USEFUL ESTIMATION PROCEDURES FOR CRITICAL GAPS

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ABSTRACT

Many different methods for the estimation of critical gaps at unsignalized intersections have been published in the international literature. This paper gives an overview of some of the more important methods. These methods are described by their characteristic properties. For comparison purposes a set of quality criteria has been formulated by which the usefulness of the different methods can be assessed. Among these one aspect seems to be of primary importance. This is the objective that the results of the estimation process should not depend on the traffic volume on the major street during the time of observation. Only if this condition is fulfilled, can the estimation be applied under all undersaturated traffic conditions at unsignalized intersections. To test the qualification of some of the estimation methods under this aspect, a series of comprehensive simulations has been performed. As a result, the maximum likelihood procedure (as it has been described by Troutbeck) and the method developed by Hewitt can be recommended for practical application.

INTRODUCTION

The estimation of critical gaps from observed traffic flow pattern is one of the most difficult tasks in empirical traffic engineering science. Miller [1], in his classic paper, could refer to nine different estimation methods, which did not cover the whole range of possible procedures to be obtained from international literature at that time. Today it would be easy to find more than 20 or 30 methods published around the world for the estimation of critical gaps. All of these methods produce different results. Therefore, the important question is: which of these procedures being recommended by different authors reveals a correct estimation? And the other question is: How can we find out if an estimation is valid or not?

Before we can answer these questions we should first discuss the fundamental definitions. Here we concentrate on the most simple case of an unsignalized intersection. This is a crossroad of two one-way streets (Figure 1). Here two movements are allowed:

- One major stream (= priority movement) of volume \( q_p \)
- One minor stream of volume \( q_m \)

According to traffic rules, each major stream vehicle can pass the intersection without any delay. A minor street vehicle, however, can only enter the conflict area if the next major vehicle is far enough away to allow the minor vehicle safe passage of the whole conflict area. “Far enough” is defined as: The next major street vehicle will arrive at the intersection at an instant that will happen \( t_r \) seconds after the previous major stream vehicle or \( t_r \) seconds after the minor vehicle’s arrival. This value \( t_r \) is called the critical gap. In other words:

\[
t_r = \text{critical gap} = \text{minimum time gap in the priority stream that a minor street driver is ready to accept for crossing or entering the major stream conflict zone}
\]

Another limiting factor for the minor street vehicles is the fact that they cannot enter the conflict area during a short while after the previous minor street vehicle has entered, owing to the physical length of the vehicles and the necessary headways. Thus, as the second variable for the characterization of minor street driver’s behavior we use the move-up time, \( t_r \):

\[
t_r = \text{move-up time} = \text{time gap between two successive vehicles from the minor street while entering the conflict area of the intersection during the same major street gap}
\]

It is obvious that \( t_r \) and \( t_r \) differ from driver to driver, from time to time, and between intersections, types of movements, and traffic situations. Due to this variability, there is no doubt that the gap acceptance process is of a stochastic nature. Thus \( t_r \) and \( t_r \) can be regarded as random variables. Moreover, the parameters of the distribution functions for these variables may be subject to different external influences. It is, therefore,
necessary to define some type of representative characteristics to model the usual behavior of drivers. Therefore, the estimation of critical gaps and move-up times tries to find out values for the variables \( t_s \) and \( t_p \) as well as for the parameters of their distributions, which represent typical driver behavior at the investigated intersection during the period of observation.

For this paper we concentrate our derivations on the critical gap, \( t_s \).

In unsignalized intersection theory, it is generally assumed that drivers are both consistent and homogeneous.

- Consistent drivers are expected to behave the same way every time in all similar situations. This means a driver with a specific \( t_s \) value will never accept a gap of less than \( t_s \) and he will accept each major stream gap larger than \( t_s \). However, within a population of several drivers, each of whom behaves consistently, different drivers could have their own \( t_s \) values. These \( t_s \) values are then treated as a random variable with a special statistical density function \( f_s(t) \) and a cumulative distribution function \( F_s(t) \).

- The population of drivers is homogeneous if each subgroup of drivers out of the population has the same functions \( f_s(t) \) and \( F_s(t) \).

It is clear that in reality drivers are neither completely consistent nor homogeneous. A completely inconsistent driver would apply a new \( t_s \) value for each gap. Further, the applied \( t_s \) value that is compared with one major stream gap is completely independent of the \( t_s \) used for the previous major stream gap by the same driver some seconds before in the same queuing situation. This is, however, not expected to be the case in reality. Instead it is assumed that a rather careful driver will always demand a rather large gap or that a risky driver will always be prepared to accept rather narrow gaps. Therefore, we assume that real driver behavior is closer to consistency than to completely inconsistent behavior.

For the estimation of critical gaps, \( t_s \), from observations, a long series of methods has been proposed. An overview on the status of English literature in the late 1960s was given by Miller [1]. Meanwhile, many more proposals have been made. For the preparation of this paper a selection of candidate procedures has been made, which does not represent by far everything that is published. The selection has been made on the basis that these procedures have been used or recommended by authors other than the original sources. In the first part of this paper these methods are described.

**ESTIMATION TECHNIQUE FOR SATURATED CONDITIONS: SIEGLOCH’S METHOD**

Siegloch [2] proposed a consistent framework for the theory of capacities at unsignalized intersections. This should be mentioned here to emphasize how the critical gaps are used within subsequent mathematical modeling. Let \( g(t) \) be the number of minor street vehicles that can enter the conflict area during one minor street gap of size \( t \). The expected number of \( t \) gaps within the major stream is \( q_p \cdot h(t) \) where \( h(t) \) is the statistical density function of all gaps (or headways) in the major stream. Thus, the amount of capacity that is provided by \( t \) gaps per hour is \( q_p \cdot h(t) \cdot g(t) \). To get the total capacity \( c \) we have to integrate over the whole range of possible major street gaps \( t \). Thus we get

\[
c = \int_{t=0}^{\infty} q_p \cdot h(t) \cdot g(t) \, dt \tag{1}
\]

This equation for the capacity of unsignalized intersections forms the foundation of the whole gap-acceptance theory. Almost all of the different analytical capacity estimation formulae found in international literature are based on this concept, even in cases where the original authors where not aware of this method.

