CAPACITY AT UNSIGNALIZED INTERSECTIONS
DERIVED BY CONFLICT TECHNIQUE

Werner Brilon and Ning Wu

Institute for Transportation and Traffic Engineering
Ruhr-University,
D-44780 Bochum, Germany

Phone: +49 234 322 5936
Fax: +49 234 3214151
e-mail:
werner.brilon@ruhr-uni-bochum.de
ning.wu@rub.de

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Abstract

A new simplified theoretical approach for the determination of capacities at unsignalized intersections has been developed based on the method of Additive Conflict Streams (ACS). The method is much easier to handle than the method of gap acceptance. It avoids many of the theoretical complications inherent in the method of gap acceptance which, under certain circumstances, seem to be rather unrealistic. The new method has been developed for potential intersection configurations where one street has priority over the other. A calibration of the model parameters is given for German conditions. The new procedure can deal with shared lanes, short lanes and flared entries and also with cases of so-called limited priority. For the estimation of traffic performance measures - like average delay and queue lengths - the classical methods can be applied.

Keywords: Capacity, unsignalized intersection, traffic flow quality

Author’s address:

Prof. Dr.-Ing. Werner Brilon, Dr.-Ing. habil. Ning Wu
Ruhr-University Bochum
D-44780 Bochum, Germany
Phone: +49 234 3225936
Fax: +49 234 3214151
e-mail: werner.brilon@rub.de & ning.wu@rub.de
1 Introduction

The capacity of unsignalized intersections is analyzed either by the empirical regression method, which is mainly applied in the context of British research results (Kimber, Coombe, 1980), or by the so-called gap acceptance procedure (GAP). The latter is used in many countries of the world (cf. Brilon, Troutbeck, Tracz, 1995) including the US (HCM, 1997 and 2000). For the HCM procedure a comprehensive investigation has been performed by Kyte et al. (1994). A rather recent state-of-the-art analysis of these theories was performed by Kyte in 1997. Other countries like Sweden also use the GAP method in their own capacity manuals. Thus it is correct to say that the theory of gap acceptance is the predominant concept for unsignalized intersection analysis in the world.

At a closer look, however, this concept appears to have a couple of drawbacks which could become a problem in practical application. These are:

- The determination of the critical gap is rather complicated (cf. Brilon, König, Troutbeck, 1997). Some details of the practical aspects of critical gap estimation were described by Tian et al. (2000) and in former publications by these authors. In fact, looking into the details of the process we find that a couple of definitions must be made which are not self-explanatory, and which contain elements of arbitrariness. Their impact on the results is not clear. Thus, it is justifiable to say that the estimation of critical gaps is a source of uncertainty within the GAP method.

- While GAP calculations look like very theoretical mathematics, they are more based on pragmatic simplifications. This applies to the whole treatment of the hierarchy of four ranks of priorities at an intersection. Here, some movements turn up twice, an approach which is based on suggestions by Harders (1968) which were confirmed by simulations by Grossmann (1991). All in all, these calculations produce results in the right order of magnitude which are, however, no more than approximations. Thus, there could be a much simpler approximation which would make the application of an estimation method much easier without losing too much reliability.

- The gap-acceptance theory will not really work well if drivers do not exactly comply with the rules of priority, forcing gaps or, conversely, polite allowing others to proceed (priority reversal). This holds true even though some approaches to solving this problem have been published (Troutbeck et al., 1997; Kita, 1997).

- The gap-acceptance theory completely loses its applicability when it is applied to pedestrians or cyclists at an intersection. For pedestrians, at least on the European continent, rather complicated rules of priority apply, such that pedestrians sometimes have the right of way over cars, and sometimes not. The whole set of rules is neither laid down explicitly in the highway code, nor is it known to many road users. As a consequence, the real behavior both of pedestrians and motor vehicle drivers is highly variable. This variability is, however, not a framework which fits in with the sophistication of the gap-acceptance theory, which needs a clearly defined ranking of priorities, and assumes that each road user will exactly comply with these rules. This aspect will be described in more detail in this paper. The same applies also to cyclists, which may either use the roadway, or separate cycle paths, or some illegal alternatives.

Therefore, it could be of interest to develop a third basic concept of analysis for the operation of unsignalized intersections. As a basis, the concept of Additive Conflict Flows (ACF) appears suitable. First developed by Gleue (1972) for signalized intersection analysis, it was modified by Wu (2000a, b) for application to All-Way-Stop-Controlled intersections (AWSC). In this paper,
the same concept will be developed for Two-Way-Stop-Controlled intersections (TWSC). In this case, the results are even easier to develop and apply than in the AWSC case. With this new procedure, it is easy to take into account
- the number of lanes of the subject, the opposite, and the conflict approach,
- the distribution of traffic flow rates on the approaches,
- the number of pedestrians crossing the legs of the intersection, and
- flared approaches.

