Corrections/Additions – For Indian Edition

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1) p. 6, Exercise 1.C.6(8), erase in the 2nd sentence the part " $\mathfrak{p}$ is a prime ideal of R and " .

2) p.10, just before Corollary 1.E.5: Write "..., and is usually denoted by $\overline{A}$." instead of "..., and is usually denoted $\overline{A}$. "

3) p.24, in the line 5 from above, replace the text " Moreover, if each if each $\mathrm{V}_K(F_i)$ is infinite, then by by Exercise 2.B.14 (4), " by the following text: " Moreover, if each $\mathrm{V}_K(F_i)$ is infinite, then the ideal $\mathfrak{I}_K(\mathrm{V}_K(F))$ of all polynomials in $K[X, Y]$ which vanish on $\mathrm{V}_K(F)$ coincides with the ideal $(\mathrm{red} F)$ because, by Exercise 2.B.14 (4), "

4) p.29, in the line 1 from above, after " such that " add " the ideal "

5) p.29, in the line 2 from below, replace " $R \cap K[Z] \neq 0$ " by " $\mathfrak{r} \cap K[Z] \neq 0$ "

6) p.33 in the line 8 from above replace " $\mathbb{K}^n \times K$ " by " $\mathbb{K}^n \times \mathbb{K}$ "

7) p.43 in the line 2 from above, replace " $\mathrm{V}(\sum_{j\in I} Rf_j)$ " by " $\mathrm{V}(\sum_{j\in J} Rf_j)$ "

8) p. 43 in the lines 16 (once), 15 (once), 14 (thrice), 11 (once), 7 (once) from below, replace " $\mathfrak{J}$ " by " $\mathfrak{I}$ "

9) p.47, at the end of Exercise 3.A.20 add the following part:

"(7) If $\varphi : R \to S$ is a ring homomorphism then $\overline{\varphi^*(\mathrm{V}(\mathfrak{b}))} = \mathrm{V}(\varphi^{-1}(\mathrm{b}))$ for any ideal $\mathfrak{b} \subseteq S$. (**Hint**: $\varphi^{-1}(\sqrt{\mathfrak{b}}) = \sqrt{\varphi^{-1}(\mathfrak{b})}$.)"

10) p.53, line 4,5 from below: Write "... exchange lemma which is proved easily by induction on n:" instead of "... exchange lemma which is proved by induction on m:"

Observe the "n" instead of "m" (and the extra "easily").

11) p.59, the 2nd diagram in the display of Exercise 3.B.42: Change the direction of the two vertical arrows.

12) p.69 in the line 2 from above, replace " $s, t \subseteq \mathbb{N}$ " by " $s, t \in \mathbb{N}$ "

13) p.76 in the lines 16 (thrice) and 17 (once) from above replace " $\mathrm{Ann}_A x$ " by " $\mathrm{Ann}_R\, x$ "

14) p.92, just before Exercise 5.A.13: Write "... see Exercise 5.A.16." instead of "... see Exercise 5.A.15."

15) p.95 in the line 10 from below replace " a weighted projective " by " the weighted projective "

16) p.107, in part (2) of Exercise 5.B.11, last line but one: Omit " In particular ", i.e. write " If $K = k$, ... " instead of " In particular, if $K = k$, ...".

17) p.117, part (11) of Exercise 6.A.11, 3rd line from below: Put " more generally " into commas: " ..., more generally, ... ".

18) p.117, part (12) of Exercise 6.A.11: Write " Let M be a finite module ... " instead of " Let M be a module ... ".

19) p.126. In Exercise 6.C.1 insert the following part (4).

" (4) Let R be a Noetherian integral domain with an algebraically closed quotient field K. Show that $R = K$. (**Hint**: Use (2) to reduce to the case that R is a field or a discrete valuation ring.) "

20) p.132, in the lines 4 (once), 5 (once), 6 (once) from below, replace " $\dim_K \Omega_{K|k}$ " by " $\mathrm{Dim}_K \Omega_{K|k}$ "

21) p.133, in the lines 2 (once), 8 (once) from above and in the lines 6 (once), 7 (once) and 12 (twice) from below, replace " $\dim_K \Omega_{K|k}$ " by " $\mathrm{Dim}_K \Omega_{K|k}$ "

22) p.135, line 6 in Exercise 6.D.15: Insert an opening bracket: i. e., write "... cyclic k-algebra (one has to prove this)." insteaed of "... cyclic k-algebra one has to prove this)."

