1 Research Interests

My aim is the mathematical understanding of interacting stochastic systems. These are systems consisting of a large number of random components. The individual components then interact with each other and with their environment. Such systems arise in the real world in a multitude of seemingly different situations.

Take the theory of solid states to start with. Look at a magnetic material. Think of the behavior of a magnetic moment of an atom an a crystal lattice that depends on the states of the neighboring atoms. Additionally it is influenced by an external magnetic field and shaken by thermal fluctuations. This adds the randomness to the picture. How can we understand the resulting state for a large number of atoms? Is there magnetic ordering that can be observed on a large scale? Does the occurrence of long-range order depend on the strength and the particular structure of the interaction? How?

Alternatively, think of the opinions of a collection of people whose opinions tend to depend on the behavior of his/her friends. How do their opinions evolve with time starting from an initial configuration? Is there an equilibrium state? What if the opinion of some of the friends act as an influence in an opposite direction?

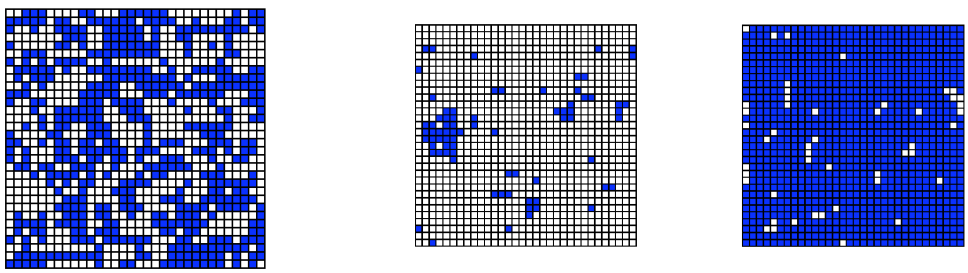
Suppose there are sick individuals in a population. They can infect other individuals they have contact with at a certain rate but may recover themselves. Or they die. What can be said about the spread of diseases? Is there persistence of the disease on a large scale? Does this depend on the rate and the structure of the network along which there can be infections?

Can one understand features of the behavior of the stock market (the atoms being traders), the spreading of rumours and fads, or the brain (the atoms being neurons) along such pictures?

Even when the interaction is local, such systems typically exhibit a complex global behavior, with a spatial long-range dependence resulting in phase transitions. In this picture phase transitions are characterized by discontinuous behavior of the possible states of the system as a function of external parameters. For specifically tuned values of the parameters there can be more one global state. An everyday example of this is the transition between fluid and vapor!

The approach to understand these phenomena is to start with simple models. Statistical mechanics here provides the conceptual ideas, in conjunction with other applied sciences. Statistical mechanics tells you: Don't look at states and motion of the individuals if you are dealing with a large number of components, look at things in a statistical way, look at averages! Moreover it guides you what the prescription of your probabilities should be. For the equilibrium behavior this leads to the study of Gibbs measures.

Below we see snapshots of a lattice model whose individual components can either be blue or white and interact with their nearest neighbors. In the left picture the strength of the interactions is too weak to result in long-range order although nearby sites show some dependence. In the two right pictures the strength of interactions is big and we see two coexisting states exhibiting long-range order.



This is the most basic picture of a phase transition to be kept in mind. There are however important models where the ordered states look much more complicated than that! The models one tries to analyse in a precise way then serve as paradigms that guide us to understand the complex structure arising also in more complicated models. Very often it turns out that there is some universality, meaning that the fundamental behavior of the systems is stable against details in the definition of the model. A very simple example of such an universality phenomenon familiar to every mathematician is already the central limit theorem!

The important and fascinating challenge is to give a mathematical treatment of the complexity of these interacting systems. Probability theory here provides the mathematical language and framework.

In the past years I myself have worked more specifically on the behavior disordered systems. Here the interactions are not assumed to be the same for all component but are random in itself. (Think of a magnet with random impurities, or a model of interacting agents having individual, random preferences.) For the analysis of these systems the challenge is to combine powerful ideas from general probability and specialized techniques from mathematical statistical mechanics. And to invent new ones, if necessary! This is not only important from the perspective of applications but also a source of fascinating mathematical developments, that are very much in progress at the moment. As a major breakthrough let us mention the recent proof of the validity of the so-called Parisi-solution for the mean-field spinglass model by Michel Talagrand. (Spinglasses are models with random interactions with *arbitrary* signs. This makes their ordered states look much more complicated then the ones we described in the pictures above.)

I would be happy to coach Master and Ph.D. thesis in this area, applications are most welcome!