2 Departure Mechanisms at TWSC Intersections

2.1 Capacity of Traffic Flows in a Conflict Group

We start our derivation by looking at a conflict between several movements (Fig. 1). Conflict arises from the intersection of several movements which have to pass the same area within an intersection. Consequently, the vehicles belonging to movements involved in a conflict have to pass the area one after the other. A set of movements involved in a particular conflict is called a conflict group.

First we concentrate on the easy case of two conflicting streams (Fig. 1). One of these movements (i=1) is assumed to have priority over the other, established by a yield sign or a stop sign for the minor movement. Then we assume that the conflict area is comparable to a queuing system where each vehicle from movement i - passing this point – takes an average service time of \( t_{B,i} \). The total time available for vehicles from both movements is 3600 s per hour. If we look at the situation where the intersection is oversaturated (i.e. \( Q_2 > C_2 \)) we find that

\[
3600 = Q_1 \cdot t_{B,1} + C_2 \cdot t_{B,2} \quad \text{[veh/h]} \tag{1}
\]

where

- \( Q_i \) = traffic demand for movement i \quad \text{[veh/h]}
- \( C_i \) = capacity for movement i \quad \text{[veh/h]}
- \( t_{B,i} \) = service time for movement i \quad \text{[s]}

with the restriction

\( Q_1 \cdot t_{B,1} \leq 3600 \)

Thus we obtain the following formula describing the capacity of the minor street movement \( i=2 \) :
\[
C_2 = \frac{3600 - Q_i \cdot t_{B_i}}{t_{B_{2}}} = \frac{3600}{t_{B_{2}}} \left(1 - \frac{Q_i \cdot t_{B_i}}{3600}\right) = C_{\text{max,2}} \cdot (1 - B_i) = C_{\text{max,2}} \cdot p_{0,i}
\]

where

\[
B_i = \frac{Q_i \cdot t_{B_i}}{3600}
\]

occupancy by movement i

\[
p_{0,i} = (1 - B_i)
\]

probability that the conflict area is not occupied by movement i

\[
C_{\text{max,2}} = \frac{3600}{t_{B_{2}}}
\]

maximum capacity of movement 1 in case of no conflicting streams

\[B_i\] is the proportion of time during which the conflict area is occupied by vehicles from movement i ("i-vehicles"). Thus \((1 - B_i)\) is the proportion of time during which the conflict area is free from i-vehicles. Thus \((1 - B_i)\) can be interpreted as an estimation for the probability \(p_{0,i}\); i.e. the probability that no i-vehicle is occupying the conflict zone. In analogy to these considerations, further \(p_0\) - values will be defined below.

Now let us look at a conflict group consisting of three movements (Fig. 2). Here, a hierarchy of priorities should apply which resembles the regulation of conflicts at an unsignalized intersection; i.e. movement 1 has highest priority, movement 2 is of intermediate priority, and movement 3 has to yield to both the other movements.

Fig. 2: Conflicts between three movements

Following the logic described above, we can again apply eq. 2 to the capacity of movement 2. For the capacity of movement 3 we can now derive:

\[
C_3 = \frac{3600 - \left( Q_1 \cdot t_{B_1} + Q_2 \cdot t_{B_2} \right)}{t_{B_{3}}}
\]

\[= C_{\text{max,3}} \cdot (1 - (B_1 + B_2))\]

\[= C_{\text{max,3}} \cdot p_{0,1/2}\]

where
\( p_{0,1/2} = [1 - (B_1 + B_2)] \) = probability that the conflict area is not occupied by vehicles from stream 1 or stream 2.

Applying the same technique to a conflict group consisting of four movements (Fig. 3) we obtain the capacity for a movement of rank 4 by:

\[
C_4 = \frac{3600 - (Q_1 \cdot t_{B,1} + Q_2 \cdot t_{B,2} + Q_3 \cdot t_{B,3})}{t_{B,4}}
\]

\[
= C_{\text{max},4} \cdot [1 - (B_1 + B_2 + B_3)]
\]

\[
= C_{\text{max},4} \cdot p_{0,1/2/3}
\]

where

\( p_{0,1/2/3} = [1 - (B_1 + B_2 + B_3)] \) = probability that the conflict area is not occupied by vehicles from stream 1, stream 2, or stream 3.

Fig. 3: Conflicts between four movements

More than four movements cannot occur on a standard cross intersection.

