INTRODUCTION TO PROBABILITY IN AN INCLUSIVE SETTING - INSIGHTS BY A STUDENT WITH LEARNING DIFFICULTIES

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Learning on a common subject is seen as fundamental for inclusive education. However, it is still unclear how to design individual learning sequences while maintaining a common subject in order to offer students the chance to benefit from diversity in cooperative learning. This contribution focuses on a learning environment which introduces seventh grade students to probability using the frequentist interpretation. Based on data from a prestudy, the insights into probability by Yasim, who is a student with difficulties in learning mathematics, are going to be presented.

INTRODUCTION

Since 2006, when Germany signed the Convention on the Rights of Persons with Disabilities (UN, 2006), increasingly more students with special needs are taught in regular schools, instead of special schools for students with disabilities. Therefore, teachers in regular schools are confronted with extremely heterogenous groups of students. If we want to learn more about teaching and learning in inclusive mathematics classrooms, a closer look at students with difficulties in learning mathematics is needed (Scherer, Beswick, DeBlois, Healy, & Moser Opitz, 2016).

The idea of inclusion is to take advantage of students' diversity in cooperative learning situations. In order to design mathematical learning environments for inclusive settings different authors have taken up the ideas of Feuser (1997) (e.g. Schöttler & Häsel-Weide, 2017). Feuser claims that learning in inclusive settings needs a common subject. This common subject is not the specific object of learning but refers to the process behind all observations. Learning on a common subject means that every student can work individually on the same mathematical phenomenon according to their individual abilities. In addition, reflecting and talking about a common subject is essential for learning mathematics (Feuser, 1997; Schöttler & Häsel-Weide, 2017).

A LEARNING ENVIRONMENT AS INTRODUCTION TO PROBABILITY

Before students are taught probability in school, they have already experienced probability on various occasions in their everyday life, often not appropriating to the formal mathematical concept of probability. In order to create a correct comprehension of formal probability students have to perform random experiments and reflect their outcomes (Fischbein, 1982). The learning environment 'racing colors' based on a betting game as referred to as 'betting king' (Prediger & Rolka, 2009) is the focus of this research. The betting context creates excitement among the students to find out, which betting strategy is the best and allows insights into the students' way of thinking. One game has three rounds of betting. In the first round the dice is rolled only once, in the second round five times and in the third round twenty times. Depending on their individual abilities, students play with a ten-sided or twenty-sided dice. The dice have red, yellow and blue colored sides (ten-sided dice: 5

red, 3 yellow, 2 blue; twenty-sided dice: 10 yellow, 5 red, 5 blue). For both dice, there is a 50-50 chance, that the most existing color on the dice wins in a single throw. The color distribution and the number of throws in the game are selected because they allow probable scores that enhance the understanding of the distinction between a long-term and short-term context (the most existing color on the dice will win with a probability of over 0.85, if the dice are rolled twenty times). That even a number of twenty throws must be considered as a short-term in the context of probability, can be reflected at the latest when the students simulate a large number of throws using the computer program. Noticing the distinction and patterns of results in long-term context is needed for a meaningful learning of probability (Schnell & Prediger, 2012).

A central cooperative moment of the learning environment is when students, who played with the ten-sided dice and students, who played with the twenty-sided dice, jointly answer the questions: ‘What is equal?’ and ‘What is different?’. The cooperative learning situation enables students to exchange their conceptions through presenting their findings to each other. Different interpretations of probability (Konold, 1991) are the common subject. The subject on which students exchange their perspective and benefit from each other. While comparing their strategies and scores, the students should understand that the most existing color on the dice wins most often if the dice are rolled twenty times. In addition, they should recognize that in the case of one or five throws every color can be the winner. For both observations it does not matter whether the game is played with the ten-sided or twenty-sided dice. If the cooperative situation is supposed to help students get a deeper understanding of probability, students need to have made some findings during their individual learning sequences beforehand. More specifically, it is necessary that students keep an overview of the colors and know which color won most often in different rounds. It could be advantagous if the students previously had the idea that the scores are depending on the color distribution of the dice. If they had not, this idea could be communicated in the interaction with other students. Many students use their individual resources in form of an ordinal intuition on probability to recognize the most likely color after a short time (Prediger & Rolka, 2009).

The learning environment allows students with different learning preconditions to experience probability from the frequentist interpretation (Konold, 1991). An aim of the learning environment is to connect the theoretical (ibid.) to the frequentist interpretation of probability. Concerning the connection of the two interpretations, the game ‘racing colors’ is played with symmetric random generators. With the help of a systematic analysis of the dice and a computer program, the stabilization of relative frequencies is used to define the probability of an event’s occurrence. Yet, further research is needed to be able to make indications about the way students with difficulties in learning mathematics can understand the connection between frequentist and theoretical interpretation. It is possible that the connection can only be a realistic learning goal for students with good performances in mathematics.

