Ruhr-Universität Bochum
Institute of Automation and Computer Control
Prof. Dr.-Ing. J. Lunze

Universitätsstrasse 150
D-44805 Bochum, Germany

Phone +49 - (0)234 32 - 28071
Fax +49 - (0)234 32 - 14101

Design of optimal switching surfaces for discretely controlled systems

Dipl.-Ing. Axel Schild
schild@atp.rub.de

1 Introduction

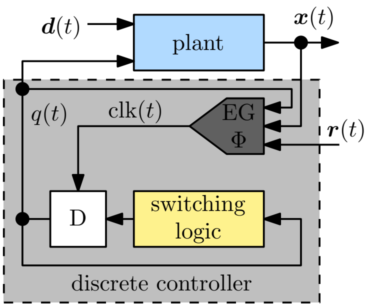

Figure 1: Structure of a discretely controlled system.

Discretely controlled continuous systems (DCCS) represent an important class of hybrid systems with many applications in power electronics, process engineering and robotics. They appear as a control loop of a continuous plant and a discrete-event controller (Fig. 1). The plant dynamics

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{d}(t), q(t)), \; \boldsymbol{x}(0) = \boldsymbol{x}_0 \qquad (1)$$

depend on the activated *mode of operation* $q(t) \in \mathcal{Q}$ and are influenced by disturbances $\boldsymbol{d}(t)$. The *discrete controller* implements a state-dependent switching law to trigger mode transitions in the plant, such that specifications $\boldsymbol{r}(t)$ imposed on the continuous state $\boldsymbol{x}(t)$ are met.
A central control task of DCCS concerns the stabilization of stationary periodic operations, which map into p-periodic limit cycles Γ in the state space (see Fig. 2). Each limit cycle Γ can be characterized by its periodic mode sequence $\mathcal{Q}_\Gamma = (\bar{q}_0^\star \dots \bar{q}_{p-1}^\star)$ and the associated switch points $\mathcal{X}_\Gamma = \{\bar{\boldsymbol{x}}^\star(\bar{q}_0^\star), \dots, \bar{\boldsymbol{x}}^\star(\bar{q}_{p-1}^\star)\}$.
In general, orbital stability must be achieved by timing mode transitions properly, as the mode order is fixed through Γ.

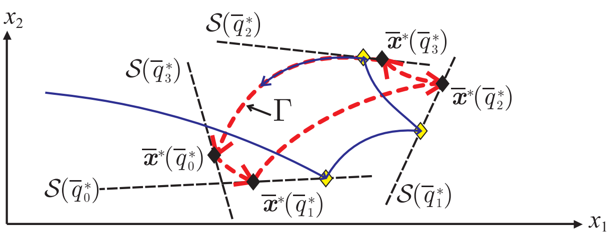

Figure 2: Stabilizing switching planes for a limit cycle Γ.

2 Project goals

During operation, the discrete controller generates a sequence $\left\{(\bar{q}_k^\star, \bar{t}(k))\right\}_{k=0}^{\infty}$ of mode-time-pairs. The mode sequence is encoded in the autonomous discrete-event switching logic, while suitable transition times $\bar{t}(k)$ are determined by the event generator that drives the latch D (Fig. 1). The latter component implements a static event function Φ, which specifies a switching surface configuration $\left\{\mathcal{S}\left(\bar{q}_k^\star\right)\right\}_{k=0}^{p-1}$ in the state space (Fig. 2). A mode transition $\bar{q}_k^\star \to \bar{q}_{k+1}^\star$ is triggered, whenever the corresponding switching condition $\Phi(\boldsymbol{x}, \bar{q}_k^\star) = 0$ is satisfied, i.e. when the state trajectory $\boldsymbol{x}(t)$ intersects with the surface

$$\mathcal{S}\left(\bar{q}_k^\star\right) = \left\{\boldsymbol{x} \; : \; \Phi(\boldsymbol{x}, \bar{q}_k^\star) = 0\right\}$$

at the switch point $\bar{\boldsymbol{x}}(k)$. The goal of this project is to determine an event function Φ, i.e. a configuration $\left\{\mathcal{S}\left(\bar{q}_k^\star\right)\right\}_{k=0}^{p-1}$, such that the control loop behaves optimally *for all* $\boldsymbol{x}_0 \in \mathcal{X}_0$ with respect to a smooth performance measure