The consequence of this equation is that, for capacity calculations, we need to know the major stream headway distribution \( h(t) \) and the function \( g(t) \). Siegloch, as a consequence of this theory, proposes a regression technique for the derivation of \( g(t) \) from field observations. For this estimation technique we need to observe saturated conditions, i.e., continuous queuing on the minor street. Only under these conditions can we observe realizations \( g \) for the function \( g(t) \) by counting the number of minor street vehicles that enter major street gaps of size \( t \). Of course, the realizations \( g \) are always integer numbers. The observation results can be plotted into a graph, as shown in Figure 2. In almost all cases investigated by the authors (and that is quite a large number), the observation points are arranged in such a way that a linear approximation for the representation of measurement points is justified. Therefore, a linear regression function is used to represent the observation data where \( t \) is the dependent variable and \( g \) is the independent variable:

\[
t = a + b \cdot g \tag{2}
\]

where the parameters \( a \) and \( b \) are the outcome of the regression analysis.

It is useful to calculate the average \( t_s \) (from the observed \( t \) values) for each observed \( g \) value before starting the regres-
sion. Thus, for every $g$ value within the sample, only one $t$ value ($= t_k$) is used. Otherwise, the more numerous observations for the smaller $g$ would govern the whole result. Experience shows that, in almost every case, the average $t_k$ values show only small deviations from a straight line.

The straight line for $t = f(g)$ would be exactly correct, if $t_c$ and $t_f$ were constant values. In that case Equation 2 could be written as

$$
g(t) = \begin{cases} 
0 & \text{for } t < t_0 \\
\frac{t - t_0}{t_f} & \text{for } t \geq t_0 
\end{cases}
$$

(3)

where

$$
t_0 = t_c - \frac{t_f}{2}
$$

Therefore, $t_c$ and $t_f$ can be evaluated from the regression technique directly. Some authors have classified this technique for critical gap estimation as deterministic, which is not correct. Instead this technique fully considers the stochastic nature of gap acceptance.

The combination of Equations 1 and 3 together with the assumption that $h(t)$ can be described by the exponential distribution leads to the well-known Siegel formula for the capacity of an unsignalized intersection of the simple type shown in Figure 1:

$$
c = \frac{3600}{t_f} \cdot e^{-\frac{t_c}{t_f}}
$$

(4)

The advantage of Siegel's procedure for the estimation of $t_c$ and $t_f$ is its close relation to the subsequent capacity theory. The drawback for practical application is the fact that this method can only be applied for saturated conditions, which are difficult to find in many practical cases.

**ESTIMATION TECHNIQUES FOR UNDERSATURATED CONDITIONS**

The Lag Method

It is more complicated to estimate the critical gap, $t_c$, from traffic observations with undersaturated conditions. One simple method could be based on lags. A lag is the time from the arrival of the minor vehicle until the arrival of the next major vehicle. We assume:

- Consistent drivers
- Independence of the minor street vehicle arrival time and the traffic situation on the major street

Then the proportion $P_{a_{lc}}(t)$ of drivers who accept a lag of size $t$ is identical to the probability that a driver has a $t_c$ value smaller than $t$. Thus we can state

$$
P_{a_{lc}} = F_c(t)
$$

(5)

From this consideration we could derive the first method of critical gap estimation for undersaturated conditions.

All lags should be measured using traffic observations at an unsignalized intersection. Whether a lag has been accepted or rejected should also be noted. Then the time scale is divided into $W$ segments of size $\Delta t$, e.g., $\Delta t = 1$ s. For each interval $i$ we look at

- $N_i$ = number of all observed lags within interval $i$
- $A_i$ = number of accepted lags within interval $i$
- $a_i = A_i / N_i$

If $t_i$ is the time at the center of interval $i$, then

$$
F_c(t_i) = a_i
$$

(6)

which is an approximation of the cumulative distribution function of critical gaps. The mean critical gap then is

$$
t_c = \sum_{i=1}^{W} t_i \cdot \left[ F_c(t_i) - F_c(t_{i-1}) \right]
$$

(7)

where

- $W$ = number of intervals of size $\Delta t$

Similarly, the standard deviation for the distribution $F_c(t)$ could be estimated.

For practical application this method has some drawbacks. For the method, in each interval, $i$, a sufficiently large sample should be available. This demands very long observation periods because with low major street traffic flow it takes a while to observe enough smaller lags, and with large major street volumes most minor street vehicles have to queue before they can enter the conflict zone. Consequently, although a large number of drivers' decisions have been observed, there will be very few lags that can be used for this estimation procedure.

An estimation procedure is needed that makes use of observed rejected and accepted gaps (i.e., not only lags) because they also contain information about the size of the critical gap for the drivers who have been observed.

Another disadvantage of this method is that it only addresses rather relaxed situations where no queuing occurs. An additional problem could be that the critical value for the lags might be systematically different from that for the gaps. As a result of all of these problematic aspects, the lag method is not used in
practice. It provides us only some insight from a theoretical point of view.

Fundamental Considerations for Further Methods

Given the reasons mentioned before, what is really needed for critical gap evaluations under undersaturated conditions is a procedure that also extracts information from those drivers who accept a gap after queuing. The proportion of accepted lags and gaps, \( p_{\text{lag}}(t) \), is no longer the distribution of \( t_c \). The reason is that a driver who accepts a gap has selected among several gaps that were provided to him. In this case, the distribution of all major stream gaps affects the distribution of accepted gaps. Moreover, among the rejected gaps those drivers with large \( t_r \) are overrepresented, since they reject many more gaps than drivers who apply small \( t_r \) values. Given this behavior, more considerations are necessary.

If we observe a driver on the minor street and his gap acceptance/rejection decisions, we can state that his \( t_r \) is greater than the maximum rejected gap and \( t_r \) is smaller than the gap he accepts. This is true if the driver behaves consistently (see above). If we observe a series of accepted gaps, \( t'_r \), then these accepted gaps can be described by an empirical statistical distribution function, \( F_x(t) \) (see Figure 3).

On the other hand, we can observe the distribution \( (F_x(t)) \) of rejected gaps. Here it is, however, a question of which types of rejected gaps are included in this distribution. Three different definitions are in use:

a) Only the largest rejected gap for each driver is taken into account. If a minor street driver was able to accept the first lag, he has not rejected any gap. In this case, this driver could be withdrawn from the sample (= Case A1) and his accepted lag would not be evaluated for the estimation procedure, or the rejected gap for this driver could be defined as 0 (= Case A2).

b) All observed rejected gaps are taken into account. In this case, if one individual minor street driver was waiting for a while, a longer series of rejected gaps would be included. The distinction of Case A must also be made here:

Case B1: Drivers accepting a lag are omitted.

