ABSTRACT

The new HCM 2010 proposes in chapter 11 two-stage curves to represent the speed-flow diagram for non-congested flow on freeways where constant speeds are postulated over a rather wide range of volumes. This division of the traffic flow in two parts plus the congested conditions is a rather pragmatic approach. Further the three steps pretend three separate traffic flow regimes, which don’t exist. A continuous function, like it is used in the other guidelines, would replicate the traffic conditions comprehensibly and realistically. Therefore, a model based on simple queueing theory analogy is presented to represent the non-congested part of traffic flow. Moreover several models which represent each state of traffic flow by one function are discussed. These different approaches are compared to real world data from the US and Germany. Based on this analysis a new single-stage model for the approximation of fundamental diagrams for freeways is recommended. The model can also be modified in order to represent the capacity drop effect.
INTRODUCTION
For use in guidelines and for many practical applications macroscopic models of traffic flow on freeways are required. These models are generally called “fundamental diagram” and represent the dependency between traffic volume v, traffic concentration k, and space mean speed s. They are in the focus of many studies since Greenshield’s first paper [1] upon this topic. The functions being proposed reach from simple linear s-v- or s-k-relationships up to exponential or logarithmic functions. The crucial point is that most of the proposed functions provide useful solutions only for a part of the whole spectrum, such that in many cases two-stage functions are in use. One objective of this paper is to study functions for the fundamental diagram which can describe the whole picture from zero to maximum concentration. The other objective is to identify useful types of functions for the more important part of fluent (non-congestion) traffic.

SPEED-FLOW DIAGRAMS FOR THE HCM
The current Highway Capacity Manual (HCM 2000) proposes speed-flow functions with nearly constant speeds up to rather large traffic flows between 1300 and 1750 pcu/h/lane. Beyond these breakpoints there is a curved decrease in speed until the concentration has reached the critical value k_{krit} = 45 pcu/mile/lane.

The draft version for the new HCM 2010 [2] chapter 11 proposes a new set of functions to represent the speed-flow diagram for US-freeways. These functions again consist of sections of straight lines stating that the free flow speed can be maintained until quite large flows of 1000 to 1800 pcu/h/lane. Beyond these breakpoints speeds are reduced according to a quadratic function (cf. table 1).

<table>
<thead>
<tr>
<th>FFS [mi/h]</th>
<th>Break-Point BP1 [pc/h/lane]</th>
<th>Segment 1: S = v ≤ BP1</th>
<th>Segment 2: S = v &gt; BP1</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>1000</td>
<td>75</td>
<td>75 – 0.000011107 (v – 1000)^2</td>
</tr>
<tr>
<td>70</td>
<td>1200</td>
<td>70</td>
<td>70 – 0.00001160 (v – 1200)^2</td>
</tr>
<tr>
<td>65</td>
<td>1400</td>
<td>65</td>
<td>65 – 0.00001418 (v – 1400)^2</td>
</tr>
<tr>
<td>60</td>
<td>1600</td>
<td>60</td>
<td>60 – 0.00001816 (v – 1600)^2</td>
</tr>
<tr>
<td>55</td>
<td>1800</td>
<td>55</td>
<td>55 – 0.00002469 (v – 1800)^2</td>
</tr>
</tbody>
</table>

The critical point seems to be the wide range of constant average speeds which is assumed here. This may be justified by the driving behavior on US-freeways – especially the rather low speed limits - which causes a rather homogeneous flow. Especially from the European perspective (with higher limits – or even no speed limit at all (Germany) and bad acceptance of the limits) a rather critical view on these assumptions may be justified.

