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Multi-criteria covering-based location of volunteer fire departments

Workingpaper no. 2012 | 01

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August 30, 2012

In Germany, a large number of volunteer fire departments (VFD) exist that have been founded in order to provide regional support for professional firefighters. The current locations of these volunteer fire departments might be suboptimal with respect to costs and availability. Due to historical developments the city of Bochum (Germany) also hosts a large number of volunteer fire departments. A mixed-integer linear program is presented in order to investigate the consequences of partial relocating or closing of volunteer fire departments. Timeliness is one of the most important objectives since it reflects the quality of emergency services such as firefighting systems. Timeliness means that in most instances the emergency can be reached within a predefined time limit after reception of the emergency call. A sufficient number of firefighting facilities have to be established and located in order to provide high quality services over a wide area with spatially distributed demands. Different criteria are taken into account in order to find optimal locations. Minimizing the average travel time and minimizing the maximum travel time are goals which are in conflict with minimizing costs for establishing facilities and for transport. A multi-criteria approach is applied and different solutions for an urban area with several volunteer firefighting departments are analyzed. The tradeoff between maximizing the quality of emergency services and cost reduction is illustrated using a real world case study with 18 existing volunteer fire departments.

Keywords: location problem, covering models, real world application, multi-criteria optimization, volunteer fire departments

1 Introduction

Emergency services such as fire departments and ambulances or police departments must provide high service levels to ensure public safety. In Germany, local fire brigades are responsible for fire fighting and rescue services. However, financial constraints and public budgets force municipalities to extensive savings. These savings are difficult to realize in the field of emergency services in so far that the quality of the service must be guaranteed. Additionally, considering large-scale disasters there is a need for volunteer fire departments in order to support professional fire fighters. In general, demand caused by mass casualty incidents increases and is therefore to be considered within the planning [15]. We investigate the possible savings for the fire brigade in the city of Bochum. In order to provide a high level of service, resources have to be used efficiently. Therefore, we consider timeliness as a criterium of major importance (see e. g.

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[18]). Timeliness can be measured in different ways: (1) Minimization of the total or average time to serve all emergency calls or (2) minimization of the maximum travel time to serve every single call (see [1]). In order to preserve a good coverage for the whole urban area as well as minimizing running costs the question of optimally placing respectively replacing of fire departments arises. For practical and economic reasons it is assumed that only the fire departments of the volunteer fire brigade can be repositioned. The main departments of the professional fire brigade are considered to be fixed because repositioning of the main fire departments is entailed with unacceptable costs. The following questions are discussed: (1) How does the actual situation differ from an optimal solution which not only take into account existing locations? (2) How sensitive is a solution that has been determined requesting that a reduced number of fire departments are to be placed and a minimum number of departments must be preserved simultaneously? Which consequences can be expected due to a variation of possible parameter constellations? Further considerations pursue three essential aims: First, the total or average travel time has to be minimized which also implies minimizing operating costs. Second, not only the accessibility of each emergency within the statutory (legal) standard service time must be ensured but also the maximum emergency response time has to be minimized. The third aim is the maximal preservation of the existing infrastructure. Thus, the existing departments should be used preferentially. We investigate these conflicting targets simultaneous within one approach. A case study for the city of Bochum is performed in order to evaluate the effects of placing, replacing and closing of locations. The article is organized as follows: Subsequent to a short literature review in section 2 a problem description is given in section 3. A multi-criteria mixed-integer linear problem (MILP) for covering based location of volunteer fire departments is formulated and a multi-criteria approach is proposed. In section 4 the previously described multi-criteria MILP is applied to a real world decision making situation for eighteen volunteer fire stations in the city of Bochum. Finally, a short conclusion and an outlook for further research are given.