Case B2: For drivers accepting a lag, the largest rejected gap is defined as 0.

Regardless of the definition a) or b) being used, we know that this distribution \( F_r(t) \), illustrated in Figure 3, must be to the left of the desired distribution \( F_x(t) \). On the other hand, the function \( F_r(t) \) of accepted gaps must be to the right of the \( F_x(t) \) distribution. This is a result of the fact that for each individual consistent driver \( t_r < t_c < t_a \).

Because the distribution function \( F_r(t) \) cannot be observed directly, it is the purpose of all of the following procedures to estimate the function \( F_r(t) \) as validly as possible or to estimate at least its typical parameters, such as the expectation, median, or variance.

Raff’s Method

The earliest method for estimating critical gaps seems to be that of Raff [3]. His definition translated into our terminology means that \( t_c \) is that value of \( t \) at which the functions

\[
1 - F_r(t) \quad \text{and} \quad F_x(t)
\]

intercept. Miller [1] gave some additional mathematical interpretations for this method. He also points out that the results of this \( t_c \) estimation are sensitive to the traffic volumes under which they have been evaluated. Raff’s method was used in many countries in earlier times; for example, Retzko’s work [4] introduced this procedure to Germany.

Ashworth’s Method

Under the assumption of exponentially distributed major stream gaps with statistical independence between consecutive gaps and normal distributions for \( t_r \) and \( t_c \), Ashworth [5, 6, 7] found that the average critical gap \( t_c \) can be estimated from \( \mu_c \) (= mean of the accepted gaps \( t'_r \); in s) and \( \sigma_c \) (= standard deviation of accepted gaps) by

\[
t_c = \mu_c - p \cdot \sigma_c^2 \tag{8}
\]

with \( p = \) major stream traffic volume (vps). If \( t_c \) is not normally distributed, the solution might become more complicated. However, for a gamma distribution or a log-normal distribution of \( t_r \) and \( t_c \), Equation 4 is still a close approximation. Miller [1] provides another correction method for the special case that the \( t_r \) are gamma distributed. Then the two equations apply

\[
t_c = \mu_c - p \cdot \sigma_c^2
\]

\[
\sigma_c = \frac{\mu_c}{\mu_r} \tag{9}
\]

from which \( t_r \) and \( \sigma_r \) are to be obtained by substitution.

For our evaluations we used Equation 8.

Harders’ Method

Harders [8] has developed a method for \( t_c \) estimation that has become rather popular in Germany. The whole practice for unsignalized intersections in Germany is still based on \( t_r \) and \( t_c \) values, which were evaluated using this technique. The method only makes use of gaps (i.e., as in Case B1 above).
method is similar to the lag method discussed in "Estimation Techniques for Undersaturated Conditions—The Lag Method." However, for Harders' procedure [8], lags should not be used in the sample. The time scale is divided into intervals of constant duration, e.g., \( \Delta t = 0.5 \) second. The center of each interval is denoted by \( t \). For each vehicle queuing on the minor street, we have to observe all major stream gaps that are presented to the driver and, in addition, the accepted gap. From these observations we have to calculate the following frequencies and relative values:

\[
N_i = \text{number of all gaps of size } i, \text{ that are provided to minor vehicles}
\]

\[
A_i = \text{number of accepted gaps of size } i
\]

\[
a_i = \frac{A_i}{N_i}
\]  

(10)

Now these \( a_i \) values can be plotted over the \( t \). The curve generated by doing this has the form of a cumulative distribution function. It is treated as the function \( F_i(t) \). However, nobody has provided any conclusive mathematical concept that this function \( a_i = \text{function of } t \) has real properties of \( F_i(t) \). Instead the approach might be a misunderstanding of the lag method.

Part of the method is that each gap \( t < 1 \) s is assumed to be rejected and that each gap \( t > 21 \) s is assumed to be accepted. For practical application, it is not guaranteed that \( a_i = \text{function of } t \) is steadily increasing over the \( t \), which should be the case for \( F_i(t) \). Therefore, the \( a_i \) values are corrected by a floating average procedure, where each \( a_i \) is also weighted with the \( A_i \) values. Finally, the estimation of \( t \) is given by the expectation of the thus formed \( F_i(t) \) distribution function. From the descriptions, this method appears to be a more pragmatic solution without a strong mathematical background.

Logit Procedures

A couple of methods have been proposed [9] that can be summarized as logit models, as they provide similarities to the classical logit models of transportation planning [10]. In each case, the models lead to a function of the logit type. One typical formulation for this family of models follows.

Each minor street driver waiting for a sufficient gap has to judge between the two alternatives

\[
i = \text{accept the gap for the crossing or merging maneuver}
\]

\[
j = \text{reject the gap}
\]

A driver, in his decision situation, \( d \), will expect a specific utility from his decision. This utility can be regarded as a combination of safety on one side and low delays on the other side. We regard the total utility \( U_{id} \) as an additive combination of a deterministic term \( V_{id} \) and a random term \( \epsilon_{id} \):

\[
U_{id} = V_{id} + \epsilon_{id}
\]

\[
U_{jd} = V_{jd} + \epsilon_{jd}
\]

(11)

We assume that the deterministic component \( V_{id} \) can be computed from attributes that can be evaluated by objective measurement techniques. Here we use as one possible solution a linear utility function.

\[
V_{id} = \alpha + \beta_1 \cdot x_{id1} + \beta_2 \cdot x_{id2} + \ldots + \beta_K \cdot x_{idK}
\]

\[
V_{jd} = \alpha + \beta_1 \cdot x_{jd1} + \beta_2 \cdot x_{jd2} + \ldots + \beta_K \cdot x_{jdK}
\]

(12)

where

\[
\alpha, \beta_1, \beta_2, \ldots, \beta_K = \text{parameters}
\]

\[
x_{idi} = \text{value of the } k\text{-th attribute in situation } d\text{ in case of acceptance}
\]

\[
x_{idj} = \text{value of the } k\text{-th attribute in situation } d\text{ in case of rejection}
\]

\[
K = \text{number of attributes}
\]

The random component \( \epsilon_{id} \) includes all influencing factors that cannot be evaluated precisely or that are a result of really random elements of the decision process.

