IMPEDANCE EFFECTS OF LEFT TURNERS FROM THE MAJOR STREET AT A TWSC INTERSECTION

by

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ABSTRACT
At a two-way stop controlled intersection without turning lane on the major street all three movements (right, through, left turners) share the same lane. To cover the effect of mutual impediments between these movements on the minor street capacity, the major street shared lane formula 17-16 is provided by the HCM. This paper shows that this formula should be replaced by an improved equation to get more realistic results. This improved formula is derived. The theoretical concept can be extended to include also the consequences of a short turning lane on the major street. An equation is derived which calculates the effect of the length of the left turn pocket on the capacity of minor movements.
1. INTRODUCTION

The estimation of capacity and quality of flow at unsignalized intersection is based on the gap acceptance theory in the guidelines of many countries, e.g. in the HCM 2000 (1), chapter 17, or in the German HBS 2001 (2), chapter 7. For these methods, as they are documented in the guidelines, a significant improvement should be made. This incorrect part of the procedures concerns the treatment of left turners from the major (priority) street (LTPS). (eq. 17-16 of the HCM 2000).

To explain the problem, first the theoretical background is highlighted. Here the abbreviations and symbols from the HCM are used. The explanations of the basics for the critical gap theory are, however, only explained quite shortly. For further explanation the reader is referred to (3). The paper, initially, is concentrated on the probability of a queue-free state for LTPS movements and the consequences on the through traffic of the major (priority) street (TTPS). On this basis the capacity for left turners from the minor street (LTMS) is estimated. The solution then is extended towards an expression which estimates the effect of the limited length of a left turn pocket on LTMS movements.

2. SHARED LANES

The purpose of calculations according to chapter 17 of the HCM 2000 is the assessment of traffic performance at an unsignalized two-way stop controlled (TWSC) intersection. As a precondition to this, the potential capacity $c_{p,k}$ for each of the twelve movements has to be estimated. This can be achieved by Harder's (4) formula (eq. 17-3 in HCM 2000) or other useful capacity estimation formulas. Also corresponding graphs (exhibit 17-6 and 17-7 in HCM 2000) can be used. For priority movements 2, 3, 5, and 6 these capacities are defined to be 1800 veh/h, without assuming that this number is completely realistic (2). They could also be estimated from field observations (1). These potential capacities $c_{p,k}$ are equal to the movement capacities $c_{m,k}$ if movement $k$ is of rank 2 (eq. 17-4 in HCM 2000).

Then the further ranking of priorities is taken into account by using impedance factors to calculate the movement capacities $c_{m,k}$ for each movement $k$ with a higher rank of priority than 2.

An impedance factor $p_{0,j}$ is the probability that a non-priority movement $j$ is in a queue-free state (eq. 17-5 in HCM 2000):

$$p_{0,j} = 1 - g_j = 1 - \frac{v_j}{c_{m,j}}$$

(1)

$$c_{m,k} = c_{p,k} \prod_{j=1}^{j} p_{0,j}$$

(2)

where

- $p_{0,j}$ = probability of a queue-free state in minor movement $j$ [-]
- $v_j$ = flow rate for minor movement $j$ [veh/h]
- $c_{m,j}$ = movement capacity for minor movement $j$ [veh/h]
- $g_j$ = degree of saturation for minor movement $j$ = $v_j/c_{m,j}$ [-]
- $c_{m,k}$ = movement capacity for minor movement $k$ [veh/h]
\[ c_{p,k} = \text{potential capacity for minor movement } k \] 

to be calculated by equation 17-3 or by any other useful capacity equation (cf. (3)) \([\text{veh/h}]\)

\[ k = \text{index for minor movements of rank 3 or 4} \ [-] \]

\[ j = \text{index for minor movements of rank 2 or 3 which have priority over movement } k \ [-] \]

(Definition of indices: cf. Figure 1)

\[ J = \text{number of minor movements of rank 2 or 3 which have priority over movement } k \ [-] \]

**FIGURE 1** Definition of movements at an unsignalized intersection

Here, e.g. at a T-junction for the LTMS-movement we have: \( k = 7 , \ j = 4 \ (J = 1) \) and \( c_{m,7} = c_{p,7} \cdot p_{10,4} \). For a crossroads intersection these calculations become more complex due to movements of rank 4, which is well described in (1) or (3).

