WORKSHOP ON

TRAFFIC AND GRANULAR FLOW

HLRZ, Forschungszentrum Jülich, Germany
October 9–11, 1995

edited by

D E Wolf
HLRZ, Forschungszentrum Jülich (KFA)

M Schreckenberg
Gerhard-Mercator-Universität Duisburg

A Bachem
Universität zu Köln

World Scientific
Singapore • New Jersey • London • Hong Kong
APPLICATION OF TRAFFIC FLOW MODELS

Werner BRILON, Martin PONZLET
Institute for Transportation, Ruhr-University Bochum, 44780 Bochum, Germany

In most of the western countries traffic demand is reaching or even exceeding the capacity of the highway system at many points and during longer periods. On the other hand, the capacity of the road networks cannot be expanded due to financial or environmental reasons. Therefore, the existing infrastructure and their capacities must be exploited as far as possible. As a consequence, a complete understanding of traffic flow and its quality is becoming more and more important in practical traffic engineering. However, traffic flow theory still has to face quite a variety of fundamental problems.

1 Describing traffic flow

Traffic flow on uninterrupted freeway and highway facilities is a phenomenon which affects nearly the whole population in the western countries:

- by participating as drivers or passengers of cars,
- by road accidents and their consequences,
- by taking advantage from economic benefits from road traffic.

Therefore, traffic flow is an everyday's experience for many people around the world. Taking this in mind, it is an amazing fact that we have not understood the mechanisms of traffic flow completely, up to now, although a really large number of wise and intelligent scientists have studied traffic flow phenomena since decades.

The reasons why the description of traffic flow is such difficult are manifold:

- Traffic on highways and motorways is not a fully controlled system such as a mechanical machinery facility where each movement is performed according to predetermined rules.
- Instead, the technical units participating in traffic flow are of widely varying performance regarding physical size, maximum speed, and acceleration.
- As an even more important aspect human perception and human behaviour is the most decisive factor in traffic flow. Behaviour patterns, personal skills, and driving attitudes, however, can vary considerably from driver to driver. Different traffic rules and highway codes have also an influence.

These are the reasons why traffic flow cannot be described by compelling rules like experiments in physics. Instead the description of traffic flow must be based on two fundamental essentials:

- The variability of vehicle performance and driver behaviour demands for a stochastic approach of description.
- The reality can only be described based on empirical investigations.
Moreover, since external parameters (like weather conditions, different predominant motivations of drivers, or even different roadway conditions) have a remarkable influence on driver behaviour, a specific description of traffic flow can only be valid for

- one specific point along the highway network,
- and for one specific period of time.

Or in other words: Traffic flow characteristics are variable between different locations and for one individual location the characteristics are varying over time.

This academic point of view, however, cannot be the last word if we consider the needs of traffic engineering practice. Practice demands for a realistic description of typical traffic flow conditions under typical external parameters like daylight conditions and dry roadway surface.

Practice of traffic engineering makes use of traffic flow characteristics for three fundamental tasks:

- design of freeways, e.g. decisions upon the number of lanes for new freeway projects or for upgrading schemes,
- on-line traffic control,
- developments of fundamental traffic rules, like speed limits or lane allocation rules.

For the first two tasks we need rather general ideas for fundamental relations in traffic flow. These fundamentals concern the question of functional relations between the quality of flow and the quantity of traffic demand. For the second and third group of tasks we need, however, in addition a very detailed understanding of microscopic mechanisms in traffic flow operations.

All of these relations between the parameters of traffic flow are based on models, since a complete mathematical theory is not possible as it has been pointed out before.

\[ q = \text{volume [veh/h]} \text{ from an observation at one point over time}, \]
\[ k = \text{density [veh/km]} \text{ from an observation at one instant over space}, \]
\[ l = \text{index for local observation at one point over time,} \]
\[ m = \text{index for momentary observation at one time instant over space,} \]
\[ E(X) = \text{expectation of random variable } X, \]
with
\[ V = \text{random variable for velocity [km/h]}, \]
\[ W = \text{random variable for skewness (tardy)} [s/km]. \]

The expectations can be described by:
\[ E_m(V) = \frac{1}{E_l(W)} \]
\[ E_l(V) = \frac{1}{E_m(V)} \cdot E_m(V^2) = E_m(V) + \frac{\sigma^2_m(V)}{E_m(V)} \]
\[ E_m(W) = \frac{1}{E_l(W)} \cdot E_l(W^2) = E_l(W) + \frac{\sigma^2_l(W)}{E_l(W)} \]

where: \( \sigma^2(X) = \text{variance of random variable } X. \)

