The so-called materially valid inferences and the logic of concepts

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Abstract

The so-called materially valid inferences have come to new prominence through the work of Robert Brandom. This paper introduces a fragment of a logic of concepts that does not reduce concepts to their extensions. Concept logic and its semantics allow us to represent the conceptual knowledge used in material inferences and thus suggest a way to deal with them.

1 Making Brandom explicit

Mary is a girl. This is true. And thus, Mary is a female child. That is sound reasoning, isn't it? But why? Validity of inference is normally regarded to be a necessary condition of soundness of inference. Which is the valid inference scheme underlying the inference from 'Mary is a girl' to 'Mary is a female child'? What is its logical form? For a guess, we could try:

$$Gx \vdash Fx \wedge Cx$$

But, obviously, this is not a valid inference in first order predicate logic. Recently, Robert Brandom claimed that it is not a valid inference scheme at all, but the meaning of the concepts *girl*, *female* and *child* that make this inference correct. Endorsing such inferences 'is part of grasping or mastering those concepts, quite apart from any specifically logical competence' (1994, 98; see also Sellars 1980). But this does not exclude the possibility that such inferences can be modelled using logical techniques. The aim of this paper is to present such a technique that allows us both to account for materially valid inferences and to represent the conceptual knowledge on the basis of which we endorse these inferences. These aims can be achieved by developing a logic of concepts. Concept logic is designed to take concepts as concepts seriously, that is, not to conflate a concept with its extension. It is not necessary to be a Platonist with regard to universals in order to use concept logic. It is sufficient to agree that concepts exist in some way, e.g. as norms (as would Brandom 1994). Concept logic is not thought to replace, but to enrich predicate logic. That should be clear from the fact that the fragment of a language of concept logic studied in this paper is an extension of first order predicate logic. Over and above the normal apparatus of predicate logic this fragment features the concept relator imp, representing the relation of conceptual implication. From the point of view of a normative account of concepts, conceptual implication can be introduced as follows:

> A concept A implies a concept B, iff: If a speaker subsumes an individual x under B, he is commited (when asked) also to subsume x under A.

Other concept relators can be employed to represent non-implication, convertibility, incompatibility, and identity of concepts; they will, however, not be dealt with in this paper. For the sake of simplicity, we will only consider a predicate logic with one-place predicates and names in this paper.

2 The fragment C*

The fragment C* to be studied here is an extension of monadic predicate logic. Like predicate logic, the alphabet of C* contains constants for individuals $(a, b, c, ...)$, variables for individuals $(x, y, z, ...)$ and the usual logical constants $(\neg, \vee, (,), \forall)$. Instead of names of predicates, C* contains names of concepts $(F, G, H, ...)$, and in addition to predicate logic C* contains the two-place concept relator imp. The usual definition of well-formed formulae (wff) in predicate logic is augmented to allow for imp-statements being wffs:

> If A and B are concepts, then $\ulcorner(A \,\mathsf{imp}\, B)\urcorner$ is a wff.

Brackets can be suppressed as usual; derived logical signs have the usual definitions. The axioms of C* have to incorporate predicate logic and to characterize the imp-relation. Following the normative account suggested above, we shall conceive conceptual implication both as a reflexive and a transitive relation, but not as a symmetric relation: if a speaker subsumes an individual under the concept *dog*, he is commited to subsume that individual also under the concept *animal*, but if a speaker subsumes an individual under the concept *animal*, he is not commited to subsume it under the concept *dog*. Finally, we add a *bridge axiom* that connects the conceptual imp-relation with a statement about extensions, expressed in predicate logic:

(A1) Every C*-instance of a theorem of predicate logic is an axiom of C*.