1. For computational purposes one continuous curve would be easier to be used for numerical calculations. The stated simplification using 2-stage curves makes things more complicated. If somebody uses the graph (for a paper and pencil solution) the type of the mathematical function does not matter. In each other case where formulas are used (either by a pocket calculator or by a computer program) one equation would be easier to apply.
2. The two-stage functions might signal to the reader that two different traffic flow regimes must be distinguished. This is not the case. In reality, there is no systematic difference between two traffic flow regimes in fluent traffic.
3. The constant speed for flow rates below the breakpoint volumes seems to be doubtful. Even if the curves are very flat, also in this area a slight decrease of the curves can be expected. Figure 2 shows three graphs with speed-flow data from Interstate I-80 and SR-51 in California. The curves give a strong indication that also in the US a decrease of speeds with increasing flow is occurring also for lower volumes.

![Graphs showing speed-flow data from Interstate I-80 and SR-51 in California.](image)

**Figure 1** Speed-flow diagrams from California (I-80, 15-min-intervals and SR-52, 5-min-intervals).

To avoid these problems an alternative solution for the basic pattern of the speed-flow curves for motorways may be desirable. This paper is discussing some alternatives to the current practice in the HCM. All parts of this paper do only intend to demonstrate potential alternatives for the mathematical form of the fundamental diagram. In the first part of the paper some equations for speed-flow curves are discussed. In the second part these approaches are compared to real world data from the US and from Germany.

**A SIMPLE QUEUEING MODEL**

In many applications only the upper part of the speed-flow relationship is of interest. This is especially valid for the use in guidelines like the HCM since here only this upper part is used. A type of function which has shown a good performance in guideline application has been proposed by Brilon, Ponzlet [3]. This model is based on a simple analogy to a queueing model.

The theoretical background is illustrated using Figure 2, which represents a longer section of a freeway. Along this freeway each point could be treated as a bottleneck for the upstream section of length $L$. 

![Graph illustrating a simple queueing model](image)
Figure 2  Virtual bottleneck.

The demand volume at the entrance to the section is \( v \). Each point at this freeway has a specific capacity \( c \). For the bottleneck we assume a very simple queuing model of type M/M/1. The queuing model M/M/1 has one server, arrivals are Poisson distributed and the service times are exponentially distributed. The storage space for queues is infinite. In such a system the expectation for the total time \( w \) which customers spend in the system is:

\[
\frac{vc}{1} = w
\]

where

\[
\begin{align*}
\text{w} & = \text{delay in front of the bottleneck including service time} \quad [\text{s}] \\
\text{c} & = \text{capacity of the M/M/1-queue} \quad [\text{pc/s}] \\
\text{v} & = \text{demand volume (< c)} \quad [\text{pc/s}]
\end{align*}
\]

This can be used as an estimation of the additional delay due to increasing traffic volumes which the vehicles suffer to pass this bottleneck. In addition the vehicles need, of course, the travel time which is at minimum the time \( r_0 \) at free flow. Thus the total travel time \( r \) for the section under concern is

\[
r = r_0 + w = r_0 + \frac{1}{c-v} = \frac{L}{s_0} + \frac{1}{c-v}
\]

where

\[
\begin{align*}
\text{r}_0 & = \text{travel time at speed } s_0 \text{ for section of length } L \quad [\text{s}] \\
\text{s}_0 & = \text{speed at a volume } \rightarrow 0 \text{ for a section of length } L \quad [\text{m/s}] \\
\text{L} & = \text{length of the freeway section under concern} \quad [\text{mile}]
\end{align*}
\]

Thus, the travel speed \( s(v) \) to cross a section of length \( L \) under traffic volume \( v \) is:

\[
s(v) = \frac{L}{r} = \frac{s_0}{1 + \frac{s_0}{L(c-v)}}
\]

where

\[
\begin{align*}
\text{s}(v) & = \text{travel speed at volume } v \text{ for a section of length } L \quad [\text{m/s}] \\
\text{c} & = \text{parameter} \quad [\text{pc/h/ln}]
\end{align*}
\]
The application of the model shows that it is not useful to estimate $L$, $c$, and $s_0$ according to ostensive constellations of the freeway. Instead $L$, $c$, and $s_0$ should be treated as parameters in this type of model which can be estimated by regression from measured speed-flow data for the fluent part of the speed-flow-diagram (upper branch). In this sense the equations have proven to be quite realistic. Therefore, they are e.g. used in the German guidelines HBS 2001 [4] to represent speed-flow diagrams for German freeways. After extensive tests they are also going to be applied for the future version of the HBS.