The cumulative distribution functions are given as:
\[ F(v) = 1 - G \left( \frac{1}{v} \right) \]
\[ G(w) = 1 - F \left( \frac{1}{w} \right) \]

where:
\[ F(v) = \text{cumulative distribution function of random variable } V, \]
\[ G(w) = \text{cumulative distribution function of random variable } W. \]

From continuity follows:
\[ \frac{\delta q}{\delta s} + \frac{\delta k}{\delta t} = 0 \]

where:
\[ s = \text{parameter for space,} \]
\[ t = \text{parameter for time.} \]

These fundamental equations must be obeyed by any valid description of traffic flow. However, they cannot describe the relation between quantity and quality of traffic flow. These relations can only be described based on models or empirical methods.

3 Traffic flow models

Traffic flow models are an approximative description of specific phenomena within traffic flow operations, normally based on a set of mathematical equations or on an analogy to other physical systems.
Traffic Flow Models

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>macroscopic</td>
<td>fundamental diagram</td>
</tr>
<tr>
<td></td>
<td>analogy to fluids</td>
</tr>
<tr>
<td>microscopic</td>
<td>headway models</td>
</tr>
<tr>
<td></td>
<td>deterministic</td>
</tr>
<tr>
<td></td>
<td>car following theory</td>
</tr>
<tr>
<td></td>
<td>psycho-physical</td>
</tr>
<tr>
<td></td>
<td>simulation</td>
</tr>
</tbody>
</table>

Table 1: Types of traffic flow models

If the initial equations are directly concerning the parameters of traffic flow (volume $q$ and/or concentration $k$) we call the model "macroscopic". If the initial equations try to approximate or generalise properties or behaviours of individual transportation units we call the model "microscopic". In this type of model, mathematical derivations or computerized experiments are performed to derive relations between macroscopic parameters. Moreover, these microscopic models should also be able to predict specific operational aspects for individual vehicles in traffic flows.

In this sense we can divide traffic flow models into the groups which are shown in Table 1. Using traffic flow models we must always be aware that a model can never be the whole truth. The outcome of each model cannot be better than the set of assumptions where it is basing on.

4 Fundamental diagram

4.1 Modern Understanding

The fundamental diagram is not so much an extensive model framework. First of all it is nothing more than an empirically based description of relations between macroscopic parameters (fig. 1). This description from empiry is given by a more or less complete set of measurement points.

Two points in the diagram (fig. 1) are conclusive:

$$k = 0 \implies q = 0,$$
$$k = k_{\text{max}} \implies v = 0 \implies q = 0.$$

Everything else between these two extremes is only based on measurements. These observations are typically performed over a specific time interval $\Delta t$. During this interval the traffic volume $q$ and the parameters of the speed distribution are observed. This enables also to determine an estimation of the average traffic concentration $k$ during that interval (eq. 1). Plotting these results from a longer observation period into a diagram like fig. 1 we should get a cloud of points which—in its best case—should cover the whole scale between $k = 0$ and $k = k_{\text{max}}$. This cloud of points indicates us an area of maximum flow which we call capacity and which should be the most important feature of the road section under concern.

It is well-known that the fundamental diagram has two branches:

- the free flow conditions.
- and the congested flow conditions.

It is known that we can transform the original fundamental diagram (i.e., the $q - k$-relation) into a speed-flow relation and a speed-density relation. The problem is that the only real truth about the traffic flow on a highway section is the observed cloud of points. And this cannot be handled in practice without complication.

Therefore, since the beginnings of traffic flow theory many attempts have been made to represent this cloud of points by mathematical functions. Here the greatest desire of scientists always was to find one simple equation which could approximate the various empirical clouds of points. This attempt was not very successful in many cases. The latest attempt into that direction was presented by van Aerde. 1

On the other side many authors preferred to describe the free-flow and the congested flow by two different equations, to make the result more realistic. Initially, for these attempts necessity was the mother of invention. If we, however, look on realistic diagrams we can clearly discern two areas within the cloud of points. And there is no clear connection between these two.

It was a long process before this became obvious that we have to discern two completely different areas of the fundamental diagram. Transitions between both are possible. We do, however, not easily understand the mechanism behind these transitions. Therefore, in our days much research emphasis is concentrated on these transition procedures.

