Assessing the linear dependence of high-dimensional time series via autocovariances and autocorrelations

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In the first part of this talk, we provide asymptotic theory for certain functions of the sample autocovariance matrices of a high-dimensional time series with infinite fourth moment. The time series exhibits linear dependence across the coordinates and through time. Assuming that the dimension increases with the sample size, we provide theory for the eigenvectors of the sample autocovariance matrices and find explicit approximations of a simple structure, whose finite sample quality is illustrated for simulated data. We also obtain the limits of the normalized eigenvalues of functions of the sample autocovariance matrices in terms of cluster Poisson point processes. In turn, we derive the distributional limits of the largest eigenvalues and functionals acting on them.

In the second part, we consider the sample correlation matrix R associated to n observations of a p-dimensional time series. In our framework, we allow that p/n may tend to 0 or a positive constant. If the time series has a finite fourth moment, we show that the sample correlation matrix can be approximated by its sample covariance counterpart for a wide variety of models. This result is very important for data analysts who use principal component analysis to detect some structure in high-dimensional time series. From a theoretical point of view, it allows to derive a plethora of ancillary results for functionals of the eigenvalues of R. For instance, we determine the almost sure behavior of the largest and smallest eigenvalues, and the limiting spectral distribution of R.

The optimal condition for the convergence of the empirical spectral distributions turns out to be slightly weaker than normal domain of attraction. In the case of time series with infinite $(2-\epsilon)$-moments, a new class of Marčenko–Pastur type laws appears as limiting spectral distributions of R.

The talk is based on joint work with Thomas Mikosch (University of Copenhagen), Alexander Aue (UC Davis), Paul Debashis (UC Davis), Jianfeng Yao (University of Hong Kong) and Richard A. Davis (Columbia University New York).