Asymptotics of Cholesky GARCH Models and Time-Varying Conditional Betas

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Motivation

Problem: Given some information set $\mathcal{F}_{t-1}$, it is often of interest in financial applications to regress $y_t$ (asset returns) on the components of a vector $x_t$ (factors).

Solution: $y_t - E(y_t \mid \mathcal{F}_{t-1}) = \beta'_{y,x,t} \{x_t - E(x_t \mid \mathcal{F}_{t-1})\} + \eta_t,$

with the dynamic conditional beta (DCB) is given by:

$$\beta_{y,x,t} = \Sigma_{x,x,t}^{-1} \Sigma_{x,y,t}$$
Practical implementations:

1. an ARCH-type model for the conditional variance:

\[
\begin{pmatrix} \Sigma_{xx,t} & \Sigma_{xy,t} \\ \Sigma_{yx,t} & \Sigma_{yy,t} \end{pmatrix}
\]

of

\[
\epsilon_t = \begin{pmatrix} x_t - E(x_t | \mathcal{F}_{t-1}) \\ y_t - E(y_t | \mathcal{F}_{t-1}) \end{pmatrix}
\]

2. or a direct specification of $\beta_{yx,t}$?
Notations

Let \( \epsilon_t = (\epsilon_{1t}, \ldots, \epsilon_{mt})' \) be a vector of \( m \geq 2 \) series satisfying a general volatility model of the form

\[
\epsilon_t = \Sigma_t^{1/2}(\vartheta_0)\eta_t
\]

where \( (\eta_t) \) is i.i.d. \((0, I_m)\) and

\[
\Sigma_t = \Sigma_t(\vartheta_0) = \Sigma(\epsilon_{t-1}, \epsilon_{t-2}, \ldots; \vartheta_0) > 0
\]

where \( \vartheta_0 \) is a \( d \times 1 \) vector
Engle (2012) DCC

\[ \Sigma_t = D_t R_t D_t = (\rho_{ijt} \sqrt{\sigma_{iit} \sigma_{jjt}}) \]

where \( D_t = \text{diag}(\sigma_{11t}^{1/2}, \ldots, \sigma_{mmt}^{1/2}) \) contains the volatilities of the individual returns and \( R_t = (\rho_{ijt}) \) the conditional correlations. The time series model needs to incorporate the complicated constraints of a correlation matrix. One often takes:

\[ R_t = (\text{diag} \ Q_t)^{-1/2} Q_t (\text{diag} \ Q_t)^{-1/2} \]

where

\[ Q_t = (1 - \theta_1 - \theta_2) S + \theta_1 u_{t-1} u'_{t-1} + \theta_2 Q_{t-1} \]

with \( u_t = (u_{1t} \ldots u_{mt})' \), \( u_{it} = \epsilon_{it}/\sqrt{\sigma_{iit}} \), \( \theta_1 + \theta_2 < 1 \)
Engle (2016) DCB

Assuming

\[
\begin{pmatrix}
  x_t \\
  y_t
\end{pmatrix}
| \mathcal{F}_{t-1} \sim N
\left(\begin{pmatrix}
  \mu_{x,t} \\
  \mu_{y,t}
\end{pmatrix},
\begin{pmatrix}
  \Sigma_{xx,t} & \Sigma_{xy,t} \\
  \Sigma_{xy,t} & \Sigma_{yy,t}
\end{pmatrix}\right)
\]

we have

\[y_t | x_t \sim N\left(\mu_{y,t} + \Sigma_{yx,t} \Sigma_{xx,t}^{-1} (x_t - \mu_x,t), \Sigma_{yy,t} - \Sigma_{yx,t} \Sigma_{xx,t}^{-1} \Sigma_{xy,t}\right)\]

\[\Rightarrow \beta_t = \Sigma_{xx,t}^{-1} \Sigma_{xy,t} \] is obtained in tow steps - by first estimating a DCC GARCH model on \((y_t, x_t)\) - NOT a natural way to specify the parameter dynamics!
Drawbacks of DCC-based DCB - 1/2

1) The **stationarity and ergodicity** conditions of the DCC are not well known.