We do, however, assume that the drivers, on average, make rational decisions, i.e.; they make those decisions which provide the highest utility for them. Thus the probability \( p_i(t) \) of acceptance of a gap by a driver is

\[
p_i(t) = p(U_{id} > U_{jd})
\]

\[
p_i(t) = p(\epsilon_{id} - \epsilon_{jd} \leq V_{id} - V_{jd})
\]

(13)

For the random component \( \epsilon_{id} \) we assume a Gumbel distribution [10]. Then the difference \( \epsilon_{id} = \epsilon_{jd} - \epsilon_{id} \) has a logistic distribution, i.e.:

\[
F_{\epsilon_i}(x) = \frac{1}{1 + e^{-\mu \cdot x}}
\]

(14)

\[
f_{\epsilon_i}(x) = \frac{\mu \cdot e^{\mu x}}{(1 + e^{\mu x})^2}
\]

(15)

where

\[
\mu = \text{parameter of the distribution}
\]

Therefore, Equations 13 and 14 can be written as

\[
p_i(t) = \frac{1}{1 + e^{-\mu \cdot (V_{id} - V_{jd})}}
\]

(16)

Within the product \( \mu \cdot (V_{id} - V_{jd}) \) the factor \( \mu \) can be included into the parameters \( \alpha \) and \( \beta \) (see Equation 12). For the special case that only one attribute is observed (\( K = 1 \)) we get
\[ p_i(t) = \frac{1}{1 + e^{\alpha + \beta \sum_{k=1}^{n} x_{ik}}} \]  \hspace{1cm} (17)

As attributes we can use the size of the present gap, time that the minor street driver has spent in the queue, speed of the major street vehicle, driving direction of the next arriving major vehicle in case of a two-way street, etc.

So far the model formulation is much the same as within the classical logit models of transportation planning. If, however, we analyze Equation 16, we see that the \( x_a \) and \( x_d \) (which could be the gap until the next arriving major street vehicle) are the same if the gap is accepted (i) or rejected (j).

Therefore, the difference of attributes is not used within the equation. Instead the attribute itself is introduced into Equation 14 or 16. Thus Equation 16 becomes:

\[ p_i(t) = \frac{1}{1 + e^{\alpha + \beta \sum_{k=1}^{n} x_{ik}}} \]  \hspace{1cm} (18)

and Equation 17 (for \( k = 1 \) and attribute \( x_d = \text{major stream gap} \) ) takes the form

\[ p_i(t) = \frac{1}{1 + e^{\alpha + \beta t}} \]  \hspace{1cm} (19)

Now, to derive the critical gap \( \tau \), we understand \( p_i(t) = \text{function (gap size \( t \))} \) (= probability that a driver in situation \( d \) accepts a gap of size \( t \) ) as a statistical density function for a random variable \( T \). Then the critical gap is defined as the median of this random variable \( T \), that is:

\[ \tau = \text{value of } T, \text{ for which } \int_0^\tau p_i(t) \, dt = 0.5 \]  \hspace{1cm} (20)

Finally the parameters \( \beta, \beta_1, \beta_2, \ldots, \beta_k \) are estimated by a maximum likelihood technique. As an example Pant and Balakrishnan [11] have used this kind of logit model with \( \alpha = 0 \) and \( K = 11 \) for different attributes.

To solve this model we have to determine the log-likelihood function. This, for the model formulation of Equation 19, is given by:

\[ L(\alpha, \beta) = \sum_{i=1}^{n} \left[ \ln \left( \frac{1}{1 + e^{\alpha + \beta \sum_{k=1}^{n} x_{ik}}} \right) + \alpha + \alpha \cdot y_d + \beta \cdot t_d - \beta \cdot y_d \cdot t_d \right] \]  \hspace{1cm} (21)

where:

\[ y_d = 1 \text{ if a driver in situation } d \text{ accepted a gap and } 0 \text{ if a driver in situation } d \text{ rejected a gap} \]

\[ n = \text{number of observed decisions} \]

\[ t_d = \text{gap size offered to a minor street driver in situation } d \]  \hspace{1cm} (s)

The maximum of \( L(\alpha, \beta) \) can be determined by forming the derivatives and setting both as zero:

\[ \frac{\partial L}{\partial \alpha} = \sum_{i=1}^{n} \left[ \ln \left( \frac{e^{\alpha + \beta \sum_{k=1}^{n} x_{ik}}}{1 + e^{\alpha + \beta \sum_{k=1}^{n} x_{ik}}} \right) + 1 - y_d \right] = 0 \]  \hspace{1cm} (22)

\[ \frac{\partial L}{\partial \beta} = \sum_{i=1}^{n} \left[ \ln \left( \frac{e^{\alpha + \beta \sum_{k=1}^{n} x_{ik}}}{1 + e^{\alpha + \beta \sum_{k=1}^{n} x_{ik}}} \right) + t_d - t_d \cdot y_d \right] = 0 \]  \hspace{1cm} (23)

These two equations could be solved iteratively. Also, Equation 21 could be maximized using a spreadsheet maximizing technique. This (using Quattro Pro, version 5) is the method that has been employed for the following analyses.

The maximization of \( L(\alpha, \beta) \) reveals values for \( \alpha \) and \( \beta \) in Equation 19. Since this is the distribution function of a logistic distribution, Equation 20 can be solved for \( \tau \) as the mean of this distribution, which is

\[ \tau = \frac{\alpha}{\beta} \]  \hspace{1cm} (24)

The variance of the critical gap thus can be estimated as

\[ \sigma_\tau^2 = \frac{\pi^2}{3\beta} \]  \hspace{1cm} (25)

Finally, it should once again be noted that this family of models allows the evaluation of other external effects on the critical gap by using Equation 18 instead of Equation 19. Then the log-likelihood function (see Equation 21) must be formed for this more complex model. As attributes we then have to include other external influencing parameters in addition to the major stream gap (see text below Equation 17).

For an example of this type of logit estimation, see Figure 4, which illustrates an evaluation using the simulations mentioned below for Case B1, i.e., situations in which the driver could accept the first lag were omitted.

Probit Procedures

Probit methods for the estimation of critical gaps have been used since the 1960s [12] (see also references given by [11]).