23) p.143, just before Example 6.E.9, add the following text immediately after " ... are coherent. " :

"Furthermore, one sets $\mathrm{Ass}\,\mathcal{F} := \{ x \in X \mid \mathrm{depth}\,\mathcal{F}_x = 0 \}$ for any coherent $\mathcal{O}_X$-module $\mathcal{F}$, cf. Exercise 6.A.11 (8)."

24) p.144, the last display: In the discription of the set use " | " instead of " : ".

25) p.147, in the 4th line from above: Write " Corollary 7.A.2 " instead of " Theorem 7.A.2 ".

26) p.148, in the 3rd line from above = last line of Corollary 6.A.17: Erase " a ", i.e. write " open and ... " instead of " a open and ... ".

27) p.154, in the proof of Theorem 7.A.1:

in the 1st line: Write " ... of positive degrees $d_0, \ldots, d_n$ which ... ".

in the 3rd line of part (1): Write " ... from $s|\mathrm{D}_+(ff_i) = 0$, ... " instead of " ... from $\mathrm{D}_+(ff_i) = 0$, ... ", i.e. " $s|$ " is missing.

in the 4th line of part (2): Twice a lower index " i " is missing. Write in the formulas " $f_i^{r_i d}$ " instead of " $f^{r_i d}$ ".

in the last line of part (2): Erase the lower index " i " in the last formula, i.e. write " ... $= s|\mathrm{D}_+(f)$ " instead of " ... $= s|\mathrm{D}_+(f_i)$ ".

28) p.161, just before Exercise 7.B.4:

Add " See Exercise 7.B.5 (3) below for an example. "

29) p.164, in the diagram of the first display:

Replace " -1 " by " - deg T_0 ". (This occurs twice.)

30) p.173, just after Exercise 7.D.5:

Start the new paragraph with " With the notations as in the last exercise, if X is a regular (=normal) .... ".

31) p.187, in the line 16 from above:

Insert " smooth ", i.e. write " ... of smooth connected projective curves ...".

32) p.188, in the 4th line from below (not counting the footnote):

Write " ... $V_{\overline{\mathbb{C}}}$ ... " instead of " ... $V_{\overline{C}}$ ... ".

33) p.189, in the line 14 of Example 7.E.18:

Write " ..., $V \subseteq Y$ open and affine, ... " instead of " ..., $V \subseteq Y$ open, ... "

2 lines later: Write " ... is a finite separable field extension!) ... " instead of " ... is a separable field extension!) ... "

22) p.190: just before " **7.E.20 Exercise** " Add the following text (starting with a new line):

If the field k is perfect the inequality $g_{\mathrm{red}}(X) \geq g_{\mathrm{red}}(Y)$ holds for an arbitrary finite morphism of normal and connected projective algebraic curves over k. For the p r o o f it suffices, by the last remark, to consider the case that $\mathcal{R}(X) = \mathcal{R}(Y)[z]$ is a purely inseparable field extension of $\mathcal{R}(Y)$ of degree $p = \mathrm{char}\, k > 0$ with $z^p \in \mathcal{R}(Y) \setminus \mathcal{R}(Y)^p$. Then $z^p \notin \mathcal{O}(Y) = \mathcal{O}(Y)^p$ and from the diagram of field extensions

$$\begin{array}{ccccccccc} \mathcal{R}(Y)^p & \subseteq & \mathcal{R}(X)^p & \subseteq & \mathcal{R}(Y) & \subseteq & \mathcal{R}(X) & = & \mathcal{R}(Y)[z] \\ & & \mid\bigcup & & \mid\bigcup & & \mid\bigcup & & \\ & & k(z^p) & = & k(z^p) & \subseteq & k(z) & , & \end{array}$$