2 Literature review

The location problems of emergency services such as fire departments are a wide and active research area. One of the first emergency facility location models discussed in literature was the Set Covering Location Problem (SCLP) proposed by Toregas et al. (1971) [20]. The goal of the SCLP is to find the minimum number of facilities and their positions to cover all demand points with predefined standard coverage distance or time constraint. In this model each demand point is treated equally because the demand in each region is not considered. The first model that takes the demand into account is the Maximal Covering Location Problem (MCLP) proposed by Church an ReVelle (1974) [6]. Given a predefined number of facilities p, the MCLP maximizes the demand coverage. Both models (SCLP and MCLP) do not take into account unavailability of the facilities due to system congestion. Two ways to overcome this drawback are discussed in literature. One is to provide multiple or redundant coverage as introduced e. g. the backup coverage problem (BACOP 1 and BACOP 2) by Hogan and ReVelle (1986) [12] or the Double Standard Model (DSM) by Gendreau et al. (1997) [19]. The second way to consider the busy probabilities and reliabilities of facilities is to incorporate a probabilistic busy fraction into the models as shown in e. g. the Maximal Expected Covering Location Model (MEXCLP) by Daskin (1983) [7] or the Maximum Availability Location Problem (MALP I and MALP II) by ReVelle and Hogan (1989) [17]. A wide overview of models for emergency response with different key aspects is given in ReVelle (1989) [16], Brotcorne, Laporte and Semet (2003) [5], Li et al. (2010) [13] and Farahani et al. (2012) [10]. A detailed taxonomy for emergency service stations location problems is given by Başar, Çatay and Ünlüyurt (2011) [3].

An investigation of the emergency medical service of the city of Bochum (Germany) is presented in Werners, Drawe and Thorn (2001) [22] and in Werners, Thorn and Hagebölling [23]. In these two papers the authors evaluate the SCLP with three different time standards ($T = 6, 8, 10$ minutes). They also solve the MCLP with real word data for 166 demand regions in the urban area of Bochum. A modified maximal covering location problem is developed to incorporate aspects from both models (SCLP and MCLP). In this modified MCLP full coverage is ensured by similar constraints as in the SCLP. The objective function in the modified MCLP maximizes the covered demand but counts the demand as often as covered. This leads to a multi-coverage of regions with high demand. The authors state that multi-criteria approaches should be used to incorporate additional criteria for real world decision making.

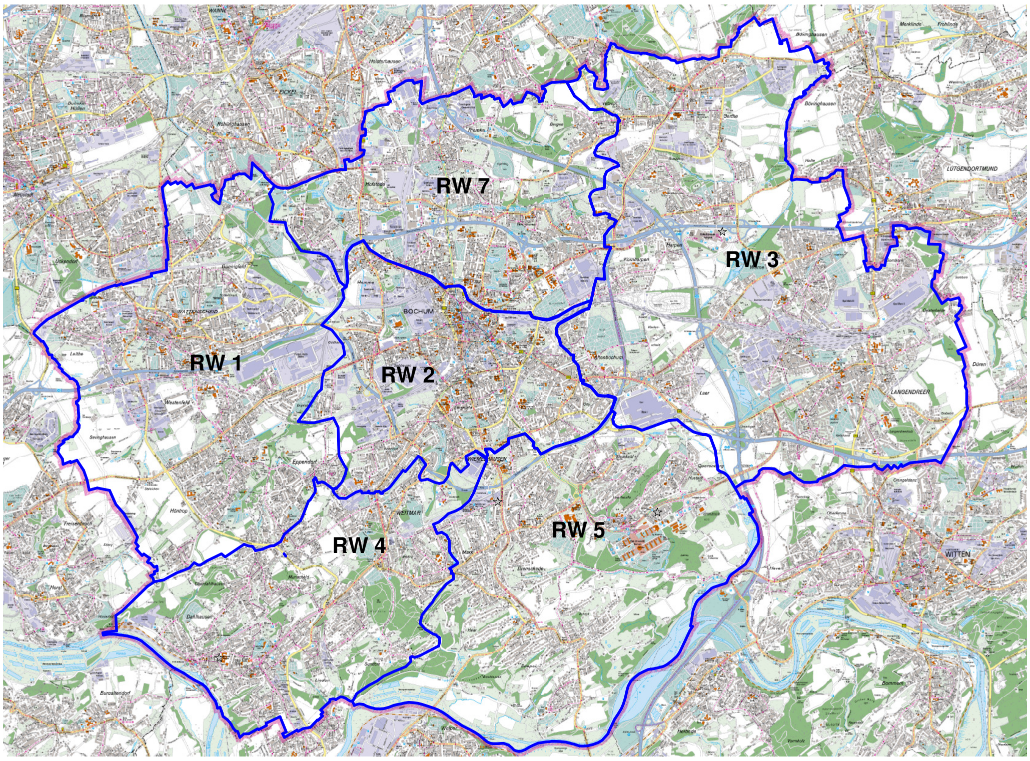
A wide survey on multiple criteria facility location problems is presented by Farahani, Steadie Seifi and Asgari (2010) [11]. Applications and solution methods of multi-criteria location planning can be found in Doerner, Focke and Gutjahr (2007) [8] and in Yang, Joenes and Young (2007) [24]. In Badri, Mortagy and Alsayed (1998) [2] a multi-criteria model for locating fire stations is presented and applied in a case study for the Dubai City Fire Department. Two of the main goals in this model are minimizing the fixed costs as well as the annual operating costs and minimizing the average and maximum time traveled. A survey on multi-objective optimization methods is given by Marler and Arora (2004) [14]. In the following section we formulate a covering model considering multiple objectives in order to locate volunteer fire stations.