Up to this point, equations were formulated for movements \( i \) having their own average service time \( t_{B,i} \). This basic assumption implies, however, that after completing the formulation of the model a calibration process is needed to arrive at meaningful estimates for the whole variety of \( t_{B,i} \)-values. In reality, however, this is impossible to do since too many sets of data from comparable situations would be needed. Once again, therefore, a simplified set of model assumptions might be useful. Some significant simplification could be achieved if - with sufficient precision - it could be assumed that the \( t_{B,i} \)-values are identical for all movements \( i \). Assuming that this is true, we arrive at the following set of equations:
\[ C_2 = \frac{3600}{t_B} - Q_1 \]
\[ C_3 = \frac{3600}{t_B} - (Q_1 + Q_2) = C_2 - Q_2 \] [veh/h] (5)
\[ C_4 = \frac{3600}{t_B} - (Q_1 + Q_2 + Q_3) = C_3 - Q_3 \]

(For negative results the corresponding capacity is always = 0).

Alternatively, an intermediate degree of simplification of equations 1 to 4 might be useful; i.e. the assumption that highest-ranked through movements (i.e. with absolute priority) have their own - usually lower - \( t_B \)-value \( t_{B1} \), whereas all non-priority and turning movements can be described by a common service time \( t_{BM} \). In this instance, the capacity equations we obtain are:
\[ C_2 = \frac{3600 - Q_1 \cdot t_{B1}}{t_{BM}} \]
\[ C_3 = C_2 - Q_2 \] [veh/h] (6)
\[ C_4 = C_3 - Q_3 \]

Calibration runs will show, however, that these simplifications are not very realistic.

In each of the equations 2 through 6, of course, negative values for capacities \( C_i \) are not allowed. Should a negative value occur, the capacity must be 0.

### 2.2 Involvement of one Stream in more than one Conflict Group

At real intersections, all flows must pass several conflict groups (see Fig. 4 for examples). In AWSC intersections, the capacity of one subject stream is the smallest capacity that can be achieved in each of the conflict groups (cf. Wu 2000a). The reason is the departure discipline according to the traffic rules: The vehicles depart in the sequence in which they arrived at their respective stop lines (first in first out, FIFO). Thus, with queues on all entrances, vehicles from all conflicting movements will wait and give way if (according to FIFO) it is a subject stream vehicle's turn to depart. On the other hand, under saturated conditions, two vehicles from non-conflicting movements will depart simultaneously after stopping, if they arrived approximately at the same time.

At TWSC-intersections we have another set of traffic rules. Priority movements operate independently from each other. Vehicles from priority movements do not stop, occupying the conflict area as soon as they arrive. The arrival process itself is random. A minor-stream vehicle, however, can only depart if all conflict group zones which it needs to cross are free (i.e. not occupied by other vehicles) at the same time. The probability that both conflict areas are free simultaneously is the product of the \( p_i \) values. With that in mind, we find that for the cases described in Fig. 4:
\[ C_x = C_{\text{max},x} \cdot p_{0,A} \cdot p_{0,B} \]

For more than two conflict groups which have to be passed within a hierarchical system of priorities we arrive at:
\[ C_i = C_{\text{max},i} \cdot \prod_{k=1}^{n_i} p_{0,k,i} \quad (7) \]

where

- \( i \) = index for a movement
- \( C_i \) = capacity of movement \( i \) [veh/h]
- \( C_{\text{max}} \) = capacity of movement \( i \) for the case that all other movements have no traffic [veh/h]
- \( k \) = index for a conflict group [-]
- \( p_{0,k,i} \) = probability that conflict group \( k \) is free for movement \( i \) [-]
- \( n_i \) = number of conflict groups which a vehicle from movement \( i \) has to pass [-]

Of course, mathematically speaking, eq. 7 is only valid if all conflict groups operate independently of each other, which is not necessarily the case for \( n_i > 2 \). Thus, to simplify matters, stochastic interdependencies between succeeding conflict groups are neglected when using eq. 7 for \( n_i > 2 \).

**Fig. 4: One movement passing through two subsequent conflict groups**

### 3 Conflict Groups at an Intersection

#### 3.1 Motor Vehicle Movements

We will now look at a simple intersection of two streets. The whole configuration of traffic movements comprises twelve streams of motor-vehicle traffic. Here, as a first approach we assume that on the four approaches of the intersection there is exactly one lane available for each of the twelve movements. In this configuration, the conflict groups outlined in Fig. 5 (5 - 8) and Fig. 6 have been identified (Wu, 2000b). The strategy was that the hierarchical system of priorities according to traffic rules should be represented together with the considerations applying to eq. 2 - 4.
Fig. 5: Four typical conflict groups at an intersection

Fig. 6: Arrangement of conflict groups at a simple cross intersection (for conflict groups 5 - 8: see Fig. 5)
For further derivation operations, the 12 movements at the intersection must be numbered. In this instance, the system applied in the German guidelines was used (Fig. 6). Using the relations formulated in Section 2 of this paper (cf. eq. 2 - 4), we can now define the involvement of each movement in conflict groups and conflicting movements as shown in Table 1. This formulation of conflict groups goes back to Wu (2000a). It has been tested against different alternative definitions, and it represents the traffic rules and the hierarchy of priorities most accurately.

