Workshop on Multiplicative Infinite Loop Space Machines
Utrecht March 12 - 16
Program

The aim of this small meeting is to bring together young researchers from Europe interested in the subject of infinity categories and their application to K-theory. More concretely, it will be a workshop with the aim to understand multiplicative infinite loop space theory in the context of ∞-categories. This will be based on the recent paper *Universality of multiplicative infinite loop space machines* by Gepner, Groth and Nikolaus [GGN15]. Further important background sources are [Lur09b] and [Lur12].

List of talks

Below you find the list of talks. Each talk is planned to be about 75 minutes long plus time for questions.

1. **Overview (Lennart Meier):** Where are we going, where are we coming from?

2. **Symmetric monoidal ∞-categories (Lyne Moser):** Recall from our last workshop what a symmetric monoidal ∞-category is and what a commutative monoid object (called E_∞-monoid in [GGN15]). The most accessible source might be version 3 of [Lur09a], with the relevant information roughly up to Proposition 2.14. Then go through Section 1 of [GGN15], possibly leaving out all remarks.

3. **Topological operads (Luca Pol):** Introduce in an example-based way what a (topological) operad and an algebra over it are. The most important examples for us are the E_n-operads (up to $n = \infty$) – other examples one might consider are the commutative or the Lie operad. A quick introduction can be found in [Bel17] and more background in [MSS02], Sections 1.2, 1.4, 2.1 and 2.2. The recognition principle for n-fold loop spaces should be mentioned (and maybe used as a motivation), but you do not have to treat it in detail.

4. **∞-operads (Gijs Heuts):** Give an introduction to ∞-operads in a way suitable for the needs of the paper [GGN15].

5. **(Pre-)additive ∞-categories (Paula Verdugo):** Treat Section 2 of [GGN15].

6. **(Smashing) localizations (Dimitar Kodjabachev):** Introduce the theory of (smashing) localizations in ∞-categories, following [GGN15], Section 3 up to Remark 3.7. Feel free to give some examples of localizations to show how ubiquitous they

are in mathematics (e.g. classical localizations in algebra or Bousfield localizations in topology – not assuming that everyone knows what Johnson–Wilson theories are).

7. **The ∞-category of presentable ∞-categories and its localizations (Jordan Williamson):** Recall what a presentable ∞-category and the ∞-category $\mathcal{P}r^L$ are. Treat the rest of Section 3 of [GGN15].

8. **Commutative monoids and groups as smashing localizations (Joost Nuiten):** Do Section 4 of [GGN15].

9. **Other models for group completion (Peter Dawson):** Give the classical construction of ΩB and perhaps $S^{-1}S$ for group completions.

10. **Canonical symmetric monoidal structures (Magdalena):** Do Section 5 of [GGN15].

11. **More functoriality and lax symmetric monoidal structures (Kay Werndli):** Do Section 6 of [GGN15]

12. **∞-categories of semirings and rings (Brice):** Do Section 7 of [GGN15]

13. **What is algebraic K-theory? (Martina Rovelli)** Give an introduction to what algebraic K-theory is and show that it is lax symmetric monoidal as a functor from symmetric monoidal ∞-categories to spectra; follow [GGN15], Section 8 up to Theorem 8.6, and whatever additional sources you find worthwhile. This talk is of particular importance as it connects our more technical work so far with the main motivations for this subject.

14. **Complements and Examples (Viktoriya Ozornova):** Treat the rest of Section 8, which particularly allows us to apply Theorem 8.6 to actual examples!

15. **Summary and outlook (Lennart Meier)**

Schedule

Monday	lunch, 2pm-5: Talks 1 and 2
Tuesday	9.30-12.30: Talks 3 and 4, lunch, 2pm-5: Talks 5 and 6
Wednesday	9.30-12.30: Talks 7 and 8, lunch, 2pm-5: Talks 9,10 and 11
Thursday	9.30-12.30: Talks 12 and 13, lunch, free afternoon
Friday	9.30-12.30: Talks 14 and 15, lunch.

References

[Bel17] Eva Belmont. A quick introduction to operads. http://math.mit.edu/~ebelmont/operads-talk.pdf, 2017. 1

[GGN15] David Gepner, Moritz Groth, and Thomas Nikolaus. Universality of multiplicative infinite loop space machines. *Algebr. Geom. Topol.*, 15(6):3107–3153, 2015. 1, 2

[Lur09a] J. Lurie. Derived Algebraic Geometry III: Commutative Algebra. *Arxiv preprint arXiv:math/0703204v3*, 2009. 1

[Lur09b] Jacob Lurie. *Higher topos theory*, volume 170 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 2009. 1

[Lur12] Jacob Lurie. Higher algebra. `http://www.math.harvard.edu/~lurie/papers/HigherAlgebra.pdf`, 2012. 1

[MSS02] Martin Markl, Steve Shnider, and Jim Stasheff. *Operads in algebra, topology and physics*, volume 96 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2002. 1