The formulation for this type of models is quite similar to the logit concept. In their original form, however, these models do not use the utility term. Instead, the size of the critical gap, \( t \), is directly randomized by an additive term, \( \epsilon \). Thus we formulate for a consistent driver \( d \):

\[ t_{c,d} = \tau + \epsilon \]  \hspace{1cm} (26)

where

\[ t_{c,d} = \text{critical gap for driver } d \]  \hspace{1cm} (s)
The probability that a driver will accept a major street gap of size \( t \) is

\[
p_s(t) = p(t \geq t_{c, e}) = p(t \geq \overline{t}_c + e_s)
\]  

(27)

For the probit model it is assumed that the random component \( e_s \) is normal distributed with mean 0 and standard deviation \( \sigma_e \). Then Equation 27 can be further developed into

\[
p_s(t) = N(t \mid \overline{t}_c, \sigma_e)
\]

(28)

where

\[
N(t \mid \overline{t}_c, \sigma_e) = \text{cumulative distribution function of a normal distribution with mean } \overline{t}_c \text{ and standard deviation } \sigma_e
\]

Using the standardized form for the normal distribution, this equation can be written as

\[
p_s(t) = \Phi\left(\frac{t - \overline{t}_c}{\sigma_e}\right)
\]

(29)

where

\[
\Phi(z) = \text{value for the standardized cumulative normal distribution function at point } z
\]

The terms \( \overline{t}_c \) and \( \sigma_e \) are parameters of the model. They can be evaluated by regression techniques [1] for the probit if the proportion of accepted lags is used as an estimate for \( p_s(t) \). With this technique, the method is nearly identical to the lag method. If gaps were also included, the technique has all the problems mentioned previously for the lag method. Therefore, Hewitt [13, 14] proposed a correction strategy to the basic probit method to account for the bias caused by multiple rejection of gaps by drivers applying a large \( t_c \) value. This technique is discussed in the following section.

Another important contribution to probit estimation techniques has been given by Daganzo [15]. Here a theory has been proposed that estimates \( t_c \) based on the whole history of rejected gaps and the accepted gap for each individual minor street driver. A normal distribution is applied for the \( t_c \) and its variance over the whole population of drivers as well as for the random term \( e_s \) (see Equation 26). The model can only be solved by special software for multinominal probit estimation techniques. However, it may not be certain that a solution for the parameters will be found. Therefore, this approach seems to be too complicated for practical application.

Mahmassani and Sheffi [20] propose a probit model that accounts for the influence of waiting time at the stop line on the gap acceptance behavior of drivers. The number of gaps that a driver has rejected before he accepts a gap is one parameter of the model. Here also a log-likelihood function is given that allows a maximum-likelihood estimation based on probit theory. The estimation leads to a solution in which the critical gap depends on the number of rejected gaps. This type of solution could be useful as input for simulation models if the concept proves to be realistic, based on ample empirical research. The solution containing the number of rejected gaps is, however, not useful for application in guidelines (e.g., Chapter 10 of the HCM) or other analytical capacity calculations. Here the theories allow only for one typical fixed value of \( t_c \) to be introduced into further calculations.

One problem with all probit approaches is that the normal distribution may not be adequate to be applied for critical gaps since a significant skewness of the \( t_c \) distribution must be expected. The concept of probit estimations has been included in our analyses via Hewitt's solution.

Hewitt's Method

Hewitt [13, 14, 16, 17] has published a series of papers on the estimation of critical gaps. For full explanation of the details of the different procedures, the reader is referred to the original sources. However, a short characterization of the method is given here.

Again the time scale is divided into intervals of constant duration, e.g., \( \Delta t = 1 \text{ second} \). The center of each interval \( i \) is denoted by \( t_i \). The method uses an iterative procedure. As a first approach for the gap acceptance function \( F(t) \), the lag method is used. However, for the purpose of analytical tractability, \( F(t) \) in the first step is estimated according to the probit method. This leads to values for the probability that \( t_i \) is inside the interval \( i \), which is denoted as \( c_{t_i} \), where the index \( 0 \) stands for the 0th step of iteration.

Subsequent theoretical derivations lead to formulae for the expected number of accepted and rejected lags and gaps, which are given in the following table.

<table>
<thead>
<tr>
<th>used</th>
<th>as lags</th>
<th>as gaps</th>
<th>for</th>
</tr>
</thead>
<tbody>
<tr>
<td>accepted</td>
<td>( \beta \cdot N \cdot c_i \cdot f_j )</td>
<td>( \beta \cdot N \cdot c_i \cdot \frac{f_j}{1 - F_i} )</td>
<td>( j \geq i )</td>
</tr>
<tr>
<td>rejected</td>
<td>( \beta \cdot N \cdot c_i \cdot f_j )</td>
<td>( \beta \cdot N \cdot c_i \cdot \frac{f_j}{1 - F_i} )</td>
<td>( j \leq i )</td>
</tr>
</tbody>
</table>
where

\[ c_i = \text{probability that the critical gap is inside interval } i \]
\[ f_i = \text{probability that a major stream gap is inside interval } i \]

(Distributions of lags and gaps are assumed to be identical as is the case for the exponential distribution.)

\[ F_i = \text{value of the cumulative distribution function for major stream gaps at the center of interval } i \]
\[ \beta = 1 \quad \text{for } j \neq i \]
\[ = 0.5 \quad \text{for } j = i \]

Applying this set of formulae, we can compute the number of accepted and rejected gaps and lags from a given set of \( \{c_i\}\).

From these putative values, a new estimation of the \( \{c_i\}\) can be computed, e.g., by using a probit estimation technique. This set of \( \{c_i\}\) is imbedding a new estimation for \( \tau_i \). Again from these new \( \{c_i\}\) (applying Equation 30), new numbers of accepted and rejected lags are calculated, which again are the basis for the \( \{c_i\}\) and so on. This iteration is repeated until the subsequent \( \tau_i \) values become nearly unchanged by the next iteration.

The only information to be extracted from observations for each time interval \( i \) of duration \( \Delta t \) are:

- Total number of gaps
- Number of rejected gaps
- Total number of lags
- Number of rejected lags

For practical application, some additional aspects have to be observed if some of the time intervals are not filled up with sufficient empirical values. Then adjacent intervals have to be amalgamated. Instead of a probit estimation procedure for the \( \{c_i\}\), which assumes a normal distribution for \( F_i(t) \), a log-normal distribution could also be applied. The whole estimation procedure is included into a set of computer programs called GAPTIM and PROBIT [18]. These programs have been used for our investigations to analyze the Hewitt method.

