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### Uwe Storch's Mathematical Work

Hubert Flenner

December 8, 2010

### Factorial domains

The PhD thesis: Q-factorial domains The work with Bingener Brieskorn singularities

#### Differential modules

Differential modules of (semi-)analytic algebras Differential dependence of ideals

#### Further results

Number of equations
The Oberseminar in Osnabrück
(Quasi-)Frobenius Algebras

### The Books

### **Bibliography**



# Biographical data

- ▶ \*12.07.1940 in Leopoldshall
- ▶ 1960/66 student University Münster and Heidelberg
- 1966 PhD Münster
- ▶ 1966/67 Assistent Münster
- 1967 Assistent University Bochum
- 1972 Habilitation at Bochum
- 1974 Full Professor Osnabrück
- ▶ 1981 Full Professor Bochum
- Retired since 2006



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### Münster: Behnke's Oberseminar 1965



Spallek Behnke Holman Vetter-Reiffen



Adviser G. Scheja

### Definition

A Krull domain A is called  $\mathbb{Q}$ -factorial (or almost factorial) if  $Cl(A)\otimes \mathbb{Q}=0$  or, equivalently, if every element in Cl(A) is torsion.

### Standard properties:

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# Q-factorial domains

#### Some results:

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- 5. If dim A = 2 then A is  $\mathbb{Q}$ -factorial iff it has a rational singularity.
- 6. Numerous examples, e.g.  $A = \mathbb{C}\{x, y, z\}/(x^r + y^s + z^t)$  (r, s, t pairwise coprime) is  $\mathbb{Q}$ -factorial iff (r, s, t) = (2, 3, 5).



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**Storch** Math. Ann. 183(1969): If A is analytic  $\mathbb{C}$ -algebra, Cl(A) is finitely generated then  $Cl(A) \to Cl(A\{T\})$  is an isomorphism. Moreover Cl(A) is finitely generated iff A is 1-rational.

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**Definition** (Danilov 70): A is said to have discrete divisor class groups if  $Cl(A) \rightarrow Cl(A[[T]])$  an isomorphism.



### Platte Günther Kuhn Flenner Bingener Lippa Wiebe



Storch

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Scheja

### Definition

An excellent  $\mathbb{Q}$ -algebra A is said to have s-rational singularities if for some resolution of singularities  $f: X \to \operatorname{Spec} A$  we have  $R^i f_*(\mathcal{O}_X) = 0$  for  $i = 1, \dots, s$ .

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Bingener, Jürgen; Storch, Uwe: Zur Berechnung der Divisorenklassengruppen kompletter lokaler Ringe. [On the computation of the divisor class groups of complete local rings] Leopoldina Symposium: Singularities (Thüringen, 1978). Nova Acta Leopoldina (N.F.) 52 (1981), no. 240, 7–63.

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- 3. N.B.:Later: —: quasihomogeneous case.
- Calculation of many examples, e.g. Brieskorn polynomials in dimension 3 but only in the case that there are no period relations.

Let A be a quasihomogeneous isolated hypersurface singularity of dimension 3 over  $\mathbb C$  and  $E=\operatorname{Proj} A$ . Main observations:

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### **Theorem**

If 
$$(\omega_A)_0 = 0$$
 then  $\operatorname{Cl}(A) \cong \mathbb{Z}^{\varrho(E)-1}$ , where  $\varrho(E) := \operatorname{rkPic}(E) = H^2(E,\mathbb{Z})$ .



# Brieskorn singularities

Storch, J. Reine Angew. Math. 350 (1984), 188-202

### **Theorem**

There is an algorithm to compute the Picard-numbers of  $t_1^{r_1} + t_2^{r_2} + t_3^{r_3} + t_4^{r_4} = 0$ .

Difficult case:  $\frac{1}{r_1} + \ldots + \frac{1}{r_4} \ge 0$ .

- Universally finite (prefinite, separated) modules of differentials (e.g. algebraic varieties, analytic algebras, Stein algebras)
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**Aim:** Generalize the theory of differentials to much wider classes of rings

- Universally finite (prefinite, separated) modules of differentials (e.g. algebraic varieties, analytic algebras, Stein algebras)
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#### **Theorem**

The Zariski-Lipman conjecture for hypersurface singularities holds.



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If chark = 0, 
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Exact sequence: 
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#### **Theorem**

Suppose that  $f: X \to Y$  is a dominant finite morphism of algebraic varieties over  $\mathbb{C}$ , where X is normal. Then a vector field V on Y admits a lifting to X if and only if it is tangent to f(C) for every component C of the critical set of f.

#### Number of equations The Oberseminar in Osnabrück (Quasi-)Frobenius Algebras

Storch, Uwe: Bemerkung zu einem Satz von M. Kneser. Arch. Math. (Basel) 23 (1972), 403–404

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Every algebraic set in  $\mathbb{A}^n$  is set-theoretically the intersection of n hypersurfaces.

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If A is a DVR: Roquette

### **Theorem**

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**Further work** on (quasi-)Frobenius algebras and complete intersections:

Explicit construction of a trace map for local complete intersections.

When are Dedekind's, Noether's and Kaehler's different equal (they describe all the ramification locus of  $A \rightarrow B$ .)



2003 Oberseminar Bochum



## Textbook of Algebra







Wiebe, Scheja 2000

# Textbook of Mathematics for mathematicians, physiscists, computer scientists





## Regular sequences and resultants/Lokale Verzweigungstheorie





## Introduction to algebraic geometry



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