<table>
<thead>
<tr>
<th>Subject Movement</th>
<th>Conflict Group</th>
<th>Higher-priority Ranking</th>
<th>Lower Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Rank</td>
<td>Movement</td>
<td>a</td>
</tr>
<tr>
<td>i</td>
<td>r</td>
<td>k</td>
<td>a</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>priority</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
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<tr>
<td>4</td>
<td>4</td>
<td>6</td>
<td>2</td>
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<td>8</td>
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<td>1</td>
<td>8</td>
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<tr>
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<td>3</td>
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<td>2</td>
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<td>9</td>
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<td>6</td>
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<td>3</td>
<td>2</td>
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<td>7</td>
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<td>7</td>
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<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>priority</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>8</td>
<td>8</td>
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<tr>
<td></td>
<td></td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>2</td>
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<tr>
<td></td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 1: Conflict groups and conflicting movements for each traffic stream at an intersection

Based on these conflicts and the derivations given in paragraph 2, the following set of equations may be formulated very easily by applying the system obtained from Table 1.

\[ C_1 = C_{\text{max,1}} \cdot (1 - B_a) \cdot (1 - B_b) \quad [\text{veh/h}] \quad (8) \]

\[ C_2 = C_{\text{max,2}} \quad [\text{veh/h}] \quad (9) \]
Using the notation from the last row in Table 1, we can express equations 8 - 19 in a more general way:

\[ C_i = C_{\text{max},i} \cdot \prod_{each \, k} \left[ 1 - \left( B_a + B_b + B_c \right) \right] \]  

\[ [\text{veh/h}] \] (20)

The conflict groups \( k \) relating to each individual movement \( i \) are given in Table 1. If columns \( a, b, \) or \( c \) for a specific conflict group \( k \) in Table 1 are empty, then the corresponding \( B = 0 \).

At this point, it should be noted that the above approach so far does not have the qualities of a theoretically precise mathematical model. Instead it is a pragmatic representation of traffic streams at an intersection, and of the mutual obstructions caused by traffic rules. Basically, the gap-acceptance theory is not better qualified.

### 3.2 Pedestrian Movements

Another group of conflicts needs to be taken into consideration if pedestrians are admitted to the intersection (see Fig. 7) as additional elements in conflict groups 1, 2, 3, and 4 at the intersection exits. Moreover, they are of some importance at the intersection entries (conflict groups 9, 10, 11, and 12).

The question is, to which degree do these pedestrians have priority over the automobile traffic? The answer may differ from country to country, depending on national traffic rules. Those laid down in the German highway code (StVO, 1998) as shown in Table 2 specify that:

- In each conflict where pedestrians are crossing the path of a vehicle going straight ahead, the vehicle has the right of way.
- At entrances to an intersection - both minor and major - vehicles have priority over pedestrians.
Pedestrians, however, have priority at the exits over all turning vehicles. Only at a zebra crossing do pedestrians have absolute priority over all vehicle movements. As some concentration is needed to understand these rules, they are not really understood by nor even known to street users, car drivers as well as pedestrians. What is more, there are even some court rulings expressing controversial opinions about these regulations.

As a consequence, the usual practice is that these rules are not enforced in the field. Instead, motor vehicle drivers and pedestrians find their own arrangement in each individual situation. Some preliminary studies by Czytich and Boer (1999) found that for each vehicle movement, pedestrians – in case of a conflict – get priority in a specific proportion A of cases. This means that, in A per cent of conflicting situations, the pedestrian goes first, while the driver waits. Some estimates for A relating to various conflicts are given in Table 2. They were obtained from Czytich and Boer (1999) and generalized by rounding. More generalized values for A based on a larger sample will be analyzed by the authors for German conditions in the next future under a research project funded by the German Federal DOT.
Vehicle Movements \( i \) in ( ) : \( A_{k,f,i} \) - value in %

<table>
<thead>
<tr>
<th>Conflict group ( k )</th>
<th>Pedestrian movement ( f )</th>
<th>Pedestrian movement (i.e. peds have to give priority to vehicles from these movements)</th>
<th>Priority to peds over vehicles from these movements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>F1</td>
<td>8 (0)</td>
<td>4 (30) 12 (70)</td>
</tr>
<tr>
<td>2</td>
<td>F3</td>
<td>11 (10)</td>
<td>3 (70) 7 (30)</td>
</tr>
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<td>F5</td>
<td>2 (0)</td>
<td>6 (70) 10 (30)</td>
</tr>
<tr>
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<td>F7</td>
<td>5 (10)</td>
<td>1 (30) 9 (70)</td>
</tr>
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<td>F2</td>
<td>1 (0) 2 (0) 3 (10)</td>
<td></td>
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<tr>
<td>10</td>
<td>F4</td>
<td>4 (50) 5 (50) 6 (50)</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>F6</td>
<td>7 (0) 8 (0) 9 (10)</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>F8</td>
<td>10 (50) 11 (50) 12 (50)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Definition of pedestrian priority according to the German Highway Code (StVO, 1998); in ( ) : percentages \( A_{k,f,i} \) of pedestrian priority for different conflicts
### Table 3: Conflict groups and conflicting movements for each traffic stream at an intersection, including pedestrians.