Up to this point the calculation assumes that each movement has its own lane with unlimited length. The realistic intersection design, however, usually provides only turning lanes of limited extension. Moreover, in many cases there are no turning lanes at all. Then several movements have to share the same lane. We call this a shared lane. Here, shared lanes on the major street and on the minor approach have a different functionality which leads to different algorithms.

For minor street approaches the capacity of a shared lane is estimated by the shared formula after Harders (3), which is also used in the HCM as eq. 17-15.

\[
c_{SH} = \frac{v_i + v_j + v_k}{c_{m,i} + c_{m,j} + c_{m,k}}
\]  

\[ \text{where} \]

\[ c_{SH} = \text{capacity of the shared lane} \ [\text{veh/h}] \]

\[ v_i, v_j, v_k = \text{traffic volume of movements } i, j, \text{ and } k \ [\text{veh/h}] \]

\[ c_{m,i}, c_{m,j}, c_{m,k} = \text{movement capacities of movement } i, j, \text{ and } k \ [\text{veh/h}] \]

\[ i, j, \text{ and } k = 7, 8, 9 \text{ or } = 10, 11, 12 \]
This equation is not completely realistic which has been argued by (5). The required slight corrections are, however, rather complex and they are of minor importance for the result. Thus, eq. 3 is still quite an adequate solution for application in guidelines.

3. SHARED LANES ON THE MAJOR STREET
This paper is mainly directed on the shared lane problem on the major street. Here we treat one approach of the major street containing movements *i*, *j*, and *k* (cf. Figure 2a). As an example we use *i* = 4, *j* = 5, *k* = 6.

Let us first concentrate on a shared lane without any turning bay spaces (cf. Figure 2b). Then equation 17-16 of the HCM 2000 (1) defines

\[ p_{0.4}^* = 1 - \frac{g_4}{1 - (g_5 + g_6)} \]  

where
\[ p_{0,4}^* = \text{probability of a queue-free state for the shared lane on the major street} \]

\[ g_j = \text{degree of saturation in movement } j = \frac{v_j}{c_{m,j}} \]

\[ g_6 = 0, \text{ if movement 6 is operating on a separate lane} \]

(For the opposite direction at a crossroads intersection indices are replaced: 4 → 1, 5 → 2, 6 → 3)

This equation also goes back to Harders (3).

In the German Manual HBS (2) even another equation is used:

\[ p_{0,4}^* = 1 - (g_4 + g_5 + g_6) \]

Both equations 4 and 5 are not a perfect solution for the TWSC-intersection model which will be demonstrated in the following. They should be replaced by the improved equation 12.

For simplicity of the derivations let us first look at the movements as they are existing at a T-junction (Figure 2b). The potential capacity of the TTPS-movement 5 is denoted by \( c_{p,5} \) (e.g. \( c_{p,5} = 1800 \text{ pcu/h} \)). During a time period when a vehicle from movement 4 is queuing, the capacity of movement 5 is equal to zero. Thus, the real movement capacity of movement 5 is reduced to

\[ c_{m,5} = c_{p,5} \cdot p_{0,4} = c_{p,5} \cdot (1 - g_4) \]

Due to this kind of queuing, over a time period of duration \( T \) (e.g. 1 hour) it has to be expected that \( x \) vehicles from movement 5 are impeded and are queued behind waiting left turners (movement 4) where

\[ x = (1 - p_{0,4}) \cdot v_5 \cdot T = g_4 \cdot v_5 \cdot T \]

The LTMS (movement 7) is inhibited to enter the intersection in the subsequent cases:

a) A vehicle from movement 4 is waiting (Figure 2b). This case is treated by the usual impediment factors (eq. 1 and 2). This means the intersection is closed for the LTMS-movement 7 during different time intervals within period \( T \) which sum up to

\[ t_{\text{close},4} = (1 - p_{0,4}) \cdot T = g_4 \cdot T \]

b) A queue from the TTPS-movement 5 (which has been formed behind a waiting left turner; cf. Figure 2b) is being discharged. Each of these 5-vehicles needs \( t_5 = \frac{1}{c_{m,5}} \) seconds to cross the entrance to the intersection. Since there are \( x \) such 5-vehicles there is a time during which the intersection is closed for 7-vehicles, of

\[ t_{\text{close},5} = x \cdot \frac{1}{c_{m,5}} = g_4 \cdot v_5 \cdot T \cdot \frac{1}{c_{m,5}} = g_4 \cdot g_5 \cdot T \]

c) A vehicle from movement 4 or from movement 5 is approaching without any queue existing on the major street and the time to its arrival at the intersection is less than the critical headway \( t_{c,7} \) of the LTMS vehicle. This case is completely covered by the capacity calculation according to eq. 17-4 of the HCM 2000 in conjunction with the definition of conflicting flows by exhibit 17-4.