These curves would also represent the new HCM (chapter 10) speed-flow curves quite well using the parameters as depicted in Table 2. This is illustrated in Figure 3.

### TABLE 2 Parameters for eq. 3 which approximate the speed-flow functions in chapter 10 of the HCM 2110

<table>
<thead>
<tr>
<th>FF</th>
<th>55</th>
<th>60</th>
<th>65</th>
<th>70</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_0$</td>
<td>55.32</td>
<td>60.98</td>
<td>66.76</td>
<td>71.98</td>
<td>78.22</td>
</tr>
<tr>
<td>$L$</td>
<td>4</td>
<td>1.5</td>
<td>0.95</td>
<td>1</td>
<td>0.7</td>
</tr>
<tr>
<td>$c$</td>
<td>2380</td>
<td>2500</td>
<td>2600</td>
<td>2550</td>
<td>2600</td>
</tr>
</tbody>
</table>

Figure 3  Fitting of eq. 3 to the HCM 2010 speed-flow functions.

### MODIFICATION OF EQ. 3

If a constant part of the curves below a breakpoint flow rate of $v_1$ should be desired then the type of the curve can also be maintained by a slight modification of eq. 3:

$$s(v) = \frac{s_0}{1 + \frac{s_0}{L(c-v_1)}}$$

where

$\begin{align*}
c &= \text{parameter (different from } c \text{ in eq. 3)} \quad \text{[pc/h/ln]} \\
v_1 &= \text{traffic volume at the breakpoint according to table 1} \quad \text{[pc/h/ln]} \\
s_0 &= \frac{Lc-s_{FF}}{Lc-s_{FF}} \quad \text{[m/s]} \\
s_{FF} &= \text{free flow speed; i.e. for a volume } < v_1 \quad \text{[mph]} \\
\end{align*}$
The units given with this eq. 4 are defined according in analogy to the units applied in the HCM 2010. Of course, the equation can be used for any useful combination of speed and flow units. Eq. 4 in combination with the parameters from table 3 yields quite a good fit to the HCM 2010 curves (Figure 4). Due the additional parameter \( v_1 \) in equation 4 the parameter \( c \) here is smaller than \( c \) in equation 3.

| TABLE 3  Parameters for eq. 4 approximating the HCM 2010 speed-flow curves |
|-------------|-----------|-----------|-----------|-----------|-----------|
| FFS         | 55        | 60        | 65        | 70        | 75        |
| \( v_1 \) = | 1800      | 1600      | 1400      | 1200      | 1000      |
| \( s_0 \) = | 64.9      | 70.8      | 77.6      | 85.8      | 92.3      |
| \( L \) =   | 0.3       | 0.28      | 0.25      | 0.2       | 0.2       |
| \( c \) =   | 1200      | 1400      | 1600      | 1900      | 2000      |

**Figure 4** Fitting of eq. 4 to the HCM 2010 speed-flow functions.

**SINGLE-STAGE FUNDAMENTAL DIAGRAM**

It would be a desire of traffic flow analysts to describe the whole fundamental diagram by one single equation. Up to now only one such approach is known, the van-Aerde function. Here two more ideas towards this kind of solution are proposed.