3 Problem description

The urban area of Bochum is divided into 166 quadratic areas with a size of 1×1 square kilometer for task allocations prior to the repositioning of the departments (see figure 1). In order to develop a graph-theoretical model the center of each area can be interpreted as a demand node in a graph. We assume that each node can be reached from any other node in the network. For this strategic decision an aggregation level is chosen so that the position of the fire department is centered in the corresponding square and only one department can be placed per square. Due to close respectively identical positioning of volunteer fire departments near the main departments, to some extent more than one department per square exists. Those close departments are considered as one department through associated complexes of buildings. Besides, some squares are excluded from consideration due to too high prices of land, a too high density of buildings or area-covering water. Additional to three fixed main fire departments there are also fixed volunteer departments. We introduce different index sets for an unambiguous identification.

3.1 Sets, Parameters and Decision Variables

The index set I denotes all planning areas (demand nodes) and J all potential sites for volunteer fire stations. The index set $\mathcal{E}$ is composed of all indices of fire departments that exist and *can* be relocated or closed ($\mathcal{E} \subset J$) whereas the index set $\mathcal{F}$ includes all indices of fixed fire departments. These fixed stations already exist but *cannot* be relocated or closed ($\mathcal{F} \subset J$). Obviously, the intersection of existing and fixed departments is empty: $\mathcal{E} \cap \mathcal{F} = \emptyset$. $\mathcal{U}$ indicates sites that are prohibited for establishing of fire stations due to financial, geographical or other reasons ($\mathcal{U} \subset J$). Exogenous parameters are given through the distance matrix $D = (d_{ij})_{i,j \in I}$, the average speed of the emergency vehicles v and the overall annual number of emergencies a_i in each planning square $i \in I$. We assume that the driving distance between the planning squares i and j is distributed symmetrically, meaning $d_{ij} = d_{ji}$ for all $i, j \in I$. The distance d_{ij} is approximated by the euclidian distance (ℓ_2-metric) between the centers of the planning squares i and j. The actual distance can be depicted reasonably well as there is a homogeneous topological structure of the urban area and an extensive road network. For rescue efforts within the whole urban area a constant (average) speed v of 25 kilometers per hour is derived from earlier empirical studies concerning operating protocols and therefore provides a good approximation (see [22] and [23]). Beyond that it is assumed that each department can handle any number of rescues. This is due to the fact that firefighters of the volunteer fire brigade only support professional firefighters. Furthermore, the simultaneous occurrence of several fire fighting operations within one planning square is very rare and thus not considered here. The number of emergencies a_i is seen as deterministic and constant over time (see figure 1). The (legal) standard service time in urban areas amounts to $T = 12$ minutes. We consider a number of sites q that must be preserved $0 \leq q \leq |\mathcal{E}|$. The overall number of sites p is limited to the number of existing facilities in the city of Bochum and thus $0 \leq p \leq |\mathcal{E} \cup \mathcal{F}|$. Two types of