<table>
<thead>
<tr>
<th>Subject Movement</th>
<th>Conflict Gr.</th>
<th>Of Higher Priority Ranking Rank r</th>
<th>Conflicting Movements</th>
<th>Lower Rank</th>
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</thead>
<tbody>
<tr>
<td>i r k a b c d e f g h m</td>
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<tr>
<td>1 3 5 8 4 9 5 8 8 9 F7 F2 4 5 11 10</td>
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<td>3 2 9 2 F3 F2 7 11</td>
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<td>4 5 10 6 5 2 8 8 F1 7 1 11 11 F4</td>
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<td>9 2 11 4 F7 F6</td>
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<tr>
<td>10 5 12 8 7 3 8 2 2 F5 1 7 5 5 F8</td>
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<tr>
<td>11 4 12 5 6 2 8 2 3 1 7 7 F8 4 4 F3</td>
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<tr>
<td>12 3 12 1 8 F1 F8 4</td>
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F1 2 1 8 4 12
F2 4 9 2 3 1
F3 5/1 2 F1
F4 6 10 6 5 4
F5 2 3 2 6 10
F6 4 11 8 9 7
F7 5/1 4 5 1 9
F8 6 12 12 11 10

Table 3: Conflict groups and conflicting movements for each traffic stream at an intersection, including pedestrians.
If we merge the rules from Table 2 into a table like Table 1, we obtain Table 3. It needs quite a lot of concentration to follow through each detail. Moreover, Table 3 also shows that the classical hierarchy of priorities is no longer easy to apply if pedestrian crossings are included. We see that F3 (and F7) are of rank 5 with respect to movement 11 (and 5). With respect to movement 3 (and 9), the same movement F3 (and F7) belongs to rank 2. These movements 3 (and 9) have priority over 11 (and 5) (cf. Fig. 8). Thus, there is no longer any clearly structured consecutive ranking of priorities if pedestrians enter into the picture. This is another reason why the classical theory of gap acceptance reaches its limitations as soon as pedestrians are included in a TWSC intersection model.

Fig. 8: Cycle of priorities: the round arrows are a symbol for "has priority over".

Regardless of this problem, Table 3 together with eq. 20 gives us under the new theory a framework for formulating an equation that calculates the capacity of each vehicle movement without neglecting the pedestrians' impact:

$$C_{i,p} = C_{\text{max},i,p} \cdot \prod_{k} \left[ 1 - \sum_{j=a}^{f} \left( \frac{A_j}{100} \cdot B_{k,j} \right) \right]$$

where

- \(C_{i,p}\) = capacity for movement \(i\) including the pedestrians' impact [veh/h]
- \(A_j\) = 100 if \(j\) is a vehicle movement (cf. remark "limited priority" below)
- \(A_j\) = \(A_{k,f,i}\) if \(j\) is a pedestrian movement \(f\) (cf. Table 2)
- \(A_{k,f,i}\) = probability of priority for pedestrians from movement \(f\) in conflict group \(k\) over vehicles from movement \(i\) [%]
- \(B_{k,j}\) = occupancy in conflict group \(k\) by movement \(j\) [-]
- \(B_{k,j}\) = \(Q_j \cdot t_{B,j} / 3600\) [-]
- \(Q_j\) = volume of movement \(j\) [veh/h or ped/h]
- \(t_{B,j}\) = 0, if the relevant cell in Table 3 is empty [s]
- \(t_{B,j}\) = average service time for one vehicle or pedestrian in movement \(j\) [s]
duration of blockage time caused on average by one vehicle or pedestrian

\( a, f, k: \) see bottom line of Table 3

The time for which a crosswalk near an unsignalized intersection is occupied by one pedestrian may depend on the pedestrian volume as well as on the width of the crosswalk. It may be estimated from empirical observations. Czytich and Boer (1999) found from a limited sample of observations made in Germany that a single pedestrian using the crosswalk takes an average service time of \( t_{B,1} = 3.2 \) s. This value appears quite low, but we must take into account that pedestrians often walk in groups. Thus, the average blocking time per pedestrian is lower than the time needed to cross the street. In the cases observed, the width of the crosswalks was not a limiting factor to the pedestrians' freedom to cross the street.

