
```

† Bitwise Boolean sum is meant here, of course.

Fig. 6.6. Four Russians' algorithm.