$$J(\boldsymbol{x}_0, \bar{\boldsymbol{t}}_N) = \sum_{k=0}^{N-1} \left(\int_{\bar{t}(k)}^{\bar{t}(k+1)} L_k(\boldsymbol{x}(t)) \mathrm{d}t + \phi_k(\bar{\boldsymbol{x}}(k)) \right) + \phi_T(\bar{\boldsymbol{x}}(N)) \;, \qquad (2)$$

which adequately reflects

1. transient performance requirements,
2. the stationary objective of local orbital stability and
3. robustness.

Minimization of (2) must be achieved under constraints

$$\bar{t}(k) \le \bar{t}(k+1) \qquad (3)$$
$$\psi(\bar{\boldsymbol{x}}(N)) \le 0 \qquad (4)$$

on the state trajectory and the switching times.

3 Approximation of optimal surfaces

As a consequence of feedback control, all switching times $\bar{t}(k) = \bar{t}(k, \Phi, \boldsymbol{x}_0, \boldsymbol{d}(t))$ depend on the event function, the initial condition and the disturbances. Hence, minimizing (2) for all possible $\boldsymbol{x}_0$ over the event function Φ constitutes an intractable endeavor.
As a remedy, we propose to approximate the optimal function $\Phi^\star(\boldsymbol{x}, q)$ in the relevant region of the state space by a piecewise affine function

$$\tilde{\Phi}^\star(\boldsymbol{x}, q) = \boldsymbol{n}_i^{\mathrm{T}}(q)\boldsymbol{x} - d_i(q), \;\; \text{if } \boldsymbol{x} \in \Omega_i \qquad (5)$$

where the partition-dependent parameters $\boldsymbol{n}_i(q)$ and $d_i(q)$ represent a locally optimal switching plane in the state space. The associated switching surface

$$\tilde{\mathcal{S}}^\star\left(\bar{q}_k^\star\right) = \left\{\boldsymbol{x} \; : \; \tilde{\Phi}^\star(\boldsymbol{x}, \bar{q}_k^\star) = 0\right\}$$

is, hence, a polygonal approximation of the true optimal surface (Fig. 3). As important properties, this polygonal representation can be stored efficiently and allows for a fast and reliable detection of switching events.

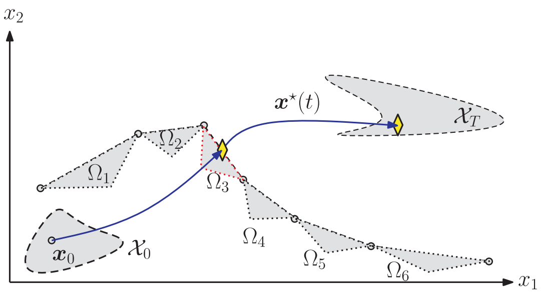

Figure 3: Polygonal representation of a switching surface.

4 Successive surface polygonization

Before the determination of $\tilde{\Phi}^\star$, the existence of a time-invariant optimal event function $\Phi^\star$ must be ensured.

Proposition 1. *The optimal control problem* (2)-(4) *admits a time-invariant event function* $\Phi^\star$*, i.e. a stationary optimal switching surface configuration, if the final time* $\bar{t}(N)$ *is a decision variable and the cost and constraint components* L_i*,* ϕ_i *and* ψ *are smooth time-invariant functions.*

Given the existence of stationary optimal surfaces $\mathcal{S}^\star\left(\bar{q}_k^\star\right)$, the sought polygonal approximations can be sequentially determined by application of numerical *predictor-corrector-continuation* methods, which aim at systematically covering a functionally specified, complex-shaped surface by a simplex mesh. Such procedures are well-suited for the design task at hand, due to the following properties:

- They only explore the relevant state space fraction.
- They enable an active precision control by adjusting the mesh distance according to the surface geometry.

