Joint circular distributions in view of higher order spectra of time series
MASANOBU TANIGUCHI (WASEDA UNIVERSITY, TOKYO)

Circular data analysis is emerging as an important component of statistics. For this half century, various circular distributions have been proposed, e.g., von Mises distribution, wrapped Cauchy distribution, among other things. Also, regarding the joint distribution, Wehrly and Johnson (1980) proposed a bivariate circular distribution which is related to a family of Markov processes on the circle. Because the sample space is on a circle, various new statistical methods have been developed. In this talk we provide a new look at circular distributions in view of spectral distributions of time series because the typical circular distributions correspond to spectral densities of time series models. For example, autoregressive AR(1) spectral density corresponds to wrapped Cauchy distribution, and von Mises distribution corresponds to exponential spectral density (Bloomfield (1973)), etc. Furthermore we introduce a class of joint circular distributions from the higher order spectra of time series, which can describe very general joint circular distributions. Hence we can develop the statistical inference for dependent observations on the circle. We present a family of distributions on the circle derived from the ARMA spectral density. It is seen that the proposed family includes some existing circular families as special cases. For these special cases, the normalizing constant and trigonometric moments are shown to have simple and closed form. We develop the asymptotic optimal inference theory based on the local asymptotic normality (LAN) on the circle. Because the observations are permitted to be dependent, the theory opens a new paradigm in the estimation for joint circular distributions.

This is joint work with Shogo Kato (Institute of Statistical Mathematics, Tokyo), Hiroaki Ogata (Tokyo Metropolitan University), and Arthur Pewsey (University of Extremadura, Spain).