Summing up these times during which the intersection is blocked for movement 7, we get the sum of all blocked periods from eq. 8 and 9 as
\[ t_{\text{close}, 4+5} = t_{\text{close}, 4} + t_{\text{close}, 5} = g_4 \cdot T + g_4 \cdot g_5 \cdot T \]  

(10)

The proportion of time during which no blockage occurs, then is

\[ p^*_{i,j} = 1 - \frac{t_{\text{close}, 4+5}}{T} = 1 - g_4 - g_4 \cdot g_5 \]  

(11)

At a crossroad intersection movement 6 has also to be taken into account which is assumed to be operating on the same lane of the major street as movements 4 and 5. With that eq. 11 can be transformed into the more general form of

\[ p^*_{0,i} = \max \left\{ (1 - g_i) - g_i \cdot (g_j + g_k) \right\} \]  

(12)

where

\[ i = 1 \text{ or } 4 \]
\[ j = 2 \text{ or } 5 \]
\[ k = 3 \text{ or } 6 \]

\[ p^*_{0,i} = \text{probability of a queue free state on the shared lane of the major street} \]

\[ g_i, g_j, g_k = \text{degree of saturation for movements } i, j, k \]

\[ v_2 = 300 \text{ veh/h} \]
\[ v_5 = 300 \text{ veh/h} \]

\[ v_2 = 300 \text{ veh/h} \]
\[ v_5 = 800 \text{ veh/h} \]

**FIGURE 3** Probability that no left turner is impeding the through traffic on a shared lane at a T-junction depending on LTPS volume \( v_4 \).
This equation should be used instead of equation 17-16 in the HCM 2000.

To compare this new equation with the former approaches (eq. 4 and 5) some numerical calculations have been performed. Figure 3 shows one example for the dependency of \( p_{0,4}^* \) on the volume of movement 4. It can be seen that eq. 5 reduces \( p_{0,4}^* \) significantly compared to the new eq. 11 or 12. Thus, eq. 5 obtained from the German guideline (2), produces a significant underestimation of the capacity for the LTMS-movement 7, since \( p_{0,4}^* \) is introduced into the further capacity calculations in the sense of eq. 2. The HCM equation 17-16 (cf. eq. 4 of this paper) is much closer to the new equation. However, for larger major street volumes there is also quite a difference with the consequence of an underestimation of the minor street capacity. There are many cases in reality where this difference may become rather relevant for the question if an unsignalized intersection offers a sufficient quality of service.

This comparison, however, does not prove which equation is the correct one. The desirable empirical evidence can not be reached due to the enormous efforts to get sufficient sample sizes required for reliable measurements of this term. Thus, the only realistic option to confirm the validity of one of the equations is microscopic simulation. Here, a time-oriented Monte-Carlo technique has been applied to simulate capacities and delays at an unsignalized T-junction. For demonstrating the results, in this paper two cases have been chosen:

a) \( v_5 = 300 \text{ veh/h} \)
b) \( v_5 = 900 \text{ veh/h} \)

All other parameters remain constant for both cases:

\( v_3 = 0 \text{ veh/h} \quad v_2 = 300 \text{ veh/h} \)

Behavioral parameters for drivers have been obtained from the HCM 2000 (1), exhibit 17-5:

\[
\begin{align*}
t_{c,4} &= 4,1 \text{ s} \\
t_{f,4} &= 2,2 \text{ s} \\
t_{c,7} &= 7,1 \text{ s} \\
t_{f,7} &= 3,5 \text{ s}
\end{align*}
\]

where

\[
\begin{align*}
t_{c,i} &= \text{critical headway for movement } i \quad [\text{s}] \\
t_{f,i} &= \text{follow-up time for movement } i \quad [\text{s}]
\end{align*}
\]

Here the LTMS capacity \( c_{m,7} \) is looked for, depending on the LTPS volume \( v_4 \). The simulation uses constant critical headways and follow-up times and consistent driver behavior.