By the way: eq. 21 could be very easily made to allow for the effect of "limited priority" (Troutbeck, Kako, 1997) by using \( A\)-values less than 100\% for vehicle traffic as well. There may be situations where car drivers typically give priority to other drivers; e.g. a minor-right turner (\( i=6 \)) being polite enough to give priority to opposite minor-left turners (\( i=10 \)) to improve their chance to depart. If this were to happen in 40\% of such conflicts, the \( A_{j=6} \)-value would be 60 in eq. 21. If this concept were to be included in a future version of the model, then the movements in columns \( g, h, \) and \( m \) of Table 4 would have to be included in the total of eq. 21.

### 4 Intersection with Single-lane Approaches

Here, we focus on an intersection where each approach has only one lane. This adds another complication, since we now also have limited entry capacities due to the mutual interaction of movements on each of the entries to the intersection.

An entry lane used by several movements is called - according to the usual concept of unsignalized intersections - a shared lane. The capacity of shared lanes can be determined by a formula first developed by Harders (1968). This concept has been extended by Wu (1997), to additionally cover lanes of limited length (short lanes). In the common case of all streams at an approach using the same shared lane, the capacity of this shared lane, \( C_s \), is given by

\[
C_s = \frac{Q_{s,L} + Q_{s,T} + Q_{s,R}}{x_{s,L} + x_{s,T} + x_{s,R}} \quad \text{[veh/h]} \quad (22)
\]

where

- \( C_s \) = capacity of the shared lane \quad \text{[veh/h]}
- \( Q_{s,L} \) = volume for left turners using the shared lane \quad \text{[veh/h]}
  (in analogy: \( T \): through movement / \( R \): right turner)
- \( C_{s,L} \) = capacity for left turners according to the equations mentioned above (i.e. for an exclusive lane) \quad \text{[veh/h]}
- \( x_{s,L} \) = \( Q_{s,L} / C_{s,L} \) = degree of saturation \quad [-]

For the case of a single-lane approach with an additional short lane near the intersection (flared entry) offering space for one right-turning vehicle, the capacity of the shared traffic lane may be calculated from (Wu, 1997):

\[
C_s = \frac{Q_{s,L} + Q_{s,T} + Q_{s,R}}{\sqrt{(x_{s,L} + x_{s,T})^2 + (x_{s,R})^2}} \quad \text{[veh/h]} \quad (23)
\]
If we use both equations (eq. 22 and 23), the following constraint must be observed

\[
(Q_{s,s} \cdot t_{b,s,s}) + (Q_{s,T} \cdot t_{b,s,T}) + (Q_{s,R} \cdot t_{b,s,R}) + (Q_F \cdot t_{b,F}) \leq 3600 \quad [s] \quad (24)
\]

Again, \((s,L), (s,R), \) and \((s, T)\) stand for the index of the left (L) and right-turning (R) movements as well as the through movement (T), respectively, on the shared lane \((s)\). \(F\) stands for the pedestrian movement involved into the entry conflict group.

### 5 Queue Length and Delay

To calculate the average delay \(d\), a classical approach may be used. For non-stationary traffic conditions (which is usually the case in the field), the formula derived by Akcelik and Troutbeck (1991), which is contained in the HCM (1997, 2000), may be applied to calculate the average delay \(d_i\) for vehicles from movement \(i\):

\[
d_i = \frac{3600}{C_i} + 900 \cdot T \cdot \left[ x_i - 1 + \sqrt{\left( x_i - 1 \right)^2 + \frac{3600}{C_i} \cdot x_i} \right] \quad [s] \quad (26)
\]

where

- \(T\) = observation period (usually 1 hour) \quad [h]
- \(C_i\) = capacity of the movement \(i\) or a shared lane \quad [veh/h]
- \(x_i\) = degree of saturation = \(Q_i / C_i\) \quad [-]

\[
x_i = Q_{s,s} + Q_{s,T} + Q_{s,R} \quad [\text{veh}]
\]

The average queue length can be obtained according to Little's rule

\[
N_i = Q_i \cdot d_i \quad \text{[veh]}
\]

only in stationary conditions. In case of non-stationary (over-saturated) conditions, the relationship

\[
N_i = \left( d_i - \frac{3600}{C_i} \right) \cdot x_i \quad \text{[veh]}
\]

applies (cf. Akcelik, 1980). The percentiles of the queue length distribution can be estimated according to Wu (1994). This means, practically speaking, that Exhibit 17-19 of HCM (2000) may be applied.

### 6 A Rough Calibration of the Model

The \(t_{b}\)-values constitute the parameters of the model. Properly selecting these values should qualify the model to represent the real world with sufficient quality. For this model, it is not useful to measure these values directly in the field, since the beginning and termination of each individual \(t_{b}\) cannot clearly be defined. Thus, direct measurements could reveal quite a range of \(t_{b}\)-values, depending on the experimenters' decisions about details of the measurement process. Instead, \(t_{b}\)-values should be estimated as statistical parameters, and only those \(t_{b}\)-values should
be selected which offer the best convergence between specific traffic performance parameters (like capacities, average delays, or average queue lengths) and the corresponding model results. Convergence could be assessed by using minimized variances between measured values and estimated model results. For these parameter estimates, the underlying observed demand volumes for all movements should be varied over a realistic range.