2) The **correlation constraints** are complicated.

3) The **asymptotic properties of the QMLE** are unknown.

4) The effects of the DCC parameters on $\beta_t$ are **hardly interpretable**.
Drawbacks of DCC-based DCBI - 2/2
We now introduce a class of Cholesky GARCH (CHAR) models that avoids all these drawbacks.

We consider the Cholesky decomposition of Pourahmadi (1999)

\[ \Sigma_t = L_t G_t L_t' \]

where \( G_t = \text{diag}(g_{1t}, \ldots, g_{mt}) \) and \( L_t \) is a lower unitriangular matrix (i.e. triangular with 1 on the diagonal) with element \( \ell_{ij,t} \) at the row \( i \) and column \( j \) for \( i > j \)
Example (static case): $\Sigma = LGL'$, $m = 3$

$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$

$G = \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix}$

$\Sigma = \begin{bmatrix} g_{11} & l_{21}g_{11} & l_{31}g_{11} \\ l_{21}g_{11} & l_{21}^2g_{11} + g_{22} & l_{21}l_{31}g_{11} + l_{32}g_{22} \\ l_{31}g_{11} & l_{21}h_{31}g_{11} + l_{32}g_{22} & l_{31}^2g_{11} + l_{32}^2g_{22} + g_{33} \end{bmatrix}$
Structural Interpretation (static case)

Let us introduce the orthogonal basis assets $\mathbf{v} = (v_1, \ldots, v_m)$ with variance $\mathbf{G}$ and such that

$$
\epsilon = \Sigma^{1/2} \eta_t = \mathbf{LG}^{1/2} \eta = \mathbf{Lv} \quad (E)
$$

Line 1 - $V(\epsilon_1) = \sigma_1^2 = g_{11}$

Line 2 - $\text{Cov}(\epsilon_1, \epsilon_2) = \sigma_{21} = \ell_{21} g_{11} \iff \ell_{21} = \frac{\sigma_{21}}{\sigma_1^2} = \beta_2$

We can start to invert $(E)$ : $\epsilon_2 = \beta_{21} \epsilon_1 + v_2 \iff v_2 = \epsilon_2 - \beta_{21} \epsilon_1$

and define:

$$
\mathbf{B} = \mathbf{L}^{-1} = \begin{bmatrix}
1 & 0 & 0 & \ldots \\
-\beta_{21} & 1 & 0 & \ldots \\
\ldots & \ldots & \ldots & \ldots
\end{bmatrix}
$$
More generally, we have for Line $i$

\[
\epsilon_i = \sum_{j=1}^{i-1} \ell_{ij} v_j + v_i
\]

\[
= \sum_{j=1}^{i-1} \beta_{ij} \epsilon_j + v_i \iff v_i = \epsilon_i - \sum_{j=1}^{i-1} \beta_{ij} \epsilon_j
\]

where $v_i$ is uncorrelated to $v_1, \ldots, v_{i-1}$, and thus uncorrelated to $\epsilon_1, \ldots, \epsilon_{i-1}$.
Structural Interpretation (static case)

In matrix form,

\[ \epsilon = L v \quad \text{and} \quad v = B \epsilon, \]

where \( L \) and \( B = L^{-1} \) are lower unitriangular and \( G := \text{var}(v) \) is diagonal.

We obtain the (static) Cholesky decomposition \( \Sigma = LL' \) (see Pourahmadi, 1999).
Structural Interpretation (dynamic setting)

Conditioning on $\mathcal{F}_{t-1}$, we get the (dynamic) Choleski decomposition $\Sigma_t = L_t G_t L_t'$ (and $\Sigma_t^{-1} = B_t' G_t^{-1} B_t$).

We thus need ...