Figure 4 shows the results. The curves illustrate that there is a good coincidence between equation 12 and the simulated results whereas equation 4 and 5 fail to meet the simulated results. In cases of a rather large capacity (e.g. FIGURE 4a) the difference of the new equation 12 to the current HCM approach (equation 4) is very small. If however the capacity is on a low level (e.g. Figure 4 b) the difference may become quite significant. This result, which was also obtained for other combinations of parameters, underlines the descriptive quality of eq. 11 and 12 compared to the existing solution. Remaining differences between the analytical solution and simulation results are due to the approximate nature of the analytical capacity estimation procedures.
FIGURE 4 LTMS capacity estimated by several types of impediment factor compared to simulated results.

4. LEFT TURNING LANE OF LIMITED LENGTH

Also a separate lane for left turners on the major street does not guarantee that the through traffic is completely unimpeded. There might be cases where the queue of left turners is exceeding the length of the left turning pocket (cf. Figure 2c). Here a method is needed to quantify this effect on the intersection capacity.

Again the derivation of the analytical solution is explained for a T-junction. The following derivations are only valid if there is just one single lane to serve the TTPS movement. It is assumed that the turning lane offers space to store \( n_L \) left turning vehicles without impeding the through traffic. The probability that the queue of waiting left turners is shorter than \( n_L \) is

\[
p_{F,4} = p(N \leq n_L) = 1 - g_4^{(n_L+1)}
\]

(13)

where

\[
N = \text{no. of waiting LTPS-vehicles} \quad [-]
\]
\[
n_L = \text{no. of spaces on the left turn lane} \quad [-]
\]

For this equation it is assumed that the queuing system of LTPS-movements is operating like an M/M/1-queue. Also if reality is slightly different from this simplifying assumption, this

\[
a)
\begin{align*}
v_2 &= 300 \text{ veh/h} \\
v_3 &= 0 \text{ veh/h} \\
v_5 &= 300 \text{ veh/h}
\end{align*}
\]

\[
b)
\begin{align*}
v_2 &= 300 \text{ veh/h} \\
v_3 &= 0 \text{ veh/h} \\
v_5 &= 800 \text{ veh/h}
\end{align*}
\]
approach has been successfully used for many theoretical derivations in unsignalized intersection research.

In analogy to eq. 1 and 2 we get from eq. 13 (cf. eq. 6) for the capacity of the TTPS movement 5:

\[
c_{m, 5} = c_{p, 5} \cdot \left(1 - g_4^{(n_L + 1)}\right)
\]

(14)

The number of vehicles from movement 5 which are queued due to a spillback of the LTPS-movement 4 beyond the length of the left turn bay during a time period of duration \(T\) is (cf. eq. 7)

\[
x = g_4^{(n_L + 1)} \cdot T \cdot v_5
\]

(15)

The LTMS-movement 7 is impeded by movements 4 or 5 from entering the intersection in the following cases:

a) like case a) in section 3

b) A queue from the TTPS movement 5 is discharged (i.e. 5-vehicles which have been queued behind an overflow of the left turning lane). With \(x\) such vehicles during period \(T\) and with \(1/c_{p, 5}\) seconds per discharging movement-5-vehicle the total duration of this discharge process during period \(T\) will consume a time of

\[
t_{\text{close, 5}} = x \cdot \frac{1}{c_{p, 5}} = g_4^{(n_L + 1)} \cdot T \cdot v_5 \cdot \frac{1}{c_{p, 5}} = g_4^{(n_L + 1)} \cdot g_5 \cdot T
\]

(16)

c) like case c) in section 3.

The sum of all these blocked times (case a) - c) ) is

\[
t_{\text{close, 4+5}} = g_4 \cdot T + g_4^{(n_L + 1)} \cdot g_5 \cdot T = \left(g_4 + g_4^{(n_L + 1)} \cdot g_5\right) \cdot T
\]

(17)

Thus, the proportion of time during which no blockage occurs is (cf. eq. 11)

\[
p_{0, 4}^* = 1 - g_4 - g_4^{(n_L + 1)} \cdot g_5
\]

(18)

Like in section 3 we now generalize this equation (which is only valid for a T-junction) to a crossroads intersection:

\[
p_{0, i}^* = \text{Max} \left\{1 - g_i - g_i^{(n_L + 1)} \left(g_j + g_k\right), 0\right\}
\]

(19)

where

\(n_L\) = no. of spaces on the left turn lane

Other variables are explained in conjunction with eq. 12.