They repeatedly execute three basic operations:

1. Determination of an anchor point $\bar{x}_0^\star(k+1) \in \mathcal{S}^\star(\bar{q}_k^\star)$,
2. Determination of tangent planes $\mathcal{T}^\star(\bar{x}_j^\star(k+1), \bar{q}_k^\star)$,
3. Projection of *candidate* *points* $\bar{x}_j(k+1) \in \mathcal{T}^\star(\bar{x}_j^\star(k+1), \bar{q}_k^\star)$.

All steps require solving the open-loop constrained transition time optimization problem (2)-(4) by second-order methods [2]. The successive unfolding is illustrated in Fig. 4. The determination of the anchor point $\bar{x}_0^\star(1)$ is followed by computing the tangent plane $\mathcal{T}^\star(\bar{x}_0^\star(1), 0)$ to the unknown optimal surface $\mathcal{S}^\star(0)$. Subsequently, candidate points $\bar{x}_j(0)$ are chosen along the tangent plane at a predetermined edge distance Δe.

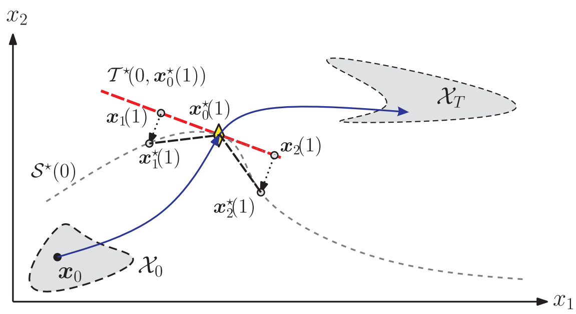

Figure 4: Unfolding of an optimal switching surface.

After the projection onto $\mathcal{S}^\star(0)$, these points become anchor points themselves and must be expanded recursively.

Once $\tilde{\mathcal{S}}^\star(0)$ covers the relevant state-space region, unfolding the surface can be stopped. Obviously, the approximate and true switching surfaces coincide at all vertices. In between, the planar approximation can be made as accurate as desired by reducing the mesh distance.

By implementing the resulting polygonal switching planes and for a suitable choice for the criterion (2), the control loop exhibits the desired properties. In particular, the switching surfaces result in an optimal loop behavior for a large set of initial states, they generate the optimal response to unknown disturbances and ensure adequate robustness of the implemented controller with respect to model uncertainties.

5 Application example

The polygonal surfaces depicted in Fig. 5 constitute approximations of the optimal switching surfaces for a hybrid thermal control process realized at the experimental manufacturing plant [1]. The primary process goals are to maintain the temperature $\theta_B(t)$ of a metal block around a desired value and to enforce a periodic stationary operation (red orbit in Fig. 5). This is to be achieved by controlling the heat energy flow via properly timed transports of two aluminium elements in between a heating unit and the block.

Figure 5 also illustrates the accomplishment of the specified objectives by implementation of the computed surfaces. The depicted sample state trajectory (blue solid line) quickly settles to a desired periodic orbit (thick red line), which is enclosed by the switching surfaces. The depicted transient evolution is optimal with respect to the chosen performance measure.

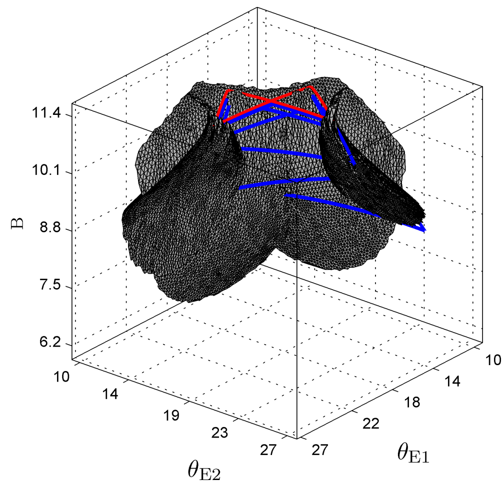

Figure 5: Optimal switching surfaces for the FESTO process. The sample trajectory (blue line) converges to the desired limit cycle (red orbit).

References

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[2] A. Schild, X. Ding, M. Egerstedt, and J. Lunze. Design of optimal switching surfaces for switched autonomous systems. In *Proc. of 42nd IEEE CDC*, 2009 (submitted).