Thus, the modern understanding of the fundamental diagram, as it has been formulated by Hall (e.g. 5) can be characterised by these essentials (fig. 2):

- Free flow can be characterised by a very conclusive functional relation between average speed and traffic volume. The form of the relation can be represented
by sections of linear approximations or by a similar closed function.

- There is a second branch of the fundamental diagram representing congested conditions.

- Measurement points being found outside these two areas of traffic conditions are transition situations, e.g., traffic leaving the head of a queue. This traffic flow cannot be larger than the capacity of the congested area, but it can be faster than congested flow. Moreover, measurement points obtained over longer observation intervals, however, could be a combination of several traffic conditions being present during that interval (cf. fig. 2). Thus, many measurement points between the two branches of the speed-flow relation are just an average of several traffic conditions. The condition which is indicated by the measurement point has not existed on the highway during one single second. This effect is also the reason why the fundamental diagram has the tendency to lower capacities if we increase the period of observation.

![Figure 2: Concept for the speed-flow-relation](image)

4.2 Queuing Model

Another model to interpret the fundamental diagram is also possible. The model assumes that each point along a freeway is a bottleneck for the upstream traffic (fig. 3). Thus, each point has a specific capacity c. The approaching traffic flow is denoted as q. Since we treat the depicted point along the road as a queuing system we should approximate it by one of the well-known standard systems. The most simple system is the M/M/1-queue.

Here the average delay is:

\[ w = \frac{1}{R} \quad \text{with} \quad R = c - q. \]  

![Figure 3: Virtual bottleneck](image)

Due to this delay a vehicle approaching this point has an increased travel time \( t \) for a highway section of length \( L \) upstream the bottleneck:

\[ r = r_0 + w = r_0 + \frac{1}{c - q} \]  

where:

- \( r_0 \): desired travel time under completely free flow.

Then the travel velocity on this section of length \( L \) is:

\[ v = \frac{L}{r} = \frac{v_0}{1 + \frac{r_0}{L(c - q)}} \]  

where: \( v_0 \): free flow speed.

This equation could be treated as an approximation for the free flow part of the \( q - v \)-diagram (fig. 2). If we invert the diagram into specific travel times per km (= tardity or slowness) it reminds us to one special application in transportation planning i.e. the capacity restraint curves for traffic assignment procedures in highway transportation planning (fig. 4).

Many papers have been published which focused the problem from that side. And, indeed, this opens a quite realistic aspect: The are two types of traffic volume:

- demand of vehicles approaching a freeway section,
- realised traffic flow at a point.

As long as demand is below capacity both types of volume are identical. If, however, demand exceeds capacity for a limited duration then some proportion of the demand can only be realised with some delay which causes a larger travel velocity. In this sense also a freeway section operates like an intersection where we can identify the bottleneck more precisely. So in many times the highway functions like a storage area. The lower part of the diagram can be well represented by a curve based on a simple dependency between the average headway and speeds, e.g.

\[ v = \frac{a \cdot q}{1 - b \cdot q} \]  

\[ a = \frac{\frac{1}{R}}{1 - b \cdot q} \]
where: \(a\) = average length occupied by one vehicle in jammed and standing traffic [m].
\(b\) = parameter.

The parameters \(a\) and \(b\) could be evaluated by regression techniques. Concluding the discussion about the divided speed-flow relationship it should be mentioned that Jacobs in Beckmann, Jacobs, et al.\(^2\) has already theoretically pointed out convincing reasons to divide the fundamental diagram into two parts which are not compatible with each other.

4.3 Breakdown situations

If we, now, accept that we have two different states of traffic flow, how can we describe the transitions between both? Several observations are available about these transitions, which we call dynamic fundamental diagrams. It is, however, difficult to find out regularities within that process. In many cases the path through that diagram occurs clockwise, i.e. the recovery from congested to free flow occurs at lower volumes than the drop from free to congested, an effect which we call the hysteresis phenomenon (fig. 5). We can obtain from this figure that the breakdown occurred much faster (4 min) than the recovery (10 min).

What we need as the most important progress in our area of science is a well sophisticated theory about these transitions between both states of traffic flow. We suggest that there are 2 basic behavioural pattern of drivers:

- free flow: go ahead and try to proceed as fast as desired,
- congested: drivers have resigned; they change behaviour to the needs of driving in a queue.