												130 (0)				
						56 (0)	68 (1)	81 (0)	94 (2)	107 (6)	119 (12)	131 (18)	141 (18)	149 (0)		
			22 (0)		45 (1)	57 (22)	69 (17)	82 (18)	95 (4)	108 (5)	120 (22)	132 (1)	142 (0)	150 (0)		
		12 (1)	23 (4)	34 (3)	46 (27)	58 (13)	70 (17)	83 (13)	96 (7)	109 (13)	121 (7)	133 (3)	143 (1)	151 (2)		
		13 (14)	24 (3)	35 (7)	47 (18)	59 (29)	71 (8)	84 (40)	97 (11)	110 (21)	122 (22)	134 (16)	144 (12)	152 (3)	157 (9)	162 (0)
1 (4)	5 (18)	14 (10)	25 (22)	36 (6)	48 (7)	60 (24)	72 (85)	85 (34)	98 (3)	111 (6)	123 (1)	135 (3)	145 (19)	153 (9)	158 (5)	163 (1)
2 (11)	6 (29)	15 (37)	26 (16)	37 (6)	49 (26)	61 (21)	73 (83)	86 (24)	99 (11)	112 (2)	124 (12)	136 (14)	146 (23)	154 (14)	159 (14)	164 (0)
3 (8)	7 (22)	16 (16)	27 (13)	38 (4)	50 (3)	62 (8)	74 (44)	87 (16)	100 (14)	113 (11)	125 (7)	137 (2)	147 (20)	155 (16)	160 (34)	165 (0)
4 (0)	8 (3)	17 (8)	28 (13)	39 (5)	51 (3)	63 (12)	75 (10)	88 (13)	101 (14)	114 (19)	126 (23)	138 (1)	148 (5)	156 (4)	161 (2)	166 (1)
	9 (1)	18 (5)	29 (2)	40 (3)	52 (9)	64 (7)	76 (11)	89 (13)	102 (17)	115 (82)	127 (9)	139 (1)				
	10 (0)	19 (3)	30 (7)	41 (4)	53 (2)	65 (6)	77 (13)	90 (3)	103 (2)	116 (0)	128 (7)	140 (1)				
	11 (4)	20 (3)	31 (6)	42 (8)	54 (1)	66 (2)	78 (3)	91 (6)	104 (3)	117 (1)	129 (8)					
		21 (10)	32 (18)	43 (4)	55 (1)	67 (3)	79 (2)	92 (1)	105 (3)	118 (2)						
			33 (6)	44 (1)			80 (0)	93 (0)	106 (1)					i (a_i)	index demand	

Abbildung 1: Map of the city of Bochum indicating the areas of responsibility of six professional fire departments (RW1,..., RW5, RW7)

decision variables are used:

$$x_{ij} := \begin{cases} 1 & \text{if demand point } i \in I \text{ is assigned to fire station } j \in J \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$y_j := \begin{cases} 1 & \text{if a fire station is located at site } j \in J \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The binary decision variable x_{ij} states whether a demand point is assigned to a fire station or not (1). Finally, the binary variable y_j indicates if a site is selected for hosting a fire station (2).

3.2 Multi-criteria MILP for locating VFD

The management of the firefighters and the population as potential victims of a fire are considered as major stakeholders. A basic multi-criteria MILP is presented in order to model their conflicting interests. The objective functions are aggregated by applying a weighted sum approach in order to find a compromise solution. The presented multi-criteria mixed-integer linear program is based on covering location models to locate a fixed number of firefighting facilities and to dedicate the demand points to the facilities.