**VAN-AERDE-FUNCTION**

One approach towards a single-stage fundamental diagram is the van-Aerde-solution \([5]\). This function is based on the approximate assumption that the average headway between two vehicles (including the length of the leading vehicle) can be described by eq. 5.

\[
\Delta x = a + \frac{b}{s_0 - s} + c \cdot s \quad [5]
\]

where
- \( \Delta x \) = distance headway between consecutive vehicles [m]
- \( a, b, c \) = parameters of the model
Then the concentration is
\[
k(s) = \frac{1000}{\Delta x} = \frac{1}{a + \frac{b}{s_0 - s} + c \cdot s}
\]  
where
\[
k = \text{concentration} \quad \text{[pc/km/ lane]}
\]

If we look on the average headway between vehicles on all lanes the model can also be calibrated for the concentration expressed in [pc/km or pc/mile]. A slight decrease of the s-k curves and, thus, also for the s-v curves is an essential property of this type of function. This equation can be solved for \( s = f(k) \). However, the solution becomes rather complex. Thus, also the equation for \( s = f(v) \) becomes too complicated for use in practice:

\[
s(v) = \frac{1}{2} \left[ s_0 + \frac{a \cdot v \cdot c \cdot v}{1-c \cdot v} \pm \sqrt{R} \right]
\]  
where
\[
R = \frac{s_0^2 - 2 \cdot a \cdot v \cdot s_0 + 2 \cdot a \cdot c \cdot v^2 \cdot s_0 + a^2 \cdot v^2 - 4 \cdot b \cdot v^4 + 4 \cdot b \cdot c \cdot v^2 - 2 \cdot c \cdot v \cdot s_0^2 + c^2 \cdot v^2}{(1-c \cdot v)^2}
\]

\[
v = \text{volume} \quad \text{[pc/h]}
\]

Also this type of fundamental diagram seems to be attractive to replace the two-stage s-v-curves in the HCM. The difference between both types of curves is so small that it can hardly be illustrated by a graph. There is, however, one sincere drawback, which is the extremely complicated form of the speed-flow function. Therefore, this approach is not very attractive to be adopted for guidelines to be used in practice.

**TWO EXPONENTIAL MODELS**

Two exponential functions have been identified which also fulfill the basic requirements for a complete fundamental diagram. The first exponential type is based on the speed-flow function as it has been proposed by Underwood [1961; 6] or Seddon [1971; 7].

\[
s = s_0 \cdot e^{-b \cdot k}
\]  
This equation is generalized into

\[
s = s_0 \cdot e^{\left(\frac{k}{b}\right)^a}
\]  
where
\[
s = \text{space mean speed} \quad \text{[km/h]}
\]
\[
s_0 = \text{space mean speed at zero flow} \quad \text{[km/h]}
\]
\[
a = \text{parameter (must be an odd integer, preferably 3 or 5)} \quad [-]
\]
\[
b = \text{parameter} \quad \text{[pc/km]}
\]

a and b are parameters to be estimated from empirical data by regression techniques. Here the critical density (density which leads to the maximum flow) is
Then the capacity is

\[ C = q_{\text{max}} = s_0 \cdot b \cdot (a \cdot e)^{(1/a)} \]  \hspace{1cm} [11]

A second exponential type is

\[ s = \frac{2 \cdot s_0}{\left(1 + e^{b \cdot \left(\frac{k}{a}\right)}\right)^a} \]  \hspace{1cm} [12]

where

- \( s \) = space mean speed \ [km/h]
- \( s_0 \) = space mean speed at zero flow \ [km/h]
- \( a \) = parameter \ [-]
- \( b \) = parameter \ [pc/km]

Again \( a \) and \( b \) are parameters to be estimated from empirical data by regression techniques – preferably in the s-k-domain. An analytical solution towards \( k_{\text{krit}} \) is not possible for eq. 12.

Eq. 9 and 12 deliver rather similar results which are shown in Figure 5 for the example from a motorway in Germany which is frequently congested. Here eq. 9, despite the fact that parameter \( a \) can only obtain the values 3 or 5, presents a similar or probably better matching.