Figure 5: Hysteresis of the speed-flow relation with 5-min-data and 1-min-data

The question, under which circumstances some of the drivers decide to resign cannot be predicted precisely. But this decision can be influenced by traffic control. The question is: Should we? We must know that the highest capacities are only reached since drivers are going with unsafe headways\(^2\).

This theory about transitions should provide access to parameters of the process which could be accessed by control techniques to enable stable flows on the outermost point of free flow and to enable recovery strategies from congestion to rather high traffic volumes. For our understanding this theory is still not available to be transferred to practice.

4.4 Applications in guidelines and reality

The application of the fundamental diagram in normal practice is still far away from these theories. The most important application is the planning and projection of freeways. To enable a predetermined traffic flow quality speed-flow-diagrams are used. In Germany the average speed of passenger cars is taken as the most substantial measure of flow quality. The diagrams (fig. 6) figure out the most recent recommendation for autobahns in Germany. The diagrams are based on long term traffic counts at several autobahn sections and on 1-hour counts\(^8\) from the BASf. It should be noted that here the average maximum traffic volume is about 1800 vph/lane whereas the US-HCM 1994\(^7\) suggests 2200 pcy/h and lane as a maximum flow.

These rather low capacities on German autobahns could be confirmed for several sections of the network quite recently by Brüion, Pomzet\(^9\), especially on autobahns with predominating long distance traffic. On the other hand, we have lots of autobahn sections where we also have traffic flows far above 2000 vph/lane every day.

What are the reasons for these differences? Some of the reasons may be supposed as following: High capacity occur only under speed limit conditions and in commuter traffic. The speed limit, however, is only a necessary condition but not a sufficient condition.
The most important aspect for high capacities is the distribution of traffic by lanes. With our German traffic rules which prohibit passing slower vehicles at the right lane on autobahns we have a bad usage of the right lane. Köhler has demonstrated this effect of lane usage on capacity in combination with speed limits. Therefore, for the exploitation of existing freeway facilities it could be useful to adjust our freeway driving rules to American traffic rules, i.e. keep in lane, as the most important rule. However, due to safety reasons overtaking on the right could not be permitted with unlimited speed.

But as an intermediate rule we could allow passing on the right under local speed limit conditions combined e.g. with better speed enforcement and with an encouragement for drivers to stay in lane. Especially on three - and more lane - autobahns this will become the only acceptable rule, since otherwise increased numbers of lanes do not contribute to an adequate increase in capacity. Wetterling has studied these effects based on simulations. What we need are practical implementations, since this is the only way to test the safety impacts. The increase in capacity will occur without any doubt.

4.5 Environmental aspects

Due to environmental aspects one other actual application of the fundamental diagram is available. This is an estimation of fuel consumption or emissions. Fig 8 as an example is based on rather rough emission data. Therefore, these figures give just on outline or a tendency.

Similar figures could be given for fuel consumption or other emissions. To make these relation more reliable, more precise emission data for different traffic flow conditions are needed.

4.6 Other aspects

Another actual remark relates to improved measurement possibilities. In future, control will not be based on local measurements but on data collected from moving cars

- by electronic licence plates as it is already performed in Houston (Texas),
- by GPS navigation combined with auto-telefon systems.

Our theories should try to include issues from that new techniques.

5 Analogy to fluids

Another set of macroscopic but more mathematical interpretations of traffic flow are those models being based on analogies to the flow of fluids. All of these models are based on the two fundamental equations, being mentioned above (eq. (8) and (1)). The first interpretation by Lighthill and Whitham gave us insight into the propagation of Shock waves in traffic flow. One very important application of this part of the theory is the prediction of the propagation of congestion and standing queues, e.g. due to an accident.

Here a series of working aids has been proposed, e.g. by Leutzbach, Willmann and other, to enable easy congestion remedial strategies for traffic control purposes on a manual basis. Moreover, computerized finite element methods have been proposed, mainly by Michalopoulos. Based on this type of approach, in our days congestion management strategies should be available on a computerized basis for on-line operation for the whole autobahn network.

The traffic fluid analogies have, of course, reached theoretical performance far beyond propagation of congestion and queues. By the introduction of further analo-
gies to the movement of fluids more detailed understanding of specific phenomena could be reached:

- Boltzmann-like kinetic gas theory analogy by Prigogine\(^{20}\),
- introduction of the Navier-Stokes-differential equation by Kühne\(^{13}\), Kerner, Konhäuser\(^{11}\) and Helbing\(^{6}\).

