# New Perspectives in hyperplane and reflection arrangements

## Abstracts

### T. Abe

*The freeness of ideal subarrangements of Weyl arrangements*

An ideal subarrangement is a hyperplane arrangement corresponding to positive roots in an ideal of the root lattice of a Weyl group. Sommers and Tymoczko conjectured in 2006 that any ideal subarrangement of the Weyl arrangement is free, and that its exponents and the height distribution are dual partitions to each other. In this talk, we will prove that the Sommers-Tymoczko conjecture holds true. When the ideal consists of all positive roots, this result coincides with the classical famous formula asserting that exponents of Weyl groups and height distributions of positive systems are dual partitions to each other. This is a joint work with Mohamed Barakat, Michael Cuntz, Torsten Hoge and Hiroaki Terao.

### N. Amend

*Inductively free restrictions of reflection arrangements*

Suppose that $W$ is a finite unitary reflection group acting on the complex vector space $V$ and let $A(W) = (A(W), V)$ be the associated reflection arrangement with intersection lattice $L = L(A(W))$. Hoge and Röhrle classified all inductively free reflection arrangements $A(W)$. Extending this work, we classify all inductively free restrictions $A(W)^X$ to elements $X$ in $L$. We will discuss this classification with particular respect to the infinite families of irreducible complex reflection groups. This is a report on joint work with Hoge and Röhrle.

### D. Garber

*On left regular bands associated to real line and conic-line arrangements*

An arrangement of curves in the real plane divides it into a collection of faces. In the simple case of line arrangements, this collection can be given a structure of a left regular band (LRB), by defining a product on this collection. In this talk, we start by studying the relation between the structure of the LRB associated to the whole arrangement and the structure of LRBs associated to some sub-arrangements induced by the original arrangement.

Another interesting problem is whether a similar structure of LRB exists for the simplest generalization of lkine arrangements - conic-line arrangements. Investigating the different algebraic structures induced on the face poset of a conic-line arrangement, we will present in the talk two possibilities for the amended product and its associated structures.

This talk is based on a joint work with Michael Friedman.

### N. Nakashima

*Construction for canonical systems of basic invariants for finite reflection groups*

A system of basic invariant is said to be canonical if the system satisfies a certain orthogonal condition. Canonical systems were introduced by Flatto and Wiener for solving mean value problems related with vertices of polytopes, and they proved that there exist canonical systems for all finite real reflection groups. Explicit formulas of canonical systems play an important part in the study for mean value problems related with polytopes. We give a construction and examples for explicit formulas of canonical systems.

**V. Ripoll**

*Complex reflection arrangements and factorisations of a Coxeter element*

To a complex reflection group W, we associate its hyperplane arrangement. The quotient of the union of hyperplanes by the action of W is called the discriminant hypersurface of W. We exploit the monodromy around the discriminant to construct geometrically certain factorisations of a Coxeter element of W. Using these factorisations and the properties of the Lyashko-Looijenga ramified covering (related to the discriminant), we obtain a geometric interpretation of a formula of Chapoton. This formula expresses the number of multichains of a given length in the noncrossing partition lattice of W, in terms of a generalised Fuss-Catalan number depending on the invariant degrees of W.

**C. Stump**

*Counting factorizations of Coxeter elements in well-generated complex reflection groups*

I will present a simple and uniform formula for the exponential generating function of factorizations of Coxeter elements in well-generated complex reflection groups into products of reflections. In the case of factorizations of minimal length, we recover a formula due to P. Deligne, J. Tits and D. Zagier for finite Coxeter groups, and to D. Bessis for complex reflection groups. For the symmetric group, our formula specializes to a formula of B. Shapiro, M. Shapiro and A. Vainshtein. I will also discuss the (case-by-case) proof of this uniform formula, and the problems occurring in approaching it uniformly. This is joint work with Guillaume Chapuy.

**D. Suyama**

*Basis construction of the derivation modules of the extended Shi and Catalan arrangements of the type $A_2$*

Let $\Phi$ be a finite crystallographic irreducible root system. The Weyl arrangement of $\Phi$ is the set of all linear hyperplanes orthogonal to positive root in $\Phi$. Extended Shi arrangements and extended Catalan arrangements are obtained by adding to the Weyl arrangement several parallel translations of hyperplanes in the Weyl arrangement. In this talk, we will construct bases for the derivation modules of extended Shi and Catalan arrangements of the type $A_2$. This is a joint work with Takuro Abe.

**H. Terao**

*Questions arising from the ideal-free theorem*

The ideal-free theorem was proved in 2013 by T. Abe, M. Barakat, M. Cuntz, T. Hoge and myself after my talk here at Bochum on Feb. 21, 2013. Almost exactly one year later, in this talk, I ask questions (and answers in some cases) which naturally arise from the theorem. The questions include: 1) does the deletion version of the multiple addition theorem(MAT) exist? 2) does the affine Weyl arrangement have a "free path"? 3) how does Sommers-Tymoczko's first conjecture relate to their second conjecture, which is nothing other than the ideal-free theorem? (The works described in this talk were jointly done with Takuro Abe.)

**M. Yoshinaga**

*Milnor fibers of real line arrangements*

The Milnor fiber of a line arrangement is a certain cyclic covering space of the complexified complement equipped with monodromy action. We will present a new algorithm computing eigen spaces of the first cohomology group with respect to the monodromy action. The algorithm uses real and combinatorial structures (chambers). I will also give some applications and several conjectures.