In this sense, estimates of 12 \( t_b \)-values only must be formed to calibrate the model. Due to the symmetry of movements coming from the north and south, the number of unknown \( t_b \)-values was reduced to 6. The \( t_b \)-value for the two major through movements was estimated at 2.5 s. For the pedestrian \( t_b \)-values, the estimate of 3.2 s mentioned above could be used as a first approach. Thus it was only necessary to calibrate 4 \( t_b \)-values. Of course, it would be desirable to be able to produce \( t_b \) estimates based on empirical studies, and this will indeed be done by the authors in the near future. As a preliminary estimate, another solution is proposed, just to demonstrate the applicability of the new method and to give an idea of the size of the \( t_b \)-values.

For this calibration process, we will look at a rather simple intersection of quite common shape, as illustrated in Fig. 9. Here, we assume one left-turning lane for each of the major street approaches. In addition, we assume that the minor street has flares on both entries, allowing additional space for one right turning vehicle. The intersection is controlled by yield-signs on the minor street, and it is assumed that the site is at a rural intersection, with a population of drivers who are familiar with the layout. This intersection was analyzed by the classical gap-acceptance theory (cf. data in Table 4) on a spreadsheet, using the German standardized procedure for unsignalized intersection analysis. This procedure is very similar to the one established in the HCM (1997, 2000). Critical gaps and follow-up times were obtained from a recent study performed on rural intersections in Germany (Weinert, 2000). For the calculations, 400 different combinations of motor-vehicle traffic volumes (which were generated randomly within a reasonable range of values, cf. Table 4) were applied. Since only the interaction of different vehicle movements had to be analyzed, no pedestrians were included in this calibration step. For these 400 combinations of volumes, the capacity of each minor movement was determined based on the GAP solution. In addition, the set of equations 8 - 20 was programmed in a spreadsheet (Corel Quattro-Pro8). Here, the optimizer tool was used to adjust the capacities obtained by both methods as closely as possible. For optimization, the sum of quadratic errors was minimized, yielding a first estimate of \( t_b \)-values, i.e. those values which fitted best the GAP results obtained with parameters characteristic for Germany.
As a result, the \( t_B \)-values given in Table 4 were obtained. They approximately reflect the order of magnitude of these parameters, and may be used as a first, very rough approach to applying this new ACS technique. Of course, some more empirical evidence is needed before this method can be definitely used, in which instance parameters might be calibrated based on optimum approximations of observed and calculated delays. This, in turn, would improve the performance of the whole procedure. Nevertheless, even the simple calibration process used here shows the practicability of this new concept.

<table>
<thead>
<tr>
<th>Movement</th>
<th>Critical Gap ( t_c )</th>
<th>Follow-up Time ( t_f )</th>
<th>Volume Range Used for Calibration</th>
<th>Resulting ( t_B )-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.5</td>
<td>2.6</td>
<td>0 - 200</td>
<td>2.9</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
<td>0 - 400</td>
<td>2.5(^1)</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>0 - 250</td>
<td>2.8(^1)</td>
</tr>
<tr>
<td>4</td>
<td>6.6</td>
<td>3.4</td>
<td>0 - 120</td>
<td>6.5</td>
</tr>
<tr>
<td>5</td>
<td>6.5</td>
<td>3.5</td>
<td>0 - 150</td>
<td>5.9</td>
</tr>
<tr>
<td>6</td>
<td>6.5</td>
<td>3.1</td>
<td>0 - 350</td>
<td>3.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>movements 7 through 12 correspond to movements 1 through 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
</tr>
</tbody>
</table>

Table 4: Parameters used for the calibration process together with calibrated \( t_B \)-values (\( t_c \)- and \( t_f \)-values are obtained from Weinert, 2000).

1) : estimated without calibration
### Table 5: Application of the new method to an intersection as sketched out in Fig. 9 (example).

<table>
<thead>
<tr>
<th>Movement</th>
<th>Volume</th>
<th>Capacity by the New Approach</th>
<th>Average Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45</td>
<td>920</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>220</td>
<td>1337</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>67</td>
<td>307</td>
<td>41</td>
</tr>
<tr>
<td>4</td>
<td>56</td>
<td>307</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>88</td>
<td>1348</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>78</td>
<td>292</td>
<td>42</td>
</tr>
<tr>
<td>7</td>
<td>76</td>
<td>932</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>240</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>56</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>45</td>
<td>292</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>120</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>45</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>F1 / F2</td>
<td>180</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>F3 / F4</td>
<td>230</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>F5 / F6</td>
<td>300</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>F7 / F8</td>
<td>250</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Application of the new method to an intersection as sketched out in Fig. 9 (example).