 Especially this sequence of improvements provided better understanding of several phenomena within traffic flow:

- e.g. the representation of periodical breakdowns in stop-ad-go-traffic by theory which can be observed in reality (fig. 9),
- or the identification of the fact that an upcoming disturbance could be detected from increasing speed variance.

 At this point it should, however, be noted that these fluid analogies are mainly based on a continuous flow model. And there is much indication that this continuous flow transitions are not real (cf. chapter 4).

6 Microscopic models

An important role in traffic flow theory has always been played by models based on supposed mechanisms to describe the process of one car following the other. The easiest assumption is to relate the headway between two cars to the speed.

For instance, standard driving rules being taught to drives and being demanded by jurisdiction tell us to keep 2s headway as a minimum, i.e. \(\Delta s = 2v \) (m) where \(v = \text{speed (m/s)}\). Transferred into a fundamental diagram this, of course, would replicate a capacity of only 1800 veh/h per lane.

Many other more or less complicated assumptions for the relation between headway and speed could be formulated (e.g. van Aerde\(^{3}\)). To transform them into a traffic flow model requires the additional assumption that all vehicles are performing with identical speeds and identical headways, which, of course, is unrealistic.

Therefore, this model basis might be useful for driver education or for discussion by journalists. In practical application this family of models could only be used to provide a type of regression line to represent congested flow conditions on a regression basis. But, by no means it is possible to transfer rough microscopic driver behaviour patterns into macroscopic traffic flow models.

A rather similar set of microscopic models is known as the car following theory. Here a more dynamic driver behaviour is represented by a differential equation (13) which produces the acceleration at one time instant \(t + T\) out of:

- relative speed,
- distance,
- and absolute speed

at a time instant \(t\).

\[
\ddot{x}_{n+1}(t + T) = \alpha - \frac{\dot{x}_n^{\text{new}}(t + T)}{[x_n(t) - x_{n+1}(t)]^2} \cdot (\dot{x}_n(t) - \dot{x}_{n+1}(t))
\]  

(13)

Here, the reaction time \(T\) tries to model the fact that a human driver always has to suffer a time lag of around 1s between stimulation and reaction. It is important to know, that the parameters within this model \(\alpha, m, \dot{t}\) have to be evaluated from observations. There is a long series of such observations. The largest set of measurements of this type in Germany has been performed by Hoehf\(^{11}\). Here one of the most interesting aspects is that the sensitivity of drivers in a contracting platoon is significantly larger than in an expanding queue of vehicles. Of course,
due to safety reasons, this is a very natural behaviour. But this could be one of the explanations for the hysteresis phenomenon.

Car following models based on these older types of calibration are, however, not realistic in that sense that they could also encompass unrealistically large accelerations and even decelerations. More recent publications by Gipps 4 and Oeaki 19 provide also calibrations for the case of limited acceleration and decelerations.

The application of these models is more directed on the investigation of driver behaviour and its consequences especially for vehicles bound in platoons. One of the important early traffic flow theory publications in Germany e.g. was produced on this field by Leutzbach, Bocellus 14.

For the simple linear car following model (i.e. with no speed and distance dependency; i.e. m = 0 and l = 0) Herman et. al. 9 have indicated criteria which describe the conditions that a stable traffic flow can be maintained.

Köhler 12 derived also stability criteria for the more complicated model based on Hoefs' parameter calibration. This study made clear, that there is a second substantial safety threshold for space headways of cars in freeway traffic. The first threshold is determined from the dynamics of a sudden emergency breaking of the leading car. This is the threshold being demanded by traffic rules. The second threshold is the limit below which headways provoke inevitable rear end collisions, if vehicles are following behind each other in a longer chain with such small distances. Each driver within a platoon who keeps a longer distance interrupts this chain of imminent danger and thus contributes to improved safety for all the others following behind him. It could be worthwhile to repeat Köhler's investigations with up-to-date calibrated parameters.

However, fig. 10 also illustrates one problem of the models. The points on the slide are observed car following processes which nearly all fall below the limit of stability. And also in reality we see that nearly nobody is keeping the distances required for stability. Nevertheless, the number of collisions is not as large as it could be expected from these model derivations. Therefore, the model is not perfectly realistic in these consequences. One reason could be that the drivers can also react on the second or third car ahead of them. Another reason could be that driver's sensitivity in the car following process could depend on the degree of risk.

