CONSOLIDATION IN SPHERICAL DOMAINS: A TIME AND FREQUENCY DOMAIN ANALYSIS

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In geomechanics the consolidation of a fluid-saturated soil or rock is of great interest and many theories have been proposed in recent years. A special case is the consolidation problem of a porous, fluid-saturated sphere under drainage and hydrostatic pressure. For this configuration Cryer [2] discussed the special effect of pore pressure response. The complexity of the original problem of Cryer is increased, taking into account a modified sphere with an undrained layer. This modification is an extension of the original problem towards more realistic situations of a porous-elastic rock with heterogeneities. These heterogeneities affect the properties of the surrounding rock, and thus, have a crucial influence on engineering applications, e.g., geothermal drilling or mining. The characterization of such media is of interest in applied geophysics and geothermal exploration, as they often contain gas or oil reservoirs.

MOTIVATION

In the context of consolidation, the primary variable is the change in porosity, which in the case of a fluid-saturated porous medium is expressed as 
\[ \phi(t) = \phi(0) - \frac{1}{K_e} \int_{0}^{t} \frac{d\pi}{dt} \, dt \]

where \( \phi(t) \) is the time-dependent porosity, \( \phi(0) \) is the initial porosity, \( K_e \) is the effective compressibility of the solid grains, and \( \pi(t) \) is the pore pressure. The time-dependent progress of \( \pi(t) \) and \( \phi(t) \) are analyzed in this study.

NUMERICAL IMPLEMENTATION AND RESULTS

Three different modeling approaches are simulated for the two models in order to analyze the differences between the solutions. At first, the Terzaghi approach, which assumes incompressible solid grains and incompressible fluid (\( K_s \rightarrow \infty, K_f \rightarrow \infty \) is applied. Second, the hybrid model using a rigid grain assumption (RGA, \( K_s \rightarrow \infty, K_f \rightarrow \infty \)) is adapted. At last no restrictions are made for both phases (Biot approach).

APPROACHES FOR THE NUMERICAL SIMULATION

Three different approaches

- **Biot**
  \[ \frac{1}{M} \frac{\partial \pi}{\partial t} + \alpha \frac{\partial \pi}{\partial R} = 0 \]

- **RGA**
  \[ \frac{1}{M} \frac{\partial \pi}{\partial t} + 2 \frac{\partial \pi}{\partial R} + \alpha \frac{\partial \pi}{\partial R} = 0 \]

- **Terzaghi**
  \[ \frac{1}{M} \frac{\partial \pi}{\partial t} + \alpha \frac{\partial \pi}{\partial R} = 0 \]

The equations are solved with the finite element program COMSOL Multiphysics. The pressure evolution at the center of the sphere for the modified Cryer problem is qualitatively similar to the one in Cryer's original problem. All solutions show the typical initial increase of pore pressure followed by pressure relaxation, known as Cryer effect [2]. This effect is caused by the immediate transmission of stresses coupled with the gradual outflow of draining water. The approach of Terzaghi shows a strong overestimation of the pressure peak in comparison to Biot's approach and an understimation of the intrinsic pressure-diffusion time.

MODEL APPROACH

mesoscopic REV

Transition of the poroelastic scale (\( \lambda \)) to the mesoscopic scale (\( \lambda_{\text{REV}} \)). A representative volume element (REV) is cut out as a sphere with radius \( R_{\text{REV}} \).

METHODOLOGY

Cryer's original model (fig. left) describes a three-dimensional fluid saturated porous rock sphere which is loaded by hydrostatic stress to its outer boundary under full drainage. The mesoscopic REV (modified Cryer model, fig. right) is an extension of the original model. Here a water-saturated sphere is surrounded by a layer of gas-saturated porous material. This configuration is also studied under hydrostatic stress, but with an undrained boundary. The two models are solved for three different approaches (see numerical results) in order to analyze the effect of compressibility and the differences between the models.

EXTENDED CRYER SPHERE

Left: Cryer's consolidation problem of a sphere with radius \( r = 1 \) m. Right: Modified Cryer's problem with radius \( R_{\text{REV}} = 1.5 \) m of the outer sphere. Both models are studied under hydrostatic stress \( f \) on the outer boundary.

REFERENCES