	Indices and sets
$\mathcal{E}$	Index set of existing (not fixed) facilities (indexed by j)
$\mathcal{F}$	Index set of fixed facility sites (indexed by j)
$\mathcal{U}$	Index set of prohibited sites for facilities (indexed by j)
J	Index set of potential facility sites (indexed by j)
I	Index set of demand points (indexed by i)
$\mathcal{K}$	Index set of individual objectives (indexed by k, ℓ)
	Parameters
a_i	Number of requests at demand point $i \in I$ per period
d_{ij}	Distance from demand point $i \in I$ to the facility at site $j \in J$ (in km)
p	Total number of available facilities
q	Total number of given facilities, which are not allowed to be changed
T	Time standard for coverage (in hours)
v	Average speed in urban area (in km per hour)
λ	Vector of weighting coefficients $\lambda = (\lambda_1, \lambda_2)$ with $\lambda_k \geq 0$ for $k \in \mathcal{K} = \{1, 2\}$

$$\min \quad \frac{1}{v\sum\limits_{i\in I} a_i}\left[\sum_{i\in I}\sum_{j\in J} d_{ij}a_i x_{ij}\right] \tag{3}$$

$$\min \quad T - t \tag{4}$$

subject to:

$$\sum_{j\in J} x_{ij} = 1 \qquad \forall i \in I \tag{5}$$

$$x_{ij} \le y_j \qquad \forall i \in I, \forall j \in J \tag{6}$$

$$\sum_{j\in J} \frac{d_{ij}}{v} x_{ij} \le T - t \qquad \forall i \in I \tag{7}$$

$$\sum_{j\in J} y_j \le p \tag{8}$$

$$\sum_{j\in \mathcal{E}} y_j \ge q \tag{9}$$

$$y_j = 1 \qquad \forall j \in \mathcal{F} \tag{10}$$

$$y_j = 0 \qquad \forall j \in \mathcal{U} \tag{11}$$

$$y_j,\, x_{ij} \in \{0,1\} \qquad \forall i \in I, \forall j \in J \tag{12}$$

$$t \ge 0 \tag{13}$$

The first objective function (3) minimizes the average travel time per request. The second objective function (4) minimizes the maximum travel time as the difference between a given time standard T and a nonnegative auxiliary decision variable $t \ge 0$. Constraint (5) states that a demand node $i \in I$ must be assigned to exactly one facility at site $j \in J$. Constraint (6) ensures that demand nodes $i \in I$ can only be assigned to located facilities. In combination with (5) constraint (7) ensures that each demand node $i \in I$ is covered by at least one server within a time standard T (if $t = 0$). Together with the objective function (4) the model maximizes t meaning minimizing the maximum travel time. Constraints (8) and (9) reflect the conflict of fixed costs for new locations and the two objectives. Constraint (8) gives the maximal number of facilities to be placed and (9) gives the minimal number of existing facilities that must be preserved. Later on we examine the tradeoff between the average travel time and the number of facilities utilized which are considered as a proxy for the available budget. Constraints (10) and (11) state whether the location of facilities is forced or prohibited. The binary decision variables y_j indicate at which site a fire station is located or not and x_{ij} shows to which fire station $j \in J$ a demand point $i \in I$ is dedicated. Finally the lower deviation of the given time limit has to be nonnegative (13). The feasible region of the multi-criteria problem consists of all points that hold constraints (5)–(13) and is denoted by X,

$$X := \left\{(x,y,t) \in \{0,1\}^{|I|\cdot|J|} \times \{0,1\}^{|J|} \times \mathbb{R}_{\ge 0} \mid (x,y,t) \text{ holds } (5)-(13)\right\}. \tag{14}$$

For a feasible solution $(x,y,t) \in X$ we define $\xi := (x,y,t)$. The objective functions z_k with $k \in \mathcal{K}$ represent different goals. The first goal can be interpreted as a public benefit or as the goal of the firefighters' management. The quality of emergency services such as firefighting systems is measured as the rate of emergencies that are served within the given time limit out of all emergencies:

$$\frac{\#\text{ operations that hold the time standard } T}{\#\text{ total operations}} \tag{15}$$

