

Remarks on non-transitive 'identity'

In his paper 'Non-Transitive Identity', Graham Priest argues that “various properties standardly taken to be possessed by identity (consistency, transitivity, substitutivity) are not to be taken for granted philosophically,” and he presents “a theory of a relationship that is naturally enough thought of as identity, but for which the properties that we have just seen to be problematic fail, though in a controlled and recoverable way.” Priest defines identity via Leibniz's Law in a second-order extension of his Logic of Paradox. As a result, the so defined binary relation fails to be transitive, the substitutivity of identicals (the replacement property) fails, and sentences of the form $a = a$ and $a \neq a$ may both be true in some model.

In first-order logic, 'real' identity as a primitive notion is proof-theoretically underdetermined by its standard axiomatization. Moreover, if the transitivity axiom is dispensed with, we are facing a dilemma. Since replacement implies transitivity, replacement must be given up as well, and the Henkin-style completeness proof for first-order logic needs modification. Without replacement, however, it is not guaranteed that the functions and relations of the canonical model are indeed well-defined. If just the first-order language of identity is considered, the objects of the canonical models can be defined in such a way that non-transitive 'identity' may be understood as the relation of non-empty intersection of certain non-empty sets. This seems to be a plausible reading in view of sorites-generating changes of entities over time.

Priest, G., 'Non-transitive identity', in: Richard Dietz & Sebastiano Moruzzi (eds.), *Cuts and Clouds: Vagueness, its Nature, and its Logic*, Oxford University Press, 2010, 406-416.