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Quotation Analysis in Sheaf-Theoretic Formal Semantics

We consider texts in some unspecified natural language, say in English, that are supposed to be written with good grace and intended for human understanding, we call further *admissible*. A text X is a finite sequence of its constituent sentences, and so it is formally identified with a graph of function. A part $U \subseteq X$ is a subsequence whose graph is a subset of the whole sequence graph.

Since the text understanding is not postponed until its final sentence, it should have meaningful parts; their meanings determine the meaning of the whole. Whether a part of an admissible text X is meaningful or not depends on some accepted *criterion of meaningfulness*. The *meaning* of some meaningful part $U \subseteq X$ is accepted as a communicative content grasped in a particular reading process following the reader's attitude, formalized by the term *sense*; the latter may be e.g. *literal, moral, allegoric,* etc. We argue that for such a criterion conveying an idealized reader's linguistic competence, the set of proper meaningful parts $\mathscr{O}(X)$ is stable under arbitrary union and finite intersection:

(i) *an arbitrary union of meaningful parts of an admissible text is meaningful;*
(ii) *a nonempty intersection of two meaningful parts of an admissible text is meaningful.*

Since an admissible text X is meaningful by the very definition, it remains to define formally the meaning of $\varnothing \subseteq X$ in order to provide X with some topology we call *phonocentric*, where the set of opens $\mathscr{O}(X)$ is constituted of all meaningful parts $U \subseteq X$ (called further *fragments*).

Let X be an admissible text, and let $\mathscr{F}$ be an adopted sense of reading. For each open $U \subseteq X$, we collect all the fragmentary meanings of U in the set $\mathscr{F}(U)$. Thus we are given a map $U \mapsto \mathscr{F}(U)$ defined on the set $\mathscr{O}(X)$ of all opens $U \subseteq X$. The precept of the hermeneutic circle "to understand a part in accordance with the understanding of the whole" defines a family of maps $\mathrm{res}_{V,U}\colon \mathscr{F}(V) \to \mathscr{F}(U)$, where $U \subseteq V$, such that $\mathrm{res}_{V,V} = \mathrm{id}_{\mathscr{F}(V)}$ and $\mathrm{res}_{V,U} \circ \mathrm{res}_{W,V} = \mathrm{res}_{W,U}$ for all nested opens $U \subseteq V \subseteq W$. Mathematically, the data $(\mathscr{F}(V), \mathrm{res}_{V,U})_{V,U \in \mathscr{O}(X)}$ is a *presheaf* over X.

According to Quine, there is no entity without identity. Any reasonable criterion of meanings' identity should satisfy:

Claim S (Separability). *Let X be an admissible text, and let U be a fragment of X. Suppose that s,t are two fragmentary meanings of U and there is an open covering $U = \bigcup_{j\in J} U_j$ such that $\mathrm{res}_{U,U_j}(s) = \mathrm{res}_{U,U_j}(t)$ for all U_j. Then $s = t$.*

The hermeneutic circle prescribes "to understand the whole by means of understandings of its parts," hence a presheaf of fragmentary meanings should satisfy:

Claim C (Compositionality). *Let X be an admissible text, and let U be a fragment of X. Suppose that $U = \bigcup_{j\in J} U_j$ is an open covering of U and there is a family $(s_j)_{j\in J}$ of fragmentary meanings, $s_j \in \mathscr{F}(U_j)$ for all U_j, such that $\mathrm{res}_{U_i,U_i\cap U_j}(s_i) = \mathrm{res}_{U_j,U_i\cap U_j}(s_j)$. Then there exists some meaning s of the whole fragment U such that $\mathrm{res}_{U,U_j}(s) = s_j$ for all U_j.*

In mathematics, a presheaf satisfying both claims **S** and **C** is said to be a *sheaf* (Tennison, 1975). Whence:

Frege's Generalized Compositionality Principle. *The presheaf of fragmentary meanings attached to any sense of reading of an admissible text endowed with phonocentric topology is really a sheaf.*

Thus the true object of study in text semantics of a natural language E should be a *category of textual spaces* $\mathbf{Logos}_E$; its objects are couples $(X, \mathscr{F})$, where X is a topological space naturally attached to a particular admissible text and $\mathscr{F}$ is a sheaf of fragmentary meanings defined on X; the morphisms are couples $(f,\theta)\colon (X,\mathscr{F}) \to (Y,\mathscr{G})$ made up of a continuous map $f\colon X \to Y$ and a natural transformation between sheaves, called f-morphism, $\theta\colon \mathscr{G} \to f_*\mathscr{F}$, where f_* is a *direct image* functor; for technical details, see (Author, 2008).

So far, we have considered the meanings of open sets (parts) in a phonocentric topology. We define now the meanings of its points (sentences). Let U, V be two neighborhoods of a sentence x and let $\mathscr{F}$ be an adopted sense. Two fragmentary meanings $s \in \mathscr{F}(U)$, $t \in \mathscr{F}(V)$ are said to induce the same contextual meaning at $x \in U \cap V$ if there exists an open neighborhood W of x, such that $W \subseteq U \cap V$ and $\mathrm{res}_{U,W}(s) = \mathrm{res}_{V,W}(t) \in \mathscr{F}(W)$. This relation 'induce the same contextual meaning at x' is clearly an equivalence relation; any equivalence class of fragmentary meanings agreeing in some open neighborhood of a sentence x is said to be a *contextual meaning* of x. The set of all equivalence classes is called a *stalk* of $\mathscr{F}$ at x and denoted by $\mathscr{F}_x$. Mathematically, the set $\mathscr{F}_x$ of contextual meanings of a sentence $x \in X$ is defined as the inductive limit $\mathscr{F}_x = \varinjlim(\mathscr{F}(U), \mathrm{res}_{V,U})_{U,V \in \mathscr{O}(x)}$.