To apply the method, the whole set of equations was programmed into another spreadsheet to calculate the capacity each non-priority movement at a 2WSC intersection shaped as sketched out in Fig. 9. Delays and percentile queue lengths may also be calculated with this spreadsheet program if the traffic volumes of the 12 vehicle movements and the pedestrian volumes are given, so that the impact of pedestrian movements on the capacity of the intersection may be evaluated as well. An example of these results is illustrated in Table 5. For other intersection layouts, the computation procedure still needs to be formulated.

### 7 Conclusion

In addition to the classical methods for TWSC intersection analysis (empirical regression and the critical-gap method), a new technique has been developed based on the procedure of Additive Conflict Flows (ACF) after Gleue. The background of this new method is easier to understand than the theory of gap acceptance. Nevertheless, it involves a series of equations which need computer application to get a solution in practice.

With this technique, it is possible to map the complicated regulations of pedestrian priority as they apply on the European continent. What is more, the real-life behavior of road users who do not comply with the rules laid down in the highway code can be simulated rather easily and flexibly. Thus, the method makes it very easy to account for so-called limited priority effects.

The $t_B$-values in the model are just parameters. It is not useful trying to measure these values in the field. They should only be calibrated based on model results, like capacities or average delays. There are various ways of calibrating $t_B$-values as model parameters. For this paper, estimates were used to show the practicability of the concept, and to demonstrate the order of
magnitude of these parameters. Of course, the method has not yet been proven empirically. Furthermore, computer programs will have to be created for all intersection layouts, but there are no theoretical complexities involved in this.

The new concept is expected to have the potential to replace the gap acceptance theory to a great extent. Particularly if pedestrian movements have to be included in an analysis, the new method provides significant advantages. An extended analysis using real-life data to show the potential of the methodology at a series of urban intersections currently being performed in Germany is scheduled for completion in 2001.
References

Akcelik, R. (1980)

Implementation of the Australian roundabout analysis method in SIDRA


Untersuchung des Fußgängerverkehrs an vorfahrtgeregelten Knotenpunkten in städtischen Gebieten (Investigation of pedestrian traffic at unsignalized urban intersections). *Ruhr-University Bochum, Germany, Study Thesis*

Gleue, A.W. (1972)
Vereinfachtes Verfahren zur Berechnung signalgeregelter Knotenpunkte (Simplified method for the calculation of signalized intersections). *Forschung Strassenbau und Strassenverkehrstechnik*, No. 136, Bonn


Harders, J. (1968)
Die Leistungsfähigkeit nicht signalgeregelter städtischer Verkehrsknoten. (The Performance of non-signalized urban junctions). *Strassenbau und Strassenverkehrstechnik*, No. 76, Bonn

HCM (1997)

HCM (2000)

The Traffic Capacity of Some Major Priority Junctions. *Transport and Road Research Laboratory*, Supplementary Report 582, Crowthorne, Berkshire

A Game Theoretical Analysis of Interaction Between Merging and Through Cars On-Ramps
Proposed Draft Computational Procedures, Capacity and Level of Service of Unsignalized Intersections (HCM 1997). *University of Idaho, Moscow*


Kyte, M., Kittelson, W., Tian, Z., Brilon, W., Troutbeck, R., Mir, Z. (1994)  

StVO (1998)  
Straßenverkehrsordnung (German Highway Code), latest edition: 1998

A Further Investigation on Critical Gap and Follow-Up Time.  
In: Brilon (ed.): *Proceedings of the 4th International Symposium on Highway Capacity, Maui/Hawaii, Transportation Research Circular E-C018, pp. 409 - 421*

Limited Priority Merge at Unsignalized Intersections.  

Weinert, A. (2000)  
Estimation of Critical Gaps and Follow-up Times at Rural Unsignalized Intersections in Germany.  
In: Brilon (ed.): *Proceedings of the 4th International Symposium on Highway Capacity, Maui/Hawaii, Transportation Research Circular E-C018, pp. 397 - 408*

Wu, N. (1994)  
An Approximation for the Distribution of Queue Lengths at Unsignalized Intersections.  

Wu, N. (1997)  
Capacity of Shared-Short Lanes at Unsignalized Intersections.  

Wu, N. (1999)  
Capacities at AWSC intersections. Technical Report No.19, *Ruhr-University Bochum, Institute for Transportation, Bochum*

Wu, N. (2000a)  
Determination of Capacity at All-Way Stop-Controlled (AWSC) Intersections.  
In: *Proceedings of the 79th Transportation Research Board Meeting, Washington, Paper-No. 00-1665*

Wu, N. (2000b)  
Capacity at All-Way Stop-Controlled and First-In-First-Out Intersections.  
In: Brilon (ed.): *Proceedings of the 4th International Symposium on Highway Capacity, Hawaii, Transportation Research Circular E-C018*