A first study with grade seven of a comprehensive school investigates the question ‘Which insights into probability does Yasim, a student with learning difficulties, show within ‘racing colors’?’.

INSIGHTS INTO PROBABILITY BY YASIM

In the following, some results of Yasim, a student with difficulties in learning mathematics, and possible interpretations will be provided in order to understand his learning pathway related to the learning environment. Yasim and Emma, a student with average mathematic skills, played the game 'racing colors' together three times and documented the results of each round. After playing the game, the students reflect their own betting strategy before talking to each other. Yasim starts to look intensively at the ten-sided dice. Spinning the dice in his hands, it seems as if Yasim is counting the different sides painted in the same color. Emma starts a conversation by presenting her betting strategy 'always bet on the most existing color' to Yasim, who has the same one ('red…because it's more, so many points'). His betting strategy is based on his findings by analyzing the dice because the protocols of the games before have shown that he always bet on different colors (Fig. left).

Round 1

Number of throws	I bet on...	This color won	Points
1	yellow	blue	0
5	yellow	blue	0
20	red	red	1

Round 2

Number of throws	I bet on...	This color won	Points
1	red	blue	0
5	blue	red/blue	1
20	red	red/yellow	1

Round 3

Number of throws	I bet on...	This color won	Points
1	blue	yellow	0
5	red	yellow	0
20	yellow	red	0

1f) For which number of throws can you bet most certainly? For which do you bet uncertainly? Write your thoughts down and give reasons.

On red I am certain/most uncertain is blue.
On 20 I am certain on red, on 5 I am certain on yellow,
on 1 I am certain on blue.

Figure left and right: Excerpts from Yasims exercise book

The sequence shows that Yasim has an ordinal intuition on probability because he explains that red is a good bet by referring to the number of red dots on the dice. Even though red is the most certain bet for each round, the probability of the event 'red wins' is also depending on the number of throws. At this point, there is no indication that Yasim and Emma understood that the number of throws has an impact. In addition, task 1f) asks specifically for a number of throws allowing a certain bet. Considering Yasims answer, an interesting way of thinking becomes obvious (Fig. right). First, he notes that a bet on red is certain and a bet on blue is most uncertain, which refers to the color distribution of the dice. So he can indicate which color is the most likely and which one is less likely but he does not draw conclusions for the probability of yellow which has less dots than red and more dots than blue. After the teacher indicated that the task asks for the number of throws on which one can bet certainly, he completes his answer. Red is the most existing color on the dice, and he concludes that he can bet certainly on red if the dice is rolled twenty times, which is the highest number of throws in the betting protocol. If the dice is rolled only once, he concludes - in accordance with the realization that the number of blue dots has least sides - that blue will be the certain bet. Yellow has more sides than blue but less than red, so he concludes that he can certainly bet on yellow if the dice is rolled five times. Yet, it is confusing that the results of the game sequence documented in the betting protocols (Fig. left) do not correspond to his idea of certain bets. Especially in rounds, in which the dice was rolled five times, yellow won only once, which is no indication for a certain bet. This indicates that Yasim does not use his previous gaming experience.

CONCLUSIONS AND OUTLOOK

Considering Yasims answers and actions it seems obvious that he has an ordinal intuition on probability. In his perception the probability of getting individual colors depends on the color distribution of the dice. After counting the dots, he only observed that the most likely color is red, and the least likely color is blue. Nevertheless, he did not use these insights in the form of the theoretical interpretation of probability. Furthermore, Yasim did not get insights into the distinction between a short-term and long-term context; neither in the individual part nor in interaction with Emma. Though, the scores provide almost perfect conditions. In order to enable Yasim to gain insights into the distinction between a short-term and long-term context, it can be suitable to suggest him to compare his answers to his scores. Another possibility could be to encourage an interaction with Emma so that he needs to argue and reflect on his answers by presenting them to her. Emma; however, has the chance to explain her findings to Yasim and stimulate his thinking about alternative answers. In the interaction about their results both can gain a better understanding of the frequentist interpretation of probability. This understanding could further be supported in cooperative learning with other students that is structured by questions concerning the differences and similarities of the two dice. To obtain a deeper comprehension of the students' learning pathways, further steps of research involve clinical interviews (Hunting, 1997) of students with different learning preconditions. Moreover, group interviews are planned to get a better idea of the opportunities of interaction. These results give important hints for the further design process of the learning environment and teaching in inclusive settings.

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