Let $(f,\theta)\colon (X,\mathscr{F}) \to (Y,\mathscr{G})$ be a morphism of textual spaces. For a given sentence $x \in X$, a natural transformation of sheaves $\theta\colon \mathscr{G} \to f_*\mathscr{F}$ induces a map $\theta(V)\colon \mathscr{G}(V) \to f_*\mathscr{F}(V)$ for every open neighborhood V of x in Y. By going to inductive limit at x, one obtains an induced map $\theta_x\colon \mathscr{G}_{f(x)} \to \mathscr{F}_x$ of the corresponding stalks. The family of maps $(\theta_x)_{x\in X}$ gives another (*bundle-theoretic*) definition of a textual spaces morphism $(f,\theta)\colon (X,\mathscr{F}) \to (Y,\mathscr{G})$. The similar considerations may be repeated with necessary modifications at the semantic level of sentence; for technical details, motivations and examples, see (Author, 2005, 2008, 2010).

Let us now analyze quotations in the frame of our sheaf-theoretic formal semantics. If one text X contains a part U' which coincides, as a sequence of sentences, with a part U of another text Y, and this origin of U' is marked by conventional typographic means, the part U' is said to be *quotation*. To distinguish quotation, one uses quote marks, italics, or prints U' in block (for a quotation longer than four lines).

If so marked subsequence U' of X is used to represent a subsequence $U \subseteq Y$ as syntactic objet regardless of its meaning as a part of Y, when this case is classified as *mention* (Cappelen & Lepore, 2012) in X of the part $U \subseteq Y$. In particular $Y = U = U'$, when the author of text X distinguishes U' as quotation in order to demonstrate to reader that this part U' of X should be considered from a syntactic point of view. The case of pure mention is not so interesting.

More interesting is the case of quotation classified as *use* (Cappelen & Lepore, 2012), when U' is considered as a part of text X whose meaning as a part of X is inherited from the meaning of the source part $U \subseteq Y$. It is clear that U is a meaningful part of Y as being worth of quoting. Let $j\colon U \hookrightarrow Y$ be the canonical injection and let $\mathscr{G}$ be a sheaf on Y. In this particular case, the *inverse image* $j^*\mathscr{G}$ of the sheaf $\mathscr{G}$ is simply defined as the following sheaf on U:

$$(j^*\mathscr{G})(V) = \mathscr{G}(j(V)) \qquad \text{for all opens } V \subseteq U;$$
$$\mathrm{res}^*{}_{W,V} = \mathrm{res}_{j(W),\,j(V)} \qquad \text{for all opens } V, W \text{ in } U \text{ such that } V \subseteq W.$$

Hence for a given textual space $(Y,\mathscr{G})$, the canonical injection of open $U \hookrightarrow Y$ defines a textual space $(U, j^*\mathscr{G})$ which is natural to consider as a subspace of $(Y,\mathscr{G})$ where all meaningful parts are taken in the context of Y understood in the sense $\mathscr{G}$. Similarly, for a given textual space $(X,\mathscr{F})$, the canonical injection $i\colon U' \hookrightarrow X$ defines a subspace $(U', i^*\mathscr{F})$ of $(X,\mathscr{F})$, because the very fact of quotation implies that U' is meaningful in X.

Quotations are used for many purposes, but always it gives rise to morphism of corresponding textual subspaces $(\mathrm{id},\theta)\colon (U', i^*\mathscr{F}) \to (U, j^*\mathscr{G})$, where $\mathrm{id}\colon U' \to U$ acting as $\mathrm{id}\colon x \mapsto x$. Following the bundle-theoretical definition of textual spaces morphism, for $f = \mathrm{id}$, it is equal to define a map $\theta_x\colon (j^*\mathscr{G})_x \to (i^*\mathscr{F})_x$ of the corresponding stalks for every sentence $x \in U$. Thus, each contextual meaning of a sentence x in a quotation $U' \subseteq X$ is really transferred as image of contextual meaning of a sentence x of quoted fragment $U \subseteq Y$.

In the paper, several types of quotation will be analyzed in the frame of sheaf-theoretic formal semantics. We consider the case of quotations that are called *maxims* or *aphorisms* often cited as the most adequate expression of its content. In the case when the senses $\mathscr{F}$ and $\mathscr{G}$ are of the same kind, (say moral, historical, etc.), it seems that we need to have $\theta_x = \mathrm{id}_{\mathscr{F}_x}$; namely for all $x \in U$, each contextual meaning of x grasped in the context of X is the same as its contextual meaning grasped in the context of Y. We also consider the case of quotation which is used to enter into a dialogue with the author of a quotation source Y in order to analyze the content of a quoted fragment $U \subseteq Y$ or to support the arguments developed in the text X by means of quotation. Here, any contextual meaning of a sentence $x \in U'$ grasped in the context of X gives rise to some its contextual meaning grasped in the context of Y, and *vice versa*.

The sheaf-theoretic approach extends to analysis of one type of *mixed quotations* studied in (Geurts & Maier, 2003), analysis of quotation within a quotation and others types of quotation interpreted as different compositions of *direct image*, *inverse image* and other related *functors* defined on the category of textual spaces.

References

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