PICARD WORKSHOP UTRECHT 2019

1. Talk: Picard Groups [Magdalena]

Introduce Picard group of a symmetric monoidal category and a symmetric monoidal ∞-category, including a reminder of the vocabulary. Present an example of a commutative ring whose Picard group is non-trivial. Discuss that the Picard group of line bundles on a paracompact space are classified by the first cohomology with $\mathbb{Z}/2$-coefficients. Introduce the notion of the Picard group for an E_∞-ring and show that it's a set. Follow [4, §2.1] and [6].

2. Talk: Picard spaces and Picard spectra [Luca]

Show that we obtain a Picard *spectrum* functor from the ∞-category of symmetric monoidal ∞-categories to connective spectra. On your way, recall what some of these words mean, and also some of the reasoning from the last workshop saying that the result is actually a spectrum. Discuss the Picard spectrum of $H\mathbb{F}_2$. You can also discuss the example of the Picard space of the symmetric monoidal ∞-category of line bundles on a paracompact space. Give a motivation for studying Picard spaces by constructing corresponding Thom spectra. Follow [4, §2.2] and [1, Defintion 1.4] (note that they denote $\mathcal{P}ic(R)$ by R-line).

3. Talk: Classical descent results [Viktoriya]

Prove [4, Proposition 2.2.3] and [4, Proposition 2.3.2]. Discuss also classical Galois and faithfully flat descent.

4. Talk: Picard group of connective spectra [Tommy]

Discuss the Picard group of connective spectra. Take the opportunity to remind us of some classical spectra ko, ku, MU and compute their Picard groups. In particular, (re)introducing MU in this way as an application of our Thom spectra construction. Follow [4, §2.4].

5. Talk: Picard spectra and orientations [Joost]

This talk should present more material on the relationship between Picard spectra, $\mathfrak{gl}_1$, Thom spectra and orientability of ring spectra. (See e.g. [1])

6. Talk: Picard group of even periodic spectra [Hadrian]

Discuss the weakly even periodic case, including proof and examples.
Follow [4, §2.4] and [2, Theorem 38] and [3, §6.2]

7. Talk: Galois extensions of ring spectra and HFPSS [Achim]

Discuss Galois extensions of ring spectra due to Rognes and Galois descent for Picard groups. Explain the *statement* of homotopy fixed point spectral sequence (HFPSS). Follow [4, §3.3]

8. Talk: Truncated spectra and truncated spaces [Lyne]

Discuss the relation of the units in a ring spectrum and the spectrum itself in a range, and the relation of truncated spectra to truncated spaces.

Follow [4, §5.1]

9. Talk: Units in a ring spectrum [Mingcong]

Discuss the example illustrating the units in a ring spectrum and their relation to Picard groups and also prove Corollary 5.2.3. If you come up with another example, you can also include it.

Follow [4, §5.2]

10. Talk: Differentials in HFPSS [Yuqing]

Prove the first comparison tool, transporting differentials from the HFPSS of the original spectrum to the HFPSS for Picard spectra [4, Comparison Tool 5.2.4], and draw the consequence Theorem E. Moreover state [4, Theorem 6.1.1], our second comparison tool, without proof.

11. Talk: Introduction to TMF and friends [Jack]

Give a reminder on classical modular forms, elliptic curves and the Landweber exact functor theorem. Construct in particular $TMF(2)$ and $TMF(3)$.

12. Talk: TMF and Galois extensions [Lennart]

Talk about TMF and why we have Galois extensions $TMF[\frac{1}{2}] \to TMF(2)$ and $TMF[\frac{1}{3}] \to TMF(3)$.

13. Talk: HFPSS for TMF [Dimitar]

Do the necessary computations to have the E_2-term of the homotopy fixed point spectral sequences computing $\pi_* TMF[\frac{1}{2}]$ and $\pi_* \mathfrak{pic}(TMF[\frac{1}{2}])$. Follow [5, Section 10.1] (as summarized in [4, Section 8.1]) and [4, Appendix A].

14. Talk: Picard Group of TMF [Lennart]

Compute the Picard group of TMF, following [4, Section 8].

References

[1] Matthew Ando, Andrew J. Blumberg, David Gepner, Michael J. Hopkins, and Charles Rezk. An ∞-categorical approach to R-line bundles, R-module Thom spectra, and twisted R-homology. *J. Topol.*, 7(3):869–893, 2014.

[2] Andrew Baker and Birgit Richter. Invertible modules for commutative $\mathbb{S}$-algebras with residue fields. *Manuscripta Math.*, 118(1):99–119, 2005.

[3] Michael A. Hill and Lennart Meier. The C_2-spectrum $\mathrm{Tmf}_1(3)$ and its invertible modules. *Algebr. Geom. Topol.*, 17(4):1953–2011, 2017.

[4] Akhil Mathew and Vesna Stojanoska. The Picard group of topological modular forms via descent theory. *Geom. Topol.*, 20(6):3133–3217, 2016.

[5] Vesna Stojanoska. Duality for topological modular forms. *Doc. Math.*, 17:271–311, 2012.

[6] Qiaochu Yuan. Picard groups. `https://qchu.wordpress.com/2014/10/19/the-picard-groups/`.