(A2) $A \,\mathsf{imp}\, A$

(A3) $(A \,\mathsf{imp}\, B \wedge B \,\mathsf{imp}\, C) \supset A \,\mathsf{imp}\, C$

(A4) $A \,\mathsf{imp}\, B \supset \forall x(Ax \supset Bx)$

Derivation rules of C* are the *modus ponens* and the generalisation rule. By these means, e.g. the following theorem can be derived in C*:

$$\neg(A \,\mathsf{imp}\, B \wedge \exists x(Ax) \wedge \neg\exists y(By))$$

Theorems like this are especially interesting, because they express logical relations between the realm of conceptual intensions (expressed by concept relators) on the one hand and the realm of extensions on the other hand (expressed by means of quantification over individuals).

3 Semantics of C*

We construct a semantics for concept logic by modelling concepts as a special kind of individuals. Thus, besides the usual universe of discourse we use a second non-empty set B, the set of concepts, members of which are not itself sets. Certain relations holding between members of B are the conceptual relations that are expressed by concept relators such as imp:

Def.: $\langle \mathsf{U}, \mathsf{B}, \mathsf{Ext}, \mathsf{IMP} \rangle$ is a *C*-frame* iff:

(1) U and B are non-empty distinct sets of individuals.

(2) Ext is a function that maps every element of B to exactly one (perhaps empty) subset of U.

(3) IMP is two-place relation on B, such that for all $X, Y, Z \in \mathsf{B}$:

(a) $\mathsf{IMP}(X, X)$.

(b) If $\mathsf{IMP}(X, Y)$ and $\mathsf{IMP}(Y, Z)$, then $\mathsf{IMP}(X, Z)$.

In this definition, (3a) matches axiom (A2) while (3b) matches axiom (A3). In the following definition of a C*-model, clause (2) matches the bridge axiom (A4):

Def.: $\langle \mathsf{B}, \mathsf{U}, \mathsf{Ext}, \mathsf{IMP}, \mathsf{Int}^{\mathsf{U}}, \mathsf{Int}^{\mathsf{B}} \rangle$ is a *C*-model* iff:

(1) $\langle \mathsf{B}, \mathsf{U}, \mathsf{Ext}, \mathsf{IMP} \rangle$ is a C*-frame.

(2) $\mathsf{Int}^{\mathsf{U}}$ is a function mapping every name of an individual exactly to one element of U, and $\mathsf{Int}^{\mathsf{B}}$ is a function mapping every name of a concept to exactly one element of B, such that if $\mathsf{IMP}(X, Y)$, then $\mathsf{Ext}(X) \subseteq \mathsf{Ext}(Y)$.

Having thus defined the concept of a C*-model, we can formulate the truth-definitions. An atomic formula like 'Fa' will be considered as true, if the individual from U designated by 'a' is a member of the extension of the concept designated by 'F'. A conceptual implication as '$F \,\mathsf{imp}\, G$' should be considered as true, if the concept designated by 'F' implies the concept designated by 'G' - i.e., if the relation IMP holds between these two concepts. The truth-definitions for negation, alternation and the universal quantification are as usual:

Def.: V_M is the *evaluation function of C**, iff V_M maps every wff of C* with respect to a C*-model M to exactly one element from $\{0, 1\}$, such that:

(1) $V_M(\ulcorner Aa \urcorner) = 1$ iff $\mathsf{Int}^{\mathsf{U}}(a) \in \mathsf{Ext}(\mathsf{Int}^{\mathsf{B}}(A))$.

(2) $V_M(\ulcorner A \,\mathsf{imp}\, B \urcorner) = 1$ iff $\mathsf{IMP}(\mathsf{Int}^{\mathsf{B}}(A), \mathsf{Int}^{\mathsf{B}}(B))$.

(3) $V_M(\ulcorner \neg\alpha \urcorner) = 1$ iff $V_M(\alpha) = 0$.

(4) $V_M(\ulcorner \alpha \vee \beta \urcorner) = 1$ iff $V_M(\alpha) = 1$ or $V_M(\beta) = 1$.

(5) $V_M(\ulcorner \forall x(\alpha[a] \urcorner) = 1$ iff for all a-alternatives M* of M: $V_{M^*}(\alpha[a]) = 1$.