The correctness of this equation can also be checked using some boundary conditions which must be fulfilled:

1. for \(g_j + g_k = 0\) (i.e.: no through or right turning traffic):
\[p_{0, i}^* = (1 - g_i)\]

2. for \(n_L = \infty\) (i.e.: an extremely long left turning lane):
\[p_{0, i}^* = (1 - g_i)\] (since \(g_i < 1\))
3. for $v_i = 0$ (i.e.: no left turning traffic)
   $p_{0,i}^* = 1$

4. for $n_L = 0$ (i.e. no left turning lane, i.e. a shared lane)
   $p_{0,i}^* = (1 - g_i) - g_i \cdot (g_j + g_k)$
   (= eq. 12)

A graph to illustrate the influence of $n_L$ (which represents the length of the turn pocket, counted in number of spaces for passenger cars) on the LTMS capacity is given in Figure 5. The function starts from the result of eq. 12 for $n_L = 0$ and it approaches $(1 - g_i)$ for large $n_L$.

If equation 19 would replace eq. 17-16 in the HCM (cf. eq. 4) then this equation would also cover the limited length $n_L$ of the left turn pocket automatically, which is not possible in the HCM $(1)$ algorithm for TWSC-intersections up to now. For the German guideline HBS $(2)$, where the current solution is completely wrong, eq. 19 provides a very important correction.

5. INCREASE OF CAPACITY CAUSED BY IMPEDED MOVEMENTS

Usually we assume that the increase of demand traffic volumes will decrease capacity and, thus, traffic performance for all of the other movements. It can, however, come to a point where further increasing traffic volumes of specific movements will cause blocking effects (as described in section 3 and 4 of this paper). These can create additional capacities for other movements.

One example is illustrated in FIGURE 6a. In this case a spillback of LTPS (movement 4) is impeding the TTPS-movement 5 to pass through the intersection. Due to this effect the opposing LTPS-movement 1 can freely discharge from the stop line. Also the RTMS (movement 12) would benefit from this blocking effect.

One might be attempted to expand equation 19 to take into account also this effect. This may be possible. However, before further derivations some concerns should be discussed:
a) Resulting equations become complicated and are difficult to apply.

b) Of course, also in the profiting direction the same blocking phenomenon could occur temporarily with the effect that then the increase of capacity would move over to the opposite direction. This means: the process of blocking could become dynamic. In these cases an iterative calculation might become appropriate. This of course, would not longer be accessible for a straightforward analytic procedure to be included into Highway Capacity Manual procedures.

c) The capacity increase by blocking effects can only occur if at least one movement is overloaded for some periods; i.e. this movement is operating on LOS E or even F. Thus, these blocking effects will never become a means to raise the whole intersection into an acceptable level of performance.

d) The dynamic effects may better be addressed by microscopic simulation.

Due to these reasons the analytical treatment of capacity increasing effects by temporal blocking of mayor movements is postponed. Effects like these might, however, also be employed to accomplish better traffic performance at TWSC-intersections in specific cases. E.g. a zebra-crossing (i.e. absolute priority for pedestrians) in one of the major street entries offers increased capacities for the LTPS (movement 4; see FIGURE 6 b). The analytical treatment of these aspects could become a matter of further research.

![FIGURE 6](image_url)  
**Increase of capacity for one left-turn movement by blocking effects in the opposing direction at a crossroads junction.**

**E) CONCLUSION**

This paper provides a new theoretical derivation of the shared lane formula to be applied in the case of shared lanes on the major street at TWSC intersections. This formula could be verified by a series of simulations. In addition, this formula can be extended to cover also the effects of a short left turning lane on the major street. This extended formula fulfills all the restrictions which are typical for the problem.
It is recommended to replace eq. 17-16 of the HCM 2000 by this new formula (eq. 19) for the new edition of the HCM. It is proposed to use this approach also for unsignalized intersection guidelines in other countries, e.g. for the German HBS (2).

Finally, it should be noted that also these derivations are not a perfect mathematical solution to the problem of unsignalized intersections. Like the whole gap acceptance theory, also the derivations submitted in this paper are more like an application of rather pragmatic mathematics, since a series of simplifying assumptions are needed to come to a solution ready for use in practice.

REFERENCES