And this aspect makes us also aware of the most severe problematic aspect of the conventional car following theory: it assumes identical behaviour pattern for each driver. Thus, this model family has a significant lack of randomness properties. Maybe, that in future investigations also car following models with randomly varied parameters come into use.

There is, however, one application where randomness is not important. That is the study of stability criteria for automated car-following vehicle control systems. In this sense these models have already been used:

- to study radar-based headway warning systems 12,
- and to study quite modern infra-red based automated follow-the-leader systems where vehicles could follow each other at distances below 5 m at high speeds.

Finally it should be mentioned for completeness, that car-following models can be transferred into macroscopic traffic flow relations 17. By choosing different values for the parameters nearly each of the conventional traffic flow models can be replicated. This step, however, is always based on unrealistic assumptions of constant speeds and headways. Therefore, let us classify this application of the car following theory as more academic.

Among the microscopic models we finally must pay attention to the so-called psycho-physical models of driver behaviour. They have been mainly formulated by Wiedemann, 22 based on empirical work from several other sources. These models transfer limits of human perception and limits of vehicle performance into an integrated model framework. To characterise these psycho-physical car-following models we should, however, mention that no attempt is known up to now to transform this theory directly into a macroscopic traffic flow model. The only known application is connected with simulation, where the Karlsruhe University made lots of contributions. Here, by the work of Rekabmirk 21 it has also been solved to express these driver behaviour models by fuzzy-set approaches.

7 Simulation models

These microscopic models lead us directly to simulation of traffic flow. This means: All the individual cars along the freeway are represented by a set of numbers in a computer. The parameters of the model can be varied in a very realistic manner.
according to well-known stochastic distributions. Also driving rules and known 
behavioural characteristics can be represented by either car-following theory mod-
els or psycho-physical models and can be included into more or less sophisticated 
computer programs. 
Each run of the computer program would produce a set of results to describe 
macroscopic parameters. Repeated applications with different flows will also pro-
duce relations between macroscopic parameters. However, also here the basic rule for 
models applies, that a model outcome can never be better than the fundamental 
assumptions. Here at the moment the most severe problem is to formulate valid 
algorithms for the lane changing behaviour. Nevertheless, simulation models which 
have been produced under various acronyms by researchers around the world seem 
to have the highest degree of trust-worthiness in our times. They are not only used 
to predict traffic patterns. Moreover, enhanced by computer simulation tools they 
are also used for estimations of fuel consumption and emissions and moreover for 
the technical layout of car components, e.g. gearboxes and automatic transmissions, 
to adjust these parts as close as possible to prevailing traffic flow conditions.

8 Conclusion
This paper tried to open a rather wide framework of model structures for application 
on traffic flow. The facets of these theories are such wide that it seems not to be 
not possible to make expertise about all the existing models present in one 
paper and even not in one head. So we could all learn from each other on our way to the 
most ideal traffic flow model which

- provides understanding for all details of vehicle movement on freeways,
- is realistic in all aspects,
- is easy to understand,
- is easy to apply.

Even if this ideal model may never be found, let us hope that this workshop provides us 
some progress into that direction.