![Graph showing speed-flow data](image)

**Figure 5** Eq. 9 and 12 fitted to speed-flow data (1-hour-counts for 1 complete year) from the German motorway A66.

The model according to eq. 12 can be extended to represent the so-called capacity-drop; i.e. a different capacity in fluent traffic compared to congested flow:
where
\[
\hat{s} = \begin{cases} 
\hat{s} + \Delta s & \text{for } k < k_{krit} \\
\hat{s} & \text{elsewhere}
\end{cases}
\] [13]

\[
\Delta s = \text{parameter} \quad [\text{km/h}]
\]

\[
k_{krit} = \text{critical density} = \text{density at maximum flow} \quad [\text{pc/km}]
\]

\[
a, b = \text{parameters}
\]

Again the parameters a, b, \(\Delta s\), and \(k_{krit}\) should be calibrated from field data. One example for the \(s\)-\(v\)-diagram using this model in comparison to eq. 12 for a German freeway is given in Figure 6. In this case eq. 13 reveals a lower RMSE of 6.0 km/h compared to 6.2 km/h obtained from eq. 12. A more detailed fitting of the equations to detector data follows in the final part of this paper.

![Figure 6](image)

**Figure 6** Eq. 12 and 13 fitted to speed-flow data (5-minute-counts for 1 week) from the German motorway A52 near Essen (Parameter estimation by minimizing RMSE in the s-k-Domain)

A well known problem of single equation models is a bad matching to observed speeds near capacity. In difference to the conventional single equation models, eq. 13 represents the speeds at capacity rather well.

CALIBRATION OF THE PROPOSED MODELS TO REAL FREEWAY DATA

To test the practicability of the models comparisons with real data from US-freeways have been performed. Here data from the PeMS-system in California were obtained. The PeMS, a cooperative effort between UC Berkeley, PATH and Caltrans, allows an online access to data from lots of detectors on freeways in California. The data present on the one hand an overview about the actual traffic conditions; on the other hand they afford the download of data recorded during recent years for research projects. The following points within the California and the German freeway network were selected for our analysis.
I-405:
The I-405 is a north-south directed freeway bypassing the city of Los Angeles in the west. It is one of the most frequented roads in the US. The detector 1201159, which has been used for calibration, is situated near the exit Sand Canyon.

I-5:
The Interstate 5 is the main Interstate Highway on the West Coast of the United States, paralleling the Pacific Ocean from Canada to Mexico. It connects some of the largest cities of the western part of the U.S., for example Seattle, Portland, Sacramento, San Francisco, Los Angeles and San Diego. The detectors, which are used for the calibration process, are located south of Los Angeles between Santa Ana and San Diego. The exact locations of the detectors are explained in table 5.

I-80:
The Interstate 80 is the second-longest Interstate Highway in the United States (after I-90). It connects downtown San Francisco, California, to Teaneck, New Jersey, a suburb of New York City. The detector 314460 is situated in the north of Sacramento, California near the exit Greenback Lane.

A 52:
The A 52 is a two lane freeway in the south of Essen, Germany. The detector MQ 177 is located downstream of the exit Essen-Haarzopf. The freeway has a posted speed limit of 120 km/h.

A 7:
Detector A7_10 is located in the north of Hamburg, Germany between the exit Hamburg-Stellingen and the three-leg interchange Hamburg-North-West. The group of detectors extends over four lanes. The freeway has a posted speed limit of 120 km/h.