we have

$$[\mathcal{R}(X) : k(z^p)] = [\mathcal{R}(X) : k(z)] \cdot [k(z) : k(z^p)] = [\mathcal{R}(X) : \mathcal{R}(Y)] \cdot [\mathcal{R}(Y) : k(z^p)]$$

which implies $[\mathcal{R}(X)^p : k(z^p)] = [\mathcal{R}(X) : k(z)] = [\mathcal{R}(Y) : k(z^p)]$ and hence $\mathcal{R}(X)^p = \mathcal{R}(Y)$ because of $\mathcal{R}(X)^p \subseteq \mathcal{R}(Y)$. It follows that X and Y are isomorphic as abstract curves over $k = k^p$ and that, in particular, $g_{\text{red}}(X) = g_{\text{red}}(Y)$.

Show that *for a finite morphism $X \to Y$ of smooth and connected projective algebraic curves over an arbitrary field k the inequality $g_{\text{red}}(X) \geq g_{\text{red}}(Y)$ holds*. (Look at the extension $X_{(\bar{k})} \to Y_{(\bar{k})}$ where $\bar{k}$ is an algebraic closure of k. The curves $X_{(\bar{k})}$ and $Y_{(\bar{k})}$ are also smooth (but not necessarily connected).)

In general, the inequality $g_{\text{red}}(X) \geq g_{\text{red}}(Y)$ does not hold. For example, let K be a field of characteristic $p > 2$ and let S be the standardly graded normal domain $S := k[X, Y, Z]/(T_1X^p + T_2Y^p + Z^p)$ over the rational function field $k := K(T_1, T_2)$.[15] By Plücker's formula (cf. Exercise 7.E.7), $Y := \text{Proj}\, S$ is a normal and connected projective curve over k with $\mathcal{O}(Y) = k$ and genus $g(Y) = g_{\text{red}}(Y) = (p-1)(p-2)/2 > 0$. Further, the canonical k-algebra homomorphism $S \to R$ with $R := k'[X, Y, Z]/(T_1^{1/p}X + T_2^{1/p}Y + Z)$, $k' := k(T_1^{1/p}, T_2^{1/p})$, is finite homogeneous and defines a finite k-morphism $X := \text{Proj}\, R = \mathbb{P}^1_{k'} \to Y$ with $g(X) = g_{\text{red}}(X) = 0$. In a similar way one constructs also examples in characteristic 2. – For further results and examples see Tate, J.: Genus Change in Inseparable Extenstions of Function Fields, Proc. Amer. Math. Soc. **3**, 400-406 (1952).

28) p.194, in the line 11 from below (not counting Footnote) replace " is normal [15] " by " is normal [16] "

29) p.194, in the footnote change the Footnote no. from

" [15] " to " [16] "

30) p.195, in the line 14 from below (not counting Footnote) replace " k-rational point P_0 [16] " by " k-rational point P_0 [17] "

31) p.195, in the footnote change the Footnote no. from

" [16] " to " [17] "

32) p.196, in the line 19 from below (not counting Footnote) replace " at P and Q [17] " by " at P and Q [18] "

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[15] The normality of S can be verified, for instance, in the following way: The singular locus of the K-algebra $A := K[T_1, T_2, X, Y, Z]/(F)$, $F := T_1X^p + T_2Y^p + Z^p$, is, by Definition 6.D.22, the zero set of the residue classes of the partial derivatives $\partial F/\partial T_1$, $\partial F/\partial T_2$, $\partial F/\partial X$, $\partial F/\partial Y$, $\partial F/\partial Z$, i. e. of X^p, Y^p, which is of codimension 2 in $\text{Spec}\, A$. Since A is a complete intersection the normality of A follows from the normality criterion 6.B.4. Now, since A is normal, S is normal too.

33) p.196, in the footnote change the Footnote no. from

" [17]) " to " [18]) "

34) p.197, in the line 5 from above, replace

" ... of rank 2 in $\mathbb{C}$ so as a group " by " ... of rank 2 in $\mathbb{C}$, (see Section 16.C in [13], for example) so as a group "