References
1. van Aerde, M., A single regime speed-flow-density relationship for freeways 
and arterials, Paper No. 950802 presented at the 74 TRB Annual meeting, 
2. Beckmann, H., Jacobs, F., Lenz, K.-H., Wiedemann, R., Zacker, H., Das 
Fundamentaldiagramm - Eine Zusammenstellung bisheriger Ergebnisse (The 
fundamental diagram - A assembly of existing results), Forschungsarbeiten 
aus dem Straßenwesen, Heft 89, Kirschbaum Verlag Bonn - Bad Godesberg, 
3. Brilon, W., Ponzet, M., Auswirkungen von zeitlich veränderlichen Leis-
tungsfähigkeiten (consequences of time-dependent capacities), Final report of the 
research project PE-Nr. 01 127 0905, for the federal DOT, Nov. 1995.
4. Gipps, P. G., A behavioural car-following model for computer simulation, 
5. Hall, F.L., Montgomery, F.O., The investigation of an alternative interpre-
tation of the speed-flow relationship for U.K. motorways, Traffic Engineering 
6. Helbing, D., Improved fluid-dynamic model for vehicular traffic, Physical 
7. HCM, Highway Capacity Manual. Transportation Research Board, Special 
8. Heidemann, D., Hotop, R., Verteilung der Plötz-Geschwindigkeiten im Netz der 
Bundesautobahnen - Modellmodifikation und Aktualisierung (Distribution of 
passenger-car-speeds in the German Autobahn network - Modification and 
updating), Straße und Auto, 1990.
11. Kerner, B. S., Konhäuser, P., Makroökopische Verkehrssimulation - 
Einsätzmöglichkeiten und numerische Lösungsverfahren (Macroscopic traffic sim-
Simulation of traffic - Working group on simulation in connection with the 
10th Symposium on simulation techniques, pp 105-130, Vieweg pub., Berlin, 
1995.
12. Köhler, U., Stabilität von Fahrzeugkolonnen, Schriftenreihe des Instituts für 
Verkehrsweisen, Universität Karlsruhe, Heft 9, 1974.
13. Kühne, R. D., Macroscopic freeway model for dense traffic, stop-start waves 
and incident detection, Proc. 9th Int. Symp. Transportation and Traffic 
14. Leutzbach, W., Bezzu, S., Probleme der Kolloidfahrt (Series Straßenbau 
15. Leutzbach, W., Willmann, G., Zustandsformen des Verkehrsablaufes: 
Übergang vom freien zum teilgebundenen Verkehr auf zweistreifigen Rich-
tungsfahrbahnen, Forschungsbericht des BMV, Institut für Verkehrsweisen, 
Universität Karlsruhe, 1975.
flow on long crowded roads, Proceedings of the Royal Society, London, Series 
17. May, A. D., Keller, H., A deterministic queueing model, Transportation 
18. Michalopoulos, P. M., Baksou, D. E., Improved continuum models of freeway 
flow, Proc. 9th Int. Symp. Transportation and Traffic Theory, VNU Science 
10th Int. Symp. Transportation and Traffic Theory, pp 349-366, Elsevier, 
(1993).

PARTICLE HOPPING VS. FLUID-DYNAMICAL MODELS FOR TRAFFIC FLOW

Kai NAGEL
Los Alamos National Laboratory, TSA-DO/SA, MS M997,
Los Alamos NM 87545, USA

and
HLRZ, KFA Jülich, 52425 Jülich, Germany

Although particle hopping models have been introduced into traffic science in the 1990s, their systematic use has only started recently. Two reasons for this are, that they are advantageous on modern computers, and that recent theoretical developments allow analytical understanding of their properties and therefore more confidence for their use. In principle, particle hopping models fit between microscopic models for driving and fluid-dynamical models for traffic flow. In this sense, they also help closing the conceptual gap between these two. This paper shows connections between particle hopping models and traffic flow theory. It shows that the hydrodynamical limits of certain particle hopping models correspond to the Lighthill-Whitham theory for traffic flow, and that only slightly more complex particle hopping models produce already the correct traffic jam dynamics, consistent with recent fluid-dynamical models for traffic flow. By doing so, this paper establishes that, on the macroscopic level, particle hopping models are at least as good as fluid-dynamical models. Yet, particle hopping models have at least two advantages over fluid-dynamical models: they straightforwardly allow microscopic simulations, and they include stochasticity.

1 Introduction

Vehicular traffic has been a widely and thoroughly researched area in the 15 and 60th. 1,2,3 Vehicular traffic theory can be broadly separated into two branches: Traffic flow theory and car-following theory. Traffic flow theory is concerned with finding relations between the three fundamental variables of traffic flow, which are velocity $v$, density $\rho$, and flow $q$. Of these three variables are independent since they are related through $q = \rho v$. Car-following theory regards traffic from a microscopic point of view: the behavior of each vehicle is modeled in relation to the vehicle ahead. As the name indicates, this theory concentrates on single lane situations where a driver reacts to the movements of the vehicle ahead of him. Mathematical car-following theories use differential delay equations. A more recent addition to the development of vehicular traffic flow theory is the particle hopping model. Imagine a one-dimensional chain of cells, each cell either empty, or occupied by exactly one particle. Movement of particles is achieved by particles jumping from one cell to another according to specific movement rules. The context of vehicular traffic, one can imagine a road represented by cells where a particle can hop exactly one car. A rough representation of car movements then is given by moving cars from one cell to another. Actually, the first proposition of such a model for vehicular traffic is from Gerlough 4 in 1956 and has been further extended by Crow and coworkers. 5

*permanent affiliation