MODULAR REPRESENTATIONS OF LIE ALGEBRAS AND HUMPHREYS' CONJECTURE

Let G be a connected reductive algebraic group defined over an algebraically closed field of characteristic $p > 0$ and suppose that the derived subgroup of G is simply connected, p is a good prime for the root system of G and the Lie algebra $\mathfrak{g} = \mathrm{Lie}(G)$ admits a non-degenerate $(\mathrm{Ad}\, G)$-invariant symmetric bilinear form. If G is a simple algebraic group of type other than A, the above assumptions mean that p is a *good prime* for G, i.e. $p > 2$ if G is of type B, C or D, $p > 3$ if G is of type G_2, F_4, E_6 or E_7, and $p > 5$ if G is of type E_8. If all components of G have type $\mathrm{A}, \mathrm{B}, \mathrm{C}$, D we set $R := \mathbb{Z}[\frac{1}{2}]$. If G has a component of exceptional type but has no components of type E_8 we set $R := \mathbb{Z}[\frac{1}{6}]$. If G has a component of type E_8 we set $R := \mathbb{Z}[\frac{1}{30}]$. Given a linear function χ on $\mathfrak{g}$ we denote by $U_\chi(\mathfrak{g})$ the reduced enveloping algebra of $\mathfrak{g}$ associated with χ. By the Kac–Weisfeiler conjecture (now a theorem), any $U_\chi(\mathfrak{g})$-module has dimension divisible by $p^{d(\chi)}$ where $2d(\chi)$ is the dimension of the coadjoint G-orbit of χ.

In my talk, based on a joint work with Lewis Topley, I'll discuss a natural question raised in the 1990s by Kac, Humphreys and myself and explain that for any $\chi \in \mathfrak{g}^*$ the reduced enveloping algebra $U_\chi(\mathfrak{g})$ has an irreducible module of dimension $p^{d(\chi)}$. Forms of finite W-algebras over the ring R and their reductions modulo good primes play a crucial role in our arguments. We also use some recent results on multiplicty-free primitive ideals of $U(\mathfrak{g}_\mathbb{C})$ associated with the rigid nilpotent orbits in complex simple Lie algebras $\mathfrak{g}_\mathbb{C}$.