Equipped with these semantical tools we can now try to get a grip on the initial problem.

4 How to analyze conceptual inferences with C*

Normally, we would consider 'Mary is a girl, thus Mary is a female child' to be a good inference. However, if we adopt a formalistic attitude to inference, we are likely to discard this inference, because it is not based on a valid inference scheme of first order predicate logic. Alternatively, we can search for hidden assumptions. As a candidate for a hidden assumption that would repair the inference, the universal statement 'All girls are female children' would do. However, this universal statement is not just an arbitrary further assumption. Philosophers would say, it is an analytic truth. On the other hand, that universal statement is of a different standing than, say, the axioms of the predicate calculus. It would be odd to add a lenghty list of such analytic universal statements to the set of axioms, because this would imply that we treat all the concepts as logical constants. Concept logic offers a third way to deal with this problem. Instead of using the syntactic tool of adding new axioms, we use the semantic tool of integrating a conceptual relation in the model. The statement 'G imp F' (read: 'The concept *girl* implies the concept *female child*') will be true in any model that adequatly mirrors our use of the words 'girl', 'female' and 'child'. And instead of treating 'All girls are female children' as an arbitrary hidden assumption, we can use the conditional statement 'If *girl* implies *child*, then all girls are female children' to make the inference – and this conditional statement is an instance of the bridge axiom (A5) and thus a logical truth in C*. Hence we have: '*Girl* implies *female child*, Mary is a girl, thus Mary is a female child.' Therefore, from the point of view of a logic of concepts, conceptual inferences like the discussed example can be considered as being instantiations of the following inference scheme that is valid in C*:

$$A \,\mathsf{imp}\, B, Ax \vdash Bx$$

5 Different Types of Conceptual Relations

In this paper, we have developed a semantical tool to deal with some simple conceptual inferences. However, our fragment C* of concept logic is not yet strong enough to deal with just any conceptual inference, as three examples given by Brandom (1994, 97-98) show. In the remaining, we will shortly discuss these three examples.

Brandom's first example is: 'Pittsburgh is to the West of Philadelphia, thus Philadelphia is to the East of Pittsburgh.' As C* is only a logic of monadic predicates, C* is not sufficient to deal with this inference. However, it is not difficult to extend C* such as to deal with dyadic predicates also. In addition to this we will need to identify inverse relations, such that the following will hold good: If a tupel $\langle x, y \rangle$ satisfies a relation, the tupel $\langle y, x \rangle$ satisfies the inverse relation.

Such extensions should also cope with Brandom's second example, which is: 'Today is Wednesday, thus tomorrow will be Thursday.' If the word 'tomorrow' means the same as the phrase 'the day after today', and if the name 'Thursday' designates the same day as the description 'the day after Wednesday', then the original example can be rephrased as: 'If today is Wednesday, the day after today will be the day after Wednesday, thus tomorrow will be Thursday.' This phrasing nicely hints at the relational logic by means of which the inference can be reconstructed.

Finally, Brandom's third example: 'Lightning is seen now, thus thunder will be heard soon.' Now this inference seems to be no *logical* inference, holding *a priori*. Many will insist that this inference is at best highly probable. But even if we acknowledge this, there is nothing to prevent our beliefs in such high probabilities from becoming part of our conceptual understanding. It is both the job of our everyday experience as well as the task of science to discover such *a posteriori* relations between concepts. But to deal with this sort of conceptual knowledge we have to introduce a second level of conceptual relations of probability. Such a second level of conceptual relations promises to be a very interesting tool both for modeling our conceptual knowledge and for the philosophy of science.

References

[Brandom 1994] Robert Brandom (1994), Making it Explicit, Cambridge MA.

[Sellars 1980] Winfred Sellars (1980), Inference and Meaning, repr. in: Pure Pragmatics and Possible Worlds, Ridgeview, Reseda CA.