**Maximum Likelihood Procedures**

Maximum likelihood techniques for the estimation of critical gaps seem to go back to Miller and Pretty [19] (for more detail see [1]). The method has been described in a more precise form by Troutbeck [20]. To understand the basic elements of this method, let us assume that for one individual minor street driver \( d \) we have observed:

\[ r_d = \text{largest rejected gap} \quad (s) \]
\[ a_d = \text{accepted gap} \quad (s) \]

The maximum likelihood method then calculates the probability of the critical gap \( \tau \) being between \( r_d \) and \( a_d \). To estimate this probability, the user must specify the general form of the distribution \( F_i(t) \) of the critical gaps for the population of drivers and then assume that all drivers are consistent. The likelihood that the driver’s critical gap will be between \( r_d \) and \( a_d \) is given by \( F_i(a_d) - F_i(r_d) \). The likelihood \( L' \) that within a sample of \( n \) observed minor street drivers, the two vectors of the \( \{r_d\} \) and \( \{a_d\} \) have been obtained is given by the product:

\[ L' = \prod_{i=1}^{n} \left( F_i(a_d) - F_i(r_d) \right) \quad (31) \]

The logarithm \( L \) of the likelihood \( L' \) is given by

\[ L = \sum_{i=1}^{n} \ln \left( F_i(a_d) - F_i(r_d) \right) \quad (32) \]

In practice, the log-normal distribution is often used as the distribution of the critical gaps \( \tau \). The mean critical gap within this distribution has been found to be an acceptable quantity for the representation of average driver behavior [1, 20].

The likelihood \( L' \) is also maximized when the logarithm \( L \) of the likelihood is maximized. Appropriate values for the critical gap distribution parameters (the mean and the variance) are found by setting the partial derivatives of \( L \) with respect to these parameters to zero. This leads to a set of two equations depending on the vectors of the observed \( \{r_d\} \) and \( \{a_d\} \). These two equations have to be solved by iterative numerical solution techniques. Troutbeck [20] describes a procedure for estimating the critical gap parameters using this maximum likelihood technique in more detail. This numerical method has been used to estimate \( \tau \) for the investigations described in this paper.

**CRITERIA FOR THE CLASSIFICATION OF ESTIMATION METHODS**

Before we can compare the different estimation procedures, we have to define the requirements for a useful procedure to estimate critical gaps. These requirements cannot be derived using mathematics. Instead we propose the following set of criteria.

**Distribution**

The critical gap \( \tau \) is not a constant value. Instead it is a variable term where a variation has to be expected between different drivers and for each individual driver over time. Therefore, the critical gaps that drivers apply for their decision-making process at unsignalized intersections are distributed like a random variable. The distribution is characterized by:

- A minimum value as the lower threshold, which is \( \geq 0 \)
- An expectation \( \mu \) (average critical gap or mean critical gap; which often is also denoted as "the critical gap")
- A standard deviation $\sigma$
- A skewness factor, which is expected to be positive, i.e., having a longer tail on the right side

This distribution or its parameters cannot be directly estimated because the critical gap cannot be observed in an individual driving situation. Only the rejected and accepted gaps can be measured. Therefore, procedures have to be established that try to estimate the distribution or their parameters as closely as possible. Normally the accepted and/or rejected gaps are used as a basis for this estimation.

Consistency
An estimation procedure should be consistent. This means that, if the minor street drivers within a specific composition of traffic streams have a given distribution of critical gaps, then the procedure should be able to reproduce this distribution rather closely. The procedure should at least reproduce the average critical gap quite reliably.

These reproduction qualities should not depend on other parameters such as traffic volumes on the major or minor street, delay experienced by the drivers, or other external influences.

Only if this consistency has been proven for a special procedure can the method then be used to study influences of the external parameters on the critical gap. Otherwise, the influences being found by empirical studies might be due to the inconsistency of the estimation procedure and those influences may not really be related to the external parameters being investigated.

For most of the well-known estimation procedures for critical gaps, this consistency has not been proven. There is a strong feeling that a great deal (if not the majority) of all relationships between critical gaps and other parameters (such as traffic volumes, time in queue, delays at the stop line as service times of the imbedded queuing system, and geometric characteristics of intersections, which normally are studied at different sites under different traffic volumes) found in the literature might not exist in reality because the differences may just be a result of this inconsistency.

Robustness of the Method
This aspect, as it is already mentioned by Miller [1], has been emphasized by Hewitt [18] in discussing the experiences described in this paper. It means that the results of estimation procedures should not be too sensitive to the assumptions being made, for example, assumptions about the distributions of critical gaps or of major stream headways.

Another factor that deserves mention is the size of the standard error of the mean critical gap. It is possible that a method that produces a slight bias but a small standard error might be preferable to an unbiased method with a large standard error [18].

Capacity Model Compatibility
The estimation of critical gaps is not an end in itself. Critical gaps are used in models for capacity computation at unsignalized intersections. Critical gaps estimated by different procedures could have different influences on the model’s capacity output. It should be guaranteed that the estimated critical gap, in conjunction with the move-up times $t_k$ (and their estimation procedure), gives a reliable and realistic estimate of the capacity independent of the external parameters, especially the major street volume.

In no case is it sensible to use a capacity computation model with critical gaps estimated in a way that is not related to the model. In other words, the estimation procedure for critical gaps and the capacity model (as well as the consecutive delay model) must form one integrated unit.

It has to be stated that this capacity model compatibility is only given for the Siegleoch method, as shown above. For each of the other estimation procedures, the method of critical gap estimation and the capacity (as well as delay) calculation methods have no theoretical connections. Some methods, however, produce results that cannot be handled by the usual theoretical concepts for capacity calculations, for example, some of the probit results (see above).

SIMULATION STUDIES
Description of the Simulation Concept
These qualities of an estimation procedure, especially consistency, can only be checked for by a simulation model, since an analytical approach is not available. To this end, an extended simulation study for testing different $t_k$ estimation procedures for consistency has been performed. The two traffic streams (see Figure 1) have been generated by randomized procedures. For each simulation run, a combination of constant traffic volumes $q_p$ and $q_s$ has been given. $q_p$ was varied between 100 and 900 vph. $q_s$ was varied between 0 and the capacity $c$, which depends on $q_p$. Each such simulation run has been performed for constant traffic volumes over 10 hours. Thus, 46 different combinations of $q_p$ and $q_s$ (where $q_s < \text{capacity}$) have been performed for two different cases, which are combinations of $t_k$ and $t_p$ values. The first case may represent minor street right turn behavior, whereas the second case could stand for driver behavior of left turners from the minor street.
The critical gaps, $t_c$, and the move-up times, $t_m$, for each simulated driver have been generated according to a shifted Erlang distribution using the parameters mentioned before. Each driver is assumed to be consistent, that is, he maintains his generated $t_c$ value until his departure. Generated $t_c$ values and $t_m$ values outside the margins of minimum and maximum values indicated above have been replaced by these given extreme values.