- a diagonal ARCH-type model for the factors vector $v_t$
- a time series model for $L_t$ (or $B_t$), without constraint

... Instead of (with the DCC model)

- a diagonal ARCH-type model for the $\epsilon_t$
- a time series model for $R_t$, with constraints
A model for $L_t$ or $B_t$? How ordering the series?

1. $B_t$ has a direct interpretation

$$B_t = L_t^{-1} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & \ldots \\
-\beta_{21,t} & 1 & 0 & 0 & \ldots \\
-\beta_{31,t} & -\beta_{32,t} & 1 & 0 & \ldots \\
-\beta_{41,t} & -\beta_{42,t} & -\beta_{42,t} & 1 & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots
\end{bmatrix}$$

2. No natural order .... must be driven by the applications
A general model for the factors

Assume

\[ \boldsymbol{v}_t = \boldsymbol{G}_t^{1/2} \boldsymbol{\eta}_t, \quad (\boldsymbol{\eta}_t) \text{ iid } (0, I_n), \]

where \( \boldsymbol{G}_t = \text{diag}(\boldsymbol{g}_t) \) follows a GJR-like equation

\[ \boldsymbol{g}_t = \omega_0 + \sum_{i=1}^{q} \left\{ \mathbf{A}_{0i,+} \boldsymbol{v}_{t-i}^{2+} + \mathbf{A}_{0i,-} \boldsymbol{v}_{t-i}^{2-} \right\} + \sum_{j=1}^{p} \mathbf{B}_{0j} \boldsymbol{g}_{t-j}, \]

with positive coefficients and

\[ \boldsymbol{v}_t^{2+} = \left( \{ v_{1t}^+ \}^2, \cdots, \{ v_{mt}^+ \}^2 \right)', \quad \boldsymbol{v}_t^{2-} = \left( \{ v_{1t}^- \}^2, \cdots, \{ v_{mt}^- \}^2 \right)'. \]
Markovian representation of the factors

Letting $z_t = \left( v_{t:(t-q+1)}^{2+}, v_{t:(t-q+1)}^{2-}, g_t^{:(t-p+1)} \right)'$, 
$h_t = \left( \omega'_0 \gamma_t^+, 0_{m(q-1)}, \omega'_0 \gamma_t^-, 0_{(q-1)m}, \omega'_0, 0_{(p-1)m} \right)'$, with 
$\gamma_t^+ = \text{diag} \left( \eta_{t}^{2+} \right)$ $\gamma_t^- = \text{diag} \left( \eta_{t}^{2-} \right)$ and obvious notations, we rewrite the model as 

$$z_t = h_t + H_t z_{t-1},$$

where, in the case $p = q = 1$, 

$$H_t = \begin{pmatrix}
\gamma_t^+ A_{01,+} & \gamma_t^+ A_{01,-} & \gamma_t^+ B_{01} \\
\gamma_t^- A_{01,+} & \gamma_t^- A_{01,-} & \gamma_t^- B_{01} \\
A_{01,+} & A_{01,-} & B_{01}
\end{pmatrix}.$$
Stationarity of the factors

In view of

\[ z_t = h_t + H_t z_{t-1}, \]

there exists a stationary and ergodic sequence \((v_t)\) satisfying \(v_t = G_t^{1/2} \eta_t\) if and only if

\[ \gamma_0 = \inf_{t \geq 1} \frac{1}{t} \mathbb{E}(\log \|H_t H_{t-1} \cdots H_1\|) < 0. \]
Stationarity of $\beta_t := -\text{vech}^0 B_t$

If $(\mathbf{v}_t)$ is stationary and ergodic ($\gamma_0 < 0$), and

$$\det \left\{ I_{m_0} - \sum_{i=1}^{s} C_{0i} z^i \right\} \neq 0 \text{ for all } |z| \leq 1,$$

then

$$\beta_t = c_0 \left( \mathbf{v}_{t-1}, \ldots, \mathbf{v}_{t-r}, g^{1/2}_{t-1}, \ldots, g^{1/2}_{t-r} \right) + \sum_{j=1}^{s} C_{0j} \beta_{t-j}.$$

defines a stationary and ergodic sequence (and thus the existence of a stationary CHAR model)
Existence of moments