This goal can be reached by minimizing the average travel time which implicates minimization of the total travel time. This leads to the fact that the travel time to regions with lower demand could be longer. For an individual victim of an accident it is more important to minimize the worst case which is the maximum travel time. For K goals there usually exist K different individual optimal solutions. In most cases no solution exists that simultaneously minimizes all of the objective functions. Therefore, a compromise solution has to be identified. Various approaches to handle multi-criteria optimization problems are discussed in literature (e. g. Ehrgott (2005) [9], Marler and Arora (2004) [14] or Werners (1987) [21]). A feasible solution $\xi^\star \in X$ of a multi-criteria minimization problem is called *Pareto optimal*, if there does not exist another solution $\xi \in X$, such that $z_k(\xi) \leq z_k(\xi^\star)$ for all $k \in \mathcal{K}$ and $z_{k'}(\xi) < z_{k'}(\xi^\star)$ for at least one objective $k' \in \mathcal{K}$ [14].

3.3 Solution approach

One of the most common approaches to handle multi-criteria decision problems is the weighted sum approach. In this approach the objectives are aggregated prior to the optimization by assigning weights to the objective functions. Usually the weights represent the decision-makers preferences in order to model the importance of a particular goal. Let $\lambda = (\lambda_1, \ldots, \lambda_K)$ with $\lambda_k > 0$ for all $k \in \mathcal{K}$ be a vector of weighting coefficients. By minimizing $\sum_{k\in\mathcal{K}} \lambda_k z_k(\xi)$ a Pareto optimal solution can be found (Marler and Arora (2004) [14]). The optimization model to find an optimal compromising solution can be formulated as follows.

$$\min \quad \lambda_1 z_1 + \lambda_2 z_2 \tag{16}$$

$$\text{s. t.} \quad z_1 = \frac{1}{v \sum\limits_{i\in I} a_i} \left[\sum_{i\in I} \sum_{j\in J} d_{ij} a_i x_{ij} \right] \tag{17}$$

$$z_2 = T - t \tag{18}$$

$$\xi = (x, y, t) \in X \tag{19}$$

The parameter $\lambda_k \in [0, 1]$ indicates the importance of a particular goal. Thus, with increasing λ_k the importance of the first (second) goal increases (decreases) and vice versa. Weights have to be chosen carefully regarding different scales of measurement within the objective functions. We chose the scalar $(v \sum_{i\in I} a_i)^{-1}$ in order to accommodate the outcomes of the two objective functions in hours. This simplifies the application of the weighted sum approach and the interpretation of the chosen weights. Implicitly, a third goal is considered regarding the number of facilities that have to be located. The costs for operating and replacing facilities are modeled in terms of the parameters p and q. The consequences of reducing the number of facilities in order to achieve municipality's cost-saving aims will be investigated using an a posteriori approach. We solve various instances with respect to different parameters (p, q) and analyze interdependencies among the different goals. The proposed multi-criteria optimization model is applied to a real-world problem setting in the following chapter. We perform a case study for an urban region using historical data from a local fire department in Bochum.

4 Case study and computational results

The city of Bochum (North Rhine-Westphalia, Germany) has a size about 145.4 square kilometers and had approximately $374,000$ residents in 2011 [4]. The urban extension is 13 km in the north-south direction, 17.1 km in the east-west direction and 17 km in the southwest-northeast direction. The average population density is 2,570 inhabitants per square kilometer (6,680 per square mile) with a maximum value of 6,580 and a minimum value of 920 inhabitants per square kilometer. In the south the city is restricted by the river Ruhr and the lake Kemnade. A map of Bochum which shows the division into 166 sites is represented in figure 1 on page 5. In the city of Bochum three main fire departments exist (at sites $\{27, 61, 145\}$) belonging to the professional fire brigade and furthermore another eighteen fire departments belonging to the volunteer fire brigade. There is one volunteer fire brigade at each site of a professional fire department and two new volunteer departments at the sites $\{32, 110\}$. The two new and the three departments belonging to the professional fire departments could be seen as fixