DMV Conference 2020: Section Discrete Mathematics and Theoretical Computer Science

Organizers: Gennadiy Averkov and Christian Stump

Schedule

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Talks

Ayush Kumar Tewari: Forbidden patterns in tropical planar curves and Panoptigons

tba

Christopher Borger: Classification of triples of lattice polytopes with small mixed volume

Given a system of d Laurent polynomials $f_1, \dots, f_d$ in d variables, a natural invariant to study is the tuple of corresponding Newton polytopes $(N(f_1), \dots, N(f_d))$. For example, the classical Bernstein-Khovanskii-Kouchnirenko theorem states that the number of common roots in the complex torus of a generic system $f_1, \dots, f_d$ can be computed as the so-called mixed volume $\mathrm{MV}(N(f_1), ..., N(f_d))$ of the Newton polytopes. We combine computational methods from discrete geometry with results from Brunn-Minkowski theory in order to completely classify inclusion-maximal triples of mixed volume at most 4. By the above correspondence, this produces a classification of general trivariate sparse polynomial systems with up to 4 solutions in the complex torus. This is joint work with Gennadiy Averkov and Ivan Soprunov.

Claus Scheiderer: On the semidefinite extension degree of convex sets

A spectrahedron is the solution set of a linear matrix inequality (LMI), or equivalently, an affine-linear section of the cone of positive semidefinite matrices of some size. Semidefinite programming allows to efficiently optimize linear functions over spectrahedra, or more generally over their images under linear maps. The semidefinite extension degree $\mathrm{sxdeg}(K)$ of a convex set K is the smallest integer d such that K is a linear image of an intersection of finitely many spectrahedra that are all described by LMIs of size at most d. We relate $\mathrm{sxdeg}(K)$ to sums of squares representations, and use this to show for all convex subsets of the plane that $\mathrm{sxdeg}(K)$ is at most 2. As a consequence, all semidefinite programs in the plane can be performed as second order cone programming.

Cordian Riener: Efficiently computing Betti numbers of symmetric semi algebraic sets

The algorithmic problem of computing Betti numbers of arbitrary semi-algebraic subsets of R^k is a central and extremely well-studied problem in algorithmic semi-algebraic geometry. In this talk we will study the particular situation, in which the semi-algebraic set is defined by a finite set $\mathcal{P}$ of symmetric polynomials of given degree d. We will show that in this situation there exists an algorithm which computes the ranks of the first $(\ell + 1)$ cohomology groups, of this symmetric semi-algebraic set whose complexity is bounded by a polynomial in k and $\mathrm{card}(\mathcal{P})$. This result contrasts with the **PSPACE**-hardness of the problem of computing just the zero-th

Betti number (i.e. the number of semi-algebraically connected components) in the general case for $d > 2$. (Joint work with Saugata Basu.)

Deniz Kus: Fusion product approach to Schur positivity

In this talk I will report on the progress in the project of trying to understand whether certain expressions of the form $s_\lambda s_\mu - s_\rho s_\nu$ are Schur positive, i.e. whether they can be written as a non-negative linear combination of Schur functions. Our approach to this problem will use the representation theory of loop algebras and identify products of Schur functions with characters of graded modules.

Frauke Liers: The non-stop disjoint trajectories problem

For all sorts of traffic routing, commodities have to move through a network without exceeding the capacities. We study the problem where commodities further have to respect a no-wait restriction after they started their path at their release date. This problem variant appears, for example, when disjoint aircraft trajectories shall be determined. We study the complexity for deciding feasibility and optimization on two generic graph classes that are frequently used where space and time are discretized simultaneously: The regular line and regular grids with and without diagonals. We show that if all commodities have a common release date, feasibility can always be decided in polynomial time on the line. For implicitly defined grids with diagonals and unit-cost, polynomial time optimization algorithms are presented. In contrast, if commodities have individual release intervals, then deciding feasibility for the line is NP-complete. If edge cost in the grid with diagonals are arbitrary and commodities with restricted turning abilities have individual release dates, optimization and approximation are not fixed-parameter tractable. This is joint work with Benno Hoch, Sarah Neumann (both FAU Erlangen-Nuremberg) and Francisco Javier Zaragoza Martinez (Universidad Autnoma Metropolitana Azcapotzalco, Mexico).

Holger Eble: Cluster partitions and fitness landscapes of the drosophila microbiome

tba

Iskander Aliev: Distance-sparsity transference for vertices of corner polyhedra

We will discuss a recently obtained transference bound for vertices of corner polyhedra that connects two well-established areas of research: proximity and sparsity of solutions to integer programs. In the knapsack scenario, it gives an exponential (in the size of support of a solution) improvement on previously known proximity estimates. In addition, for general integer linear programs we obtain a resembling result that connects the minimum absolute nonzero entry of an optimal solution with the size of its support. This is a joint work with Martin Henk, Marcel Celaya (TU Berlin) and Aled Williams (Cardiff University).

Ivan Soprunov: Maximizing the volume of the Minkowski sum in terms of the mixed volume

Recently Esterov and Gusev showed that the problem of classifying generic sparse polynomial systems which are solvable in radicals reduces to the problem of classifying collections of lattice polytopes of mixed volume up to 4. Given the value of mixed volume m and dimension n, there exist only finitely many collections $(P_1, \dots, P_n)$ of n-dimensional lattice polytopes of mixed volume m, up to unimodular transformations. One reason for this is that the volume of the Minkowski sum $P_1 + \cdots + P_n$ is bounded above by $O(m^{2^n})$, as follows by a direct application of the Aleksandrov-Fenchel inequality. I will show how one can employ more relations between mixed volumes to improve the bound to $O(m^n)$, which is asymptotically sharp. We will also see the exact sharp bound in dimensions 2 and 3. This is joint work with Gennadiy Averkov and Christopher Borger.

Jacinta Torres: A positive combinatorial formula for Kostka-Foulkes polynomials I: Rows

Kostka-Foulkes polynomials, originally defined in type A, are defined as transition coefficients between the Schur basis and the monomial basis of the ring of symmetric functions. Moreover, they are known to have positive coefficients. The elegant charge formula of Lascoux-Schützenberger assigns a positive integer to a semistandard Young tableau in such a way that the Kostka-Foulkes polynomials are generating functions over the set of semistandard Young tableaux with respect to this statistic. In general type, although Kostka-Foulkes polynomials are known to have positive coefficients (they are affine Kazhdan-Lusztig polynomials), there is so far no positive combinatorial formula describing them. In type C, there is a conjectural formula in terms of a certain statistic on symplectic tableaux due to Lecouvey. We reformulate his definition in terms of a simple, intuitive algorithm in the case of row shapes of general weight, and thereby prove the conjecture in this case.

Karl Heuer: Characterising the existence of odd dijoins via cut minors

tba

Katharina Jochemko: Generalized permutahedra: Minkowski linear functionals and Ehrhart positivity

Generalized permutahedra form a rich class of polytopes that naturally appear in many areas such as combinatorics, optimization and statistics. We study functions on generalized permutahedra that behave linearly under dilation and taking Minkowski sums. We give a complete classification of all such functions that are additionally positive, translation-invariant and invariant under permutations of the coordinates: they form a simplicial cone and we explicitly describe the generators.

We apply our results to Ehrhart polynomials of generalized permutahedra. This is joint work with Mohan Ravichandran.

Mareike Dressler: Global optimization via the dual SONC cone and linear programming

In this talk we report on recent developments in optimization: Using the dual cone of sums of nonnegative circuits (SONC), we provide a relaxation of the global optimization problem to minimize an exponential sum and, as a special case, a multivariate real polynomial. This approach builds on two key observations. First, that the dual SONC cone is contained in the primal one. Hence, containment in this cone is a certificate of nonnegativity. Second, we show that membership in the dual cone can be verified by a linear program. Finally we present initial experimental results comparing our method to existing approaches. Joint work with Janin Heuer, Helen Naumann, and Timo de Wolff.

Martin Skrodzki: Combinatorial and asymptotical results on the neighborhood grid

In 2009, Joselli et al. introduced the Neighborhood Grid data structure for fast computation of neighborhood estimates in point clouds. Even though the data structure has been used in several applications and shown to be practically relevant, it is theoretically not yet well understood. The purpose of this talk is to present a polynomial-time algorithm to build the data structure. Furthermore, it is investigated whether the presented algorithm is time-optimal. This investigations leads to several combinatorial questions for which partial results are given.

Martin Winter: Spectral graph theory for edge-transitive polytopes

tba

Masahiko Yoshinaga: A q-deformation of the Aomoto complex

The operation replacing integers by q-integers is sometimes called a q-deformation. One may naively define a q-deformation of a cochain complex (of Z-modules) by replacing each entry of the matrix representation of the coboundary map by q-integers. However, in general, the resulting object is no longer a cochain complex. It is a matter of the choice of basis. In this talk, I will discuss the above "naive q-deformation" for the Aomoto complex of arrangements. In particular, for real line arrangements, there exists a good basis which makes the naive q-deformation a cochain complex, and specialization of q at root of 1 computes local system cohomology groups.

Matthias Beck: The Arithmetic of Coxeter Permutahedra

Ehrhart theory measures a polytope P discretely by counting the lattice points inside its dilates P, $2P$, $3P$, ... We compute the Ehrhart quasipolynomials of the standard Coxeter permutahedra for the classical Coxeter groups, expressing them in terms of the Lambert W function. A central tool is a description of the Ehrhart theory of a rational translate of an integer zonotope. This is joint work with Federico Ardila (SF State & Los Andes) and Jodi McWhirter (Washington University St. Louis).

Matthias Walter: Persistency for linear programming relaxations for the stable set problem

The Nemhauser-Trotter theorem states that the standard LP formulation for the stable set problem has a remarkable property, also known as (weak) persistency: for every optimal LP solution that assigns integer values to some variables, there exists an optimal integer solution in which these variables retain the same values. While the standard LP is defined by only non-negativity and edge constraints, a variety of stronger LP formulations have been studied and one may wonder whether any of them has the this property as well. We show that any stronger LP formulation that satisfies mild conditions cannot have the persistency property on all graphs, unless it is always equal to the stable-set polytope.

Michael Cuntz: Frieze patterns with coefficients

Friezes with coefficients are maps assigning numbers to the edges and diagonals of a regular polygon such that all Ptolemy relations for crossing diagonals are satisfied. These are relevant for example for the study of cluster algebras, in a special case they may also be viewed as root systems of certain quantum groups. In this talk I will report on recent results on subpolygons of friezes. Depending on the domain of the entries of the friezes, these subpolygons satisfy interesting arithmetic obstructions. This is joint work with Thorsten Holm and Peter Jorgensen.

Paul Mücksch: Accurate arrangements

In 1987 Orlik, Solomon and Terao conjectured that for every $1 \leq d \leq \ell$, the first d exponents of the reflection arrangement $\mathscr{A}(W)$ of a Coxter group – when listed in increasing order – are realized as the exponents of a free restriction of $\mathscr{A}(W)$ to some intersection of reflecting hyperplanes of $\mathscr{A}(W)$ of dimension d.

This conjecture does follow from rather extensive case-by-case studies by Orlik and Terao from 1992 and 1993, where they show that all restrictions of Coxeter arrangements are free.

We call a general free arrangement with this property involving their free restrictions *accurate*.

An order ideal in the root poset of a Weyl group W gives rise to a subarrangement of $\mathscr{A}(W)$ which is called an ideal subarrangement.

In this talk, I will present joint work with Gerhard Röhrle generalizing Orlik and Terao's results to all ideal subarrangements. Our principal theorem implies in a uniform way that all ideal arrangements are accurate. In particular, this gives as a special case the first uniform proof of Orlik, Solomon and Terao's conjecture for Weyl groups.

Raman Sanyal: Inscribable fans, hyperplane arrangements, and reflection groupoids

Steiner posed the question if any 3-dimensional polytope had a realization with vertices on a sphere. Steinitz constructed the first counter examples and Rivin gave a complete complete answer to Steiner's question. In dimensions 4 and up, Mnev's universality theorem renders the question for inscribable combinatorial types hopeless. In this talk, I will address the following refined question: Given a polytope P, is there a continuous deformation of P into an inscribed polytope that keeps corresponding faces parallel? This is a property of the normal fan of P, which we call *inscribable* if the answer is yes.

It turns out that the study of inscribable fans reveals a rich interplay of algebra, geometry, and combinatorics. In particular, inscribable fans give rise to reflection groupoids and deciding if a fan is inscribable is polynomial time decidable.

In the second part of the talk, we will focus on the important class of fans given by linear hyperplane arrangements and their associated zonotopes. It turns out that inscribable hyperplane arrangements are rare and intimately related to reflection groups and Grünbaum's quest for simplicial arrangements. This is based on joint work with Sebastian Manecke.

Thomas Kahle: Symmetric chains in algebra and discrete geometry

We describe a framework to study chains of symmetric geometric objects (e.g. cones or monoids). Each such chain has a natural limit object in countable-dimensional space. We discuss under which properties the limit object is finitely generated "up to symmetry", that is, by finitely many orbits under the action of the union of all finite symmetric groups or the monoid of strictly increasing maps. Joint work with Dinh Van Le and Tim Römer.

Timm Oertel: Sparse representation of vectors in lattices and semigroups

In this talk we consider sparse solutions to systems of linear Diophantine equations with and without non-negativity constraints, i.e., minimizing the number of non-zero entries of solutions. The main results are new improved bounds on this number. Joint work with I. Aliev, G. Averkov, and J. De Loera.

Volkmar Welker: Combinatorics of compositions and the topology of spaces of polynomials with restricted root multiplcities

We show how to encode homological and topological information about spaces of real monic polynomials with restricted root multiplicites (e.g. no 2 different roots of multiplicity ≥ 5) in a post structure on compositions. We show how to use the poset structure to calculate invariants of the topological spaces. Joint work with Gebriel Katz and Boris Shaprio.