Moreover, to achieve a realistic pattern of headways, the so-called hyper-Erlang distribution [21], [22] has been applied for the major stream traffic flow generation where traffic on one single lane has been assumed. Also, the arrivals of minor stream vehicles have been generated according to this type of distribution. These complicated distributions generate traffic situations of great variability, which are similar to realistic conditions.

Within each simulation run the critical gaps, $t_c$, have been estimated out of the simulated flow patterns according to the abovementioned estimation methods. A series of 46 estimations for $t_c$ has been obtained for each method.

Results for the Critical Gap Analysis

Figure 5 illustrates the results of the critical gap analysis. It presents all the $t_c$ estimated by Hewitt's method for different $q_o$ values. Here they are plotted over the major stream traffic volume $q_o$, which was generated by the simulation. This type of relation, after some comparisons, turned out to be the most sensible as results in relation to minor stream flows $q_o$ showed much less variation. Each cross represents one $t_c$ value. The crosses plotted one above the other were produced for different $q_o$. For both traffic flows the simulated volumes (not the predetermined values for the generation of headways) were used in the following graphs.

We see that the outcome of the Hewitt estimation process reveals $t_c$ values between 5.63 and 5.98 seconds for a correct value of 5.8 seconds and between 7.08 and 7.52 seconds for a correct value of 7.2 seconds. Thus, a rather large variation of the results must be stated. For the $t_c$ values, a regression analysis regarding their relation to $q_o$ has been performed. In this case the regression line was rather horizontal and it was close to the correct $t_c$ value in both cases. That means that the Hewitt method on average reproduces the correct $t_c$ value very well and its results do not depend on major street volumes. Thus, this method fulfills our most important quality criterion, consistency, with a rather high performance. We shall see that this is not the case with most of the other methods.

From all the other methods studied, only the maximum likelihood method (Figure 6, denoted as the Troutbeck method) comes up to the same performance as the Hewitt method. Here again the regression line shows no relation to major street volumes and the correct $t_c$ values are on average close to the estimation results. Again, a rather large variation of the results obtained for different minor street volumes can be recognized from the plots. Nevertheless, these two methods fulfill the requirements formulated above, especially the most important, consistency.

Similar plots for the illustration of simulation results have also been produced for the other methods studied. For the logit method (Figure 7) we see that the results become very different if we either use gaps and lags or only gaps. In our example (only $t_c = 5.8$ seconds has been evaluated) only the exclusive use of gaps (no lags have been used in the second sample) reproduces on average the correct $t_c$. The inclusion of lags into the estimation process leads to a remarkable underestimation of the critical gap. However, in each of the approaches, the results show a significant dependency on the major street traffic volume. Thus the logit procedure fails the consistency criterion, and consequently cannot be recommended as a useful estimation procedure in the form as it is described above. It could be that a correction to the procedure might become available.

For the Ashworth method (Figure 7) we see that most of the estimation results are smaller than the correct values. Again, as the results do not seem to be correct, this procedure (at least in its uncorrected version as shown in Equation 5) is not recommended for application.

A rather disastrous result has evolved from the simulations both for the Raff and the Harders procedures. The Raff method meets the correct values only for median major street traffic volumes, whereas Harders' method leads to a tremendous overestimation of critical gaps. Only if Harders' method includes the lags in the evaluation process are some improvements for the average size of the resulting $t_c$ values obtained. The even more important problem is, however, that the results coming
out of both types of procedures have a strong relationship to the major street traffic volumes, a feature that should be avoided. Therefore, neither method should be used for reliable critical gap estimation.

For the estimation method according to Siegloch we find better performance. As with the Hewitt and Troutbeck methods, both example values of $t_e$ are on average a correct replication of the true $t_e$ values. In addition, the regression lines are not too much related to major street traffic volumes; i.e. the major street flow does not systematically influence the estimation results. Therefore, the Siegloch method seems to be a useful estimation technique.

The simulation for the Siegloch method allows us to evaluate the capacity with the predetermined $t_e$ and $t_f$ values because a continuous queue has to be generated for the simulation. As a result, the number of minor street vehicles able to pass the conflict zone is exactly the capacity of the intersection. The results from a series of 10-hour simulation runs have been noted. They were described by a regression function (see Figure 10):

$$ c = A \cdot e^{-k_p} \quad (33) $$

which is of the same type as Siegloch’s capacity function (Equation 4). The values for $A$ and $B$ have been estimated according to the principle of least squares using a spreadsheet technique without prior linear transformation. This type of function allows another type of estimation of $t_e$ and $t_f$:

$$ t_f = \frac{3600}{A} \quad t_e = B \cdot \frac{t_f}{2} \quad (34) $$

The simulated capacity results and the regression function (“simulated”) are represented in Figure 10. The regression has an extremely high $r$ value, which indicates a very good correlation. However, the $t_e$ and $t_f$ value obtained from the regression line are different from the true values. The relation between capacity and major flow, as it would be calculated from Siegloch’s original formula with the true values for $t_e$ and $t_f$, is also indicated in the graph of Figure 10 (“calculated”). Both curves for the capacity show significant differences. Therefore, the robustness criterion is not fulfilled. The reason is that the Siegloch formula (Equation 4) is only exactly valid for constant $t_e$ and $t_f$ values and Poisson major street traffic flows, whereas the simulation has been performed for more realistic circumstances, especially with non-Poisson major stream arrival patterns. These results indicate that the Siegloch estimation technique is sensitive to the major street headway distribution.

Figure 11 gives an overview of all the regression lines obtained.

Results for the Variance of Estimated Critical Gaps

The simulated 10 hours of constant flows do not represent a sample that could be observed in reality. Under practical circumstances, measurements can only be taken for one or two hours. Therefore, we also studied the convergence behavior of some of the methods. Here the results for the maximum likelihood method are mentioned. Figure 12 shows the range of minimum and maximum $t_e$ values obtained from observation intervals of different durations. We see that for 1-hour (and longer) intervals, the variability is less than 0.2 second.