### TABLE 4 Selected data for the calibration process

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Detector Nr.</th>
<th>Interstate</th>
<th>Lanes</th>
<th>Lane</th>
<th>Direction</th>
<th>Country</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1201159</td>
<td>I-405</td>
<td>4</td>
<td>2</td>
<td>N</td>
<td>US</td>
<td>Sand Canyon</td>
</tr>
<tr>
<td>2</td>
<td>1108723</td>
<td>I-5</td>
<td>4</td>
<td>1</td>
<td>S</td>
<td>US</td>
<td>Leucadia</td>
</tr>
<tr>
<td>3</td>
<td>1108723</td>
<td>I-5</td>
<td>4</td>
<td>2</td>
<td>S</td>
<td>US</td>
<td>Leucadia</td>
</tr>
<tr>
<td>4</td>
<td>1108477</td>
<td>I-5</td>
<td>4</td>
<td>2</td>
<td>S</td>
<td>US</td>
<td>Santa Fe</td>
</tr>
<tr>
<td>5</td>
<td>1108475</td>
<td>I-5</td>
<td>4</td>
<td>2</td>
<td>S</td>
<td>US</td>
<td>Birmingham</td>
</tr>
<tr>
<td>6</td>
<td>1108477</td>
<td>I-5</td>
<td>4</td>
<td>1</td>
<td>S</td>
<td>US</td>
<td>Santa Fe</td>
</tr>
<tr>
<td>7</td>
<td>314460</td>
<td>I-80</td>
<td>4</td>
<td>1</td>
<td>E</td>
<td>US</td>
<td>EB Greenback Lane</td>
</tr>
<tr>
<td>8</td>
<td>A52_177</td>
<td>A 52</td>
<td>2</td>
<td>2</td>
<td>S</td>
<td>Germany</td>
<td>Essen</td>
</tr>
<tr>
<td>9</td>
<td>A7_10</td>
<td>A 7</td>
<td>4</td>
<td>2</td>
<td>N</td>
<td>Germany</td>
<td>Hamburg</td>
</tr>
<tr>
<td>10</td>
<td>A7_10</td>
<td>A 7</td>
<td>4</td>
<td>3</td>
<td>N</td>
<td>Germany</td>
<td>Hamburg</td>
</tr>
</tbody>
</table>

### Calibration Process

The calibration of the models is performed by a regression type of parameter optimization. The parameters in the models have been evaluated by minimizing the mean square error. Minimization was performed by the Solver function within MS-Excel. The quality of the
calibration is mainly tested by the root mean square error (RMSE). The root mean square error is the common method to quantify the amount by which a model differs from measurement data, which are used for the calibration. Other statistical measures like the RMSPE, ME and MPE have also been calculated (for more details: see relevant statistical textbooks, e.g. Cramer, Kamps 2008 [9] or Fomby, T. 2006 [10].

The calibration is divided in two parts. In a first part the HCM 2010-function is compared to the results of equation 3. The second part tests the different equations, which describe the whole fundamental diagram by one single equation.

**Part 1:**

In the first part of the calibration the HCM 2010-equation and eq. 3 are tested for the most exact replication of the traffic conditions on the test sites. Further analysis indicated a more general tendency within this set of parameters. It turned out that the parameters within eq. 3 and 4 respectively are clearly depending on Free Flow Speed (FFS) by the following equations:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>eq. 3</th>
<th>eq. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_0$</td>
<td>$s_0 = FFS \cdot 1.04$</td>
<td>$s_0 = FFS \cdot 1.21 + 4$ (mph)</td>
</tr>
<tr>
<td>$L$</td>
<td>$L = FFS / 130$</td>
<td>$L = FFS / 400$ (mile)</td>
</tr>
<tr>
<td>$c$</td>
<td>$c = 30000 / \sqrt{FFS}$</td>
<td>$c = 25000 / \sqrt{FFS}$ (pc/h/lane)</td>
</tr>
</tbody>
</table>

Table 5 shows the mean error for both models and the ten detectors.

<table>
<thead>
<tr>
<th>Detector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-405</td>
<td>6.48</td>
<td>5.89</td>
<td>2.57</td>
<td>2.78</td>
<td>2.46</td>
<td>6.38</td>
<td>1.22</td>
<td>3.16</td>
<td>3.58</td>
<td>4.84</td>
<td>3.94</td>
</tr>
<tr>
<td>I-5</td>
<td>2.05</td>
<td>1.66</td>
<td>0.984</td>
<td>2.26</td>
<td>1.94</td>
<td>2.53</td>
<td>1.68</td>
<td>1.92</td>
<td>4.82</td>
<td>4.81</td>
<td>2.47</td>
</tr>
</tbody>
</table>

Table 5 illustrates how the two kinds of models perform if a general parameter-set is used for all measurement point. We see a clear advantage for the eq.-3-model. The difference between the results from the HCM 2010 and eq. 3 is particularly justified by the speed at capacity. The results show that the speed reduction due to increasing volumes on the freeway sections under investigation emerges as not so sharp as table 1 (last column) would indicate. Of course, this is a study with a rather limited sample which cannot really doubt the well supported rules in table 1.