If in addition

\[ E\| \eta_1 \|^{2k_1} < \infty \quad \text{and} \quad \varrho(EH_1^{\otimes k_1}) < 1, \]

for some integer \( k_1 > 0 \), and

\[ \| c_0(x) - c_0(y) \| \leq K \| x - y \|^a \]

for some constants \( K > 0 \) and \( a \in (0, 1] \), then the CHAR model satisfies \( E \| \epsilon_1 \|^{2k_1} < \infty \).
A simpler triangular parameterization

A tractable submodel is

\[ g_{it} = \omega_0 + \gamma_0 + \left( \epsilon_{1,t-1}^+ \right)^2 + \gamma_0 - \left( \epsilon_{1,t-1}^- \right)^2 + \sum_{k=2}^{i} \alpha_{0i}^{(k)} v_{k,t-1}^2 + b_{0i} g_{i,t-1} \]

with positivity coefficients, and

\[ \beta_{ij,t} = \omega_{0ij} + \varsigma_{0ij} + \epsilon_{1,t-1}^+ + \varsigma_{0ij} - \epsilon_{1,t-1}^- + \sum_{k=2}^{i} \tau_{0ij}^{(k)} v_{k,t-1} + c_{0ij} \beta_{ij,t-1} \]

without positivity constraints. Notice the triangular structure and note that the asymmetry is introduced via the first (observed) factor only.
Stationarity for the previous specification

There exists a strictly stationary and ergodic solution to the CHAR model when

1) $E \log \left\{ \omega_{01} + \gamma_{01} + \left( \eta_{1,t-1}^+ \right)^2 + \gamma_{01} - \left( \eta_{1,t-1}^- \right)^2 + b_{01} \right\} < 0,$

2) $E \log \left\{ \alpha_{0i}^{(i)} \eta_{it}^2 + b_{0i} \right\} < 0$ for $i = 2, \ldots, m,$

3) $|c_{0ij}| < 1$ for all $(i, j).$

Moreover, the stationary solution satisfies $E\|\epsilon_1\|^{2s_0} < \infty,$ $E\|g_1\|^{s_0} < \infty,$ $E\|v_1\|^{s_0} < \infty,$ $E\|\beta_1\|^{s_0} < \infty$ and $E\|\Sigma_1\|^{s_0} < \infty$ for some $s_0 > 0$
Full QMLE of the general CHAR

A QMLE of the CHAR parameter $\vartheta_0$ is

$$\hat{\vartheta}_n = \arg \min_{\vartheta \in \Theta} \tilde{O}_n(\vartheta), \quad \tilde{O}_n(\vartheta) = n^{-1} \sum_{t=1}^{n} \tilde{q}_t(\vartheta),$$

where $\tilde{\Sigma}_t(\vartheta) = \Sigma (\epsilon_{t-1}, \ldots, \epsilon_1, \tilde{\epsilon}_0, \tilde{\epsilon}_{-1}, \ldots; \vartheta)$ and

$$\tilde{q}_t(\vartheta) = \epsilon_t' \tilde{B}_t(\vartheta) \tilde{G}_t^{-1}(\vartheta) \tilde{B}_t(\vartheta) \epsilon_t + \sum_{i=1}^{m} \log \tilde{g}_{it}(\vartheta).$$

