On inductively free and additionally free arrangements

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Let $\mathcal{A}(W)$ be the hyperplane arrangement consisting of all the reflecting hyperplanes of a finite complex reflection group W acting on a vectorspace V. The class of inductively free arrangements \mathcal{IF} got introduced by Orlik and Terao in [OT]. In [CH] it was shown to be a combinatorial property. Abe showed in [A] that the Addition-Deletion-Theorem is combinatorial too and defined the class of additionally free arrangements \mathcal{AF} . Those properties are the same if the dimension of V is lower than 4.

Hoge and Röhrle gave two examples of additionally free arrangements that are not inductively free. To construct the first example \mathcal{B} they took an inductively free subarrangement \mathcal{A} of $\mathcal{A}(E_7)$ containing 32 hyperplanes and exponents $\{1, 5, 5, 5, 5, 5, 6\}$. Removing one further hyperplane provides an arrangement \mathcal{B} that is additionally free with exponents $\{1, 5, 5, 5, 5, 5, 5\}$, but fails to be inductively free. They also show that the rank 6 restriction of \mathcal{B} with matching exponents $\{1, 5, 5, 5, 5, 5\}$ is additionally free without being inductively free. To construct an other example \mathcal{D} they restrict \mathcal{B} to the intersection of ker (x_1) and ker (x_6) . The arrangement \mathcal{D} is free with exponents $\{1, 5, 5, 5, 5\}$ and up to isomorphism only one restriction of \mathcal{D} got matching exponents to use the Addition-Deletion-Theorem on. This restriction is free, but fails to be additionally free (in particular inductively free), because none of its restrictions got matching exponents. Removing a hyperplane $H \in \mathcal{D}$ such that the restriction on H is free and got matching exponents results in an inductively free arrangement. In particular the arrangement \mathcal{D} satisfies $\mathcal{D} \in \mathcal{AF} \setminus \mathcal{IF}$. Those examples of additionally free arrangements that fail to be inductively free are the only ones known for now.

In this talk I will present results obtained in my Master Thesis concerning the coincidence of inductive and additional freeness of subarrangements of $\mathcal{A}(W)$ where W is one of the groups $G_{29}, H_4, G_{31}, G_{32}$ or G(r, r, 4).

References

- [A] T. Abe; Deletion theorem and combinatorics of hyperplane arrangements, Math. Ann. 373 (2019), no. 1-2, 581–595.
- [CH] M. Cuntz and T. Hoge; *Free but not recursively free arrangements*, Proc. Amer. Math. Soc. 143 (2015), 35-40.
- [HR] T. Hoge and G. Röhrle; Some Remarks On Free Arrangements, Tôhoku Math. J. **73**, no. 2, (2021).

- [KU] J. P. S. Kung; Critical Problems
 in: J. E. Bonin, J. G. Oxley und B. Servatius; Theory of Matroids
 Amer. Math. Soc., Providence, RI, 1996.
- [LT] G. I. Lehrer and D. E. Taylor; *Unitary Reflection Groups*, Springer-Verlag, Berlin 1992.
- [OT] P. Orlik and H. Terao; Arrangements of Hyperplanes, Springer-Verlag, Berlin 1992.
- [OX] J. Oxley; *Matroid Theory*, Oxford University Press, USA 2011.
- [T] S. Tohaneanu; Free arrangements with low exponents, https://arxiv.org/abs/1707.07091.