**Part 2:**

In the second part of the calibration process those equations which describe the whole fundamental diagram by one single equation are compared. Figure 7, as an example, illustrate the result for the four equations compared to detector data from the I-5 given for 5-minute intervals. In addition, Figure 8 illustrates a comparison of four models to empirical data for eight sites mentioned in Table 4. For the two German motorways (Figure 8, number 8 and 9) the flow depicts the average volume per lane over the lanes at the detector point.
The results of the parameter optimization regarding the measurement sites in Table 4 are given in Table 6.

### TABLE 6 Results of the calibration part 2: RMSE in the s-k-domain [mph]

<table>
<thead>
<tr>
<th>Detector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-5</td>
<td>2.17</td>
<td>4.17</td>
<td>2.71</td>
<td>1.89</td>
<td>3.41</td>
<td>3.01</td>
<td>2.37</td>
<td>2.87</td>
<td>5.05</td>
<td>7.72</td>
<td><strong>3.54</strong></td>
</tr>
<tr>
<td>I-5</td>
<td>2.67</td>
<td>5.74</td>
<td>4.09</td>
<td>3.31</td>
<td>2.57</td>
<td>5.92</td>
<td>6.90</td>
<td>2.84</td>
<td>5.08</td>
<td>4.76</td>
<td><strong>4.39</strong></td>
</tr>
<tr>
<td>I-5</td>
<td>2.71</td>
<td>5.70</td>
<td>4.44</td>
<td>3.25</td>
<td>2.88</td>
<td>5.68</td>
<td>5.42</td>
<td>2.90</td>
<td>3.97</td>
<td>3.86</td>
<td><strong>4.08</strong></td>
</tr>
<tr>
<td>I-80</td>
<td>1.86</td>
<td>2.69</td>
<td>2.82</td>
<td>2.42</td>
<td>1.69</td>
<td>3.81</td>
<td>3.33</td>
<td>2.64</td>
<td>3.89</td>
<td>3.64</td>
<td><strong>2.88</strong></td>
</tr>
</tbody>
</table>

In the last column of Table 6 the errors of the nine sites are averaged. We see that eq. 6 and 13 provide the best fit. This, to some extent, may also be explained by the fact that they contain more parameters than eq. 9 and 12.

In nearly all cases eq. 13, which also represents the capacity-drop, fits best to real world data. It is the only function which offers the potential to characterize the larger volumes at higher speeds.
CONCLUSION

Three basic types of models to represent speed-flow-curves for freeway traffic have been compared. Together with this the constant sections of the HCM 2010 speed-flow relationships have been looked at with a critical view.

The analysis of field data for freeways in California showed a flat speed curve for lower flow rates. Nevertheless, also a slight decrease of speeds with increasing traffic volumes seems to be typical for California freeways. Therefore, the curves according to eq. 3 seem to provide a good representation of speed-flow-relations on US-freeways for the uncongested part of the speed-flow diagram. They could be used to replace the current functions in the HCM on the longer run. Of course, further analysis of comprehensive data from the US is desirable.

For covering the speed-flow-concentration relations by one single function two new types of formulas are presented by eq. 9 and 12. They are compared to the well-known van-
Aerde solution. Here eq. 12 seems to offer an advantageous solution for a single-stage approach to model the fundamental diagram. This model is flexible enough to enable a close calibration to data sets from freeways of different kind, i.e. for speed limit conditions as in the US or for unlimited speeds as in Germany. Also the potential to include an estimation of the capacity drop effect into the model offers advantages for practical application.

REFERENCES