- Does not require matrix inversion
- CAN under general regularity conditions
Consider the triangular model. In a first step, the parameter 
\( \vartheta^{(1)}_{0} = (\omega_{01}, \gamma_{01+}, \gamma_{01-}, b_{01}) \) is estimated by

\[
\hat{\vartheta}^{(1)}_{n} = \arg \min_{\vartheta^{(1)} \in \Theta^{(1)}} \sum_{t=1}^{n} \tilde{q}_{1t}(\vartheta^{(1)}),
\]

where

\[
\tilde{q}_{1t}(\vartheta^{(1)}) = \frac{\epsilon_{1t}^{2}}{\tilde{g}_{1t}(\vartheta^{(1)})} + \log \tilde{g}_{1t}(\vartheta^{(1)}),
\]

and 
\( \tilde{g}_{1t}(\vartheta^{(1)}) = \omega_{1} + \gamma_{1+} (\epsilon_{1t}^{+})^{2} + \gamma_{1-} (\epsilon_{1t}^{-})^{2} + b_{1} \tilde{g}_{1,t-1}(\vartheta^{(1)}). \)
EbE second step

Let \( \vartheta_{0}^{(2)} = (\varphi_{0}^{(2)}, \theta_{0}^{(2)}) \), where \( \tilde{\beta}_{21, t} = \tilde{\beta}_{21, t}(\varphi_{0}^{(2)}) \) and \( \tilde{g}_{2 t} = \tilde{g}_{2 t}(\theta_{0}^{(2)}) \).

Independently or in parallel to \( \vartheta_{0}^{(1)} \), one can estimate \( \vartheta_{0}^{(2)} \) by

\[
\hat{\vartheta}_{n}^{(2)} = \arg \min_{\vartheta^{(2)} \in \Theta^{(2)}} \sum_{t=1}^{n} \tilde{q}_{2 t}(\vartheta^{(2)}),
\]

where, for \( t = 1, \ldots, n \),

\[
\tilde{q}_{2 t}(\vartheta^{(2)}) = \frac{\tilde{v}_{2 t}^{2}(\varphi^{(2)})}{\tilde{g}_{2 t}(\vartheta^{(2)})} + \log \tilde{g}_{2 t}(\vartheta^{(2)}),
\]

\[
\tilde{g}_{2 t}(\vartheta^{(2)}) = \omega_{2, t-1} + \alpha_{2}^{(2)} \tilde{v}_{2, t-1}(\varphi^{(2)}) + b_{2} \tilde{g}_{2, t-1}(\varphi^{(2)}),
\]

\[
\tilde{v}_{2 t}(\varphi^{(2)}) = \epsilon_{2 t} - \tilde{\beta}_{21, t}(\varphi^{(2)}) \epsilon_{1 t},
\]

\[
\tilde{\beta}_{21, t}(\varphi^{(2)}) = \omega_{21, t-1} + \tau_{21}^{(2)} \tilde{v}_{2, t-1}(\varphi^{(2)}) + c_{21} \tilde{\beta}_{21, t-1}(\varphi^{(2)}).
\]
EbE remaining steps

For \( i \geq 3 \), \( \tilde{\beta}_{ij,t} \) depends on \( \varphi_0^{(i)} = (\varphi_0^{(i)}, \varphi_0^{(-i)}) \), where \( \varphi_0^{(-i)} \) has been estimated in the previous steps. The volatility \( \tilde{g}_{it} \) depends on \( \vartheta_0^{(+i)} = (\theta_0^{(i)}, \varphi_0^{(+i)}) \), and \( \vartheta_0^{(i)} = (\theta_0^{(i)}, \varphi_0^{(i)}) \) can be estimated by

\[
\tilde{\vartheta}_n^{(i)} = \arg \min_{\vartheta^{(i)} \in \Theta^{(i)}} \sum_{t=1}^{n} \tilde{q}_{it}(\vartheta^{(i)}, \tilde{\varphi}_n^{(-i)}), \quad \tilde{q}_{it}(\vartheta^{(i)}) = \frac{\tilde{V}_{it}^2(\varphi^{(+i)})}{\tilde{g}_{it}^{(i)}} + \log \tilde{g}_{it}(\vartheta^{(+i)}),
\]

\[
\tilde{g}_{it}(\vartheta^{(+i)}) = \omega_{i,t-1} + \sum_{k=2}^{i} \alpha_{i}^{(k)} \tilde{V}_{k,t-1}^{2}(\varphi^{(+k)}) + b_i \tilde{g}_{i,t-1}(\vartheta^{(+i)}),
\]