The traffic volume should also be regarded as an additional influencing factor. Figure 13 shows the standard deviation in relation to the true value as a function of the number of minor street vehicles simulated. On the horizontal axis a logarithmic scale has been used. Above a sample size of 100 minor street vehicles, the standard deviation is less than 0.3 second. The relation shown in Figure 13 seems to be more generally valid than that shown in Figure 12.

CONCLUSION

A review of publications about estimation of critical gaps reveals many different proposed solutions. It is difficult to understand which procedure is reliable and which is not. From the sample of methods tested for this paper, the maximum likelihood procedure and Hewitt’s method gave the best results. Both were valid for the two cases studied. This explains the selection of the maximum likelihood method for the evaluation of critical gaps for the next edition of the HCM, Chapter 10 [23].

The investigation of the different theoretical concepts shows that principles of the various methods could also be combined. In the future, even more estimation techniques for critical gaps might be proposed.

References


16. Hewitt, R. H. Analysis of Critical Gaps Using Probit Analysis. Paper presented at the Second International Workshop on Unsignalized Intersections in Bochum. 1988. (The English version of the paper is unpublished, a copy is available from the authors of this paper on request; a German version has been published: Hewitt, 1993)


Figure 1. Illustration of the Basic Queuing System

Figure 2. Illustration of Siegloch’s Method
(The points illustrate the observed values for \( g \). The circles represent the average \( t \) values for each \( g \). The line indicates the regression equation: \( t = 4.8 + 2.9 \cdot t \), from which we can obtain the estimations \( t_c = 6.25 \) s, \( t_f = 2.9 \) seconds.)

Figure 3. The Distribution Function \( F_c(t) \) of the Critical Gaps (Must be situated between the distribution functions of rejected gaps \( F_r(t) \) (here Case A2). Lags are treated as \( t_c = 0 \) and the distribution function \( F_a(t) \) for accepted gaps.)

Figure 4. Example for the Logit Estimation (Obtained from the simulation runs mentioned below for \( q_b = 800 \) vph and \( q_i = 200 \) vph. The resulting values are: \( a = 6.61, b = -1.01, t_c = 6.54 \) seconds.)
Figure 5. Results from the Simulation Runs for the Hewitt Method for Two $t_c$ Values

Hewitt's method for $t_c = 5.8$ s

Hewitt's method for $t_c = 7.2$ s

Figure 6. Results from the Simulation Runs for Troutbeck’s Maximum Likelihood Method and for Two $t_c$ Values

Troutbeck 5.8 s

Troutbeck 7.2 s

Figure 7. Results from the Simulation Runs for the Logit Method and Ashworth’s Method. (For logit only, the case for $t_c = 5.8$ seconds has been evaluated.)

logit model (lags and gaps) $t_c = 5.8$

logit model (only gaps) $t_c = 5.8$

Ashworth $t_c = 5.8$ s

Ashworth $t_c = 7.2$ s
Figure 8. Results from the Simulation Runs for the Raff and Harders Methods and for Two $t_c$ Values

Raff $t_c = 5.8 \text{ s}$

Raff $t_c = 7.2 \text{ s}$

Harders $t_c = 5.8 \text{ s}$

Harders $t_c = 7.2 \text{ s}$

Harders (with lags) $t_c = 5.8 \text{ s}$

Harders (with lags) $t_c = 7.2 \text{ s}$

Figure 9. Results from the Simulation Runs for Siegloch’s Method

Siegloch $t_c = 5.8 \text{ s}$

Siegloch $t_c = 7.2 \text{ s}$
Figure 10. Simulated Capacities Compared with Results Calculated from Siegloch’s Formula (Equation 4) (using the given critical gap and move-up time.)

Figure 11. Comparison of the Regression Lines for the Relation Between Major Street Traffic Flow and Estimated $t_c$ Values for $t_c = 5.8$ s (a) and $t_c = 7.2$ s (b)

$t_c = 5.8$ s

$t_c = 7.2$ s
Figure 12. Minimum and Maximum Estimates for the Critical Gap in Relation to the Duration of the Observation Period (for the maximum likelihood method and a given $t_i = 5.8$ seconds.)

**Definition of variables:**

- $A_i$ = number of accepted gaps of size $i$ (Harders method) or the number of accepted lags of size $i$ (lag method)
- $a_i = a_i = \frac{A_i}{N_i}$ = rate of acceptance for gaps of size $t_i$ (Harders method)
- $a_i = \frac{A_i}{N_i}$ = rate of acceptance for lags of size $t_i$ (lag method)
- $c$ = capacity = maximum number of minor street vehicles, which can cross the major stream during one hour (veh/h)
- $\Delta t$ = length of time interval for the lag method or Harders method
- $f(t)$ = statistical density function for the accepted gaps
- $F_i(t)$ = cumulative distribution function for the accepted gaps
- $f_i(t)$ = statistical density function for the critical gaps
- $F_i(t)$ = cumulative distribution function for the critical gaps
- $f(t)$ = statistical density function for the rejected gaps
- $F(t)$ = cumulative distribution function for the rejected gaps
- $g$ = observed value for $g(t)$
- $g(t)$ = number of minor street vehicles, which can enter into a major street gap of size $t$
- $h(t)$ = statistical density function for gaps (headways) between vehicles in the major stream
- $L$ = Log-likelihood function
- $L'$ = Likelihood function
- $\mu_x$ = mean of the accepted gaps $t_i$ (s)
- $\mu_x$ = average critical gap or the expectation within the distribution function $F_i(t)$ (s)
- $n$ = number of observed minor street drivers
- $N_i$ = number of all gaps of size $i$, which are provided to minor vehicles (Harders method) or the number of all lags of size $i$, which are provided to minor vehicles (lag method)
- $p$ = major (= "priority") street traffic volume = $q / 3600$ (veh/s)
- $q_a$ = minor street traffic volume (vph)
- $q_p$ = major (= "priority") street traffic volume (vph)
- $s_a$ = standard deviation of accepted gaps $t_i$ (s)
- $s_e$ = standard deviation of critical gaps (s)
- $s_c$ = standard deviation of critical gaps (s)
- $t = time$ (s)
- $t_i$ = accepted gap (s)
- $t_r$ = critical gap (s)
- $t_m$ = move-up time (s)
- $t_t = average t-value for each g (Siegloch method)$ (s)
- $t_i$ = center of the i-th time interval for Harders method (s)
- $t_r$ = rejected gap (s)
- $W$ = number of time intervals (lag method) (-)

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