\[
\tilde{V}_{kt}(\varphi^{(+k)}) = \epsilon_{kt} - \sum_{j=1}^{k-1} \tilde{\beta}_{kj,t}(\varphi^{(+k)}) \epsilon_{jt},
\]

\[
\tilde{\beta}_{ij,t}(\varphi^{(+i)}) = \omega_{ij,t-1} + \sum_{k=2}^{i} \tau_{ij}^{(k)} \tilde{V}_{k,t-1}(\varphi^{(+k)}) + c_{ij} \tilde{\beta}_{ij,t-1}(\varphi^{(+i)}),
\]
QML vs. EbE

1) If $m = 2$, the one-step full QMLE and the two-step EbEE are exactly the same.

2) For $m \geq 3$, the two estimators are generally different.

3) The QML and EbE estimators are CAN under similar assumptions.

4) The EbEE is simpler, but is not always less efficient than the full QMLE.
Applications - Asset Pricing for Industry Portfolios

We consider the 12 industry portfolios returns $r_{j,t}$ used by Engle (2016) examined in the context of the Fama French 3 factor model

$$r_{j,t} = \alpha_j + \beta_{j,m,t}r_{m,t} + \beta_{j,hml,t}r_{hml,t} + \beta_{j,smb,t}r_{smb,t} + u_{j,t}$$

where $r_{smb,t}$ (Small Minus Big) is the average return on small portfolios minus the average return on big portfolios $r_{hml,t}$ (High Minus Low) is the average return on value portfolios minus the average return on growth portfolios.
Let $\epsilon_t = (x_t, y_t)'$ with $x_t = (r_{m,t}, r_{hml,t}, r_{smb,t})'$ and $y_t = r_{j,t}$

**CCC-GARCH(1,1)** and **DCC-GARCH(1,1)**. In-sample and out-of-sample one-step-ahead forecasts of conditional betas estimates of the DCB models obtained using $\Sigma_{yx,t} \Sigma_{xx,t}^{-1}$ and $\Sigma_{yx,t+1|t} \Sigma_{xx,t+1|t}^{-1}$.

**CHAR** with constant betas $\beta_{ij,t} = \beta_{ij} \forall t$ and time varying betas

$\beta_{ij,t} = \omega_{ij} + \tau_{ij} V_{i,t-1} V_{j,t-1} + c_{ij} \beta_{ij,t-1}$
Buseq: Business Equipment

Motivation

Cholesky-GARCH models

Theoretical Results

Estimation

Applications
Buseq: One-step ahead forecasts

\[ Z_{k,t+1|t} = \beta_{k,MKT,t+1|t} MKT_{t+1} + \beta_{k,SMB,t+1|t} SMB_{t+1} + \beta_{k,HML,t+1|t} HML_{t+1}, \]

\[ TE_{k,t+1} = r_{k,t+1} - Z_{k,t+1|t} \]
Motivation Cholesky-GARCH models Theoretical Results Estimation Applications

Transaction costs: \[ \frac{\Delta \beta_{\text{CHAR}}}{\Delta \beta_{\text{DCC-DCB}}} \]

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\[ \Delta \beta_{k,j} = \sum_{t=2}^{1,678} |\beta_{k,j,t+1} - \beta_{k,j,t}|_{t-1}. \]

For each column, the figures correspond to the ratio between the value of \( \Delta \beta_{k,j} \) obtained for the CHAR and the DCC-DCB models.
Conclusion

Compare to other multivariate GARCH, the Cholesky-GARCH models introduced here have several advantages.

1) Precise **stationarity and moment conditions** exist
2) The parameters are **directly interpretable** in terms of DCB
3) There is no complicated **correlation constraint**
4) The estimation can be done without **matrix inversion**
5) The **asymptotic theory** of the QMLE is available
6) **EbE estimation** is possible for triangular models
7) The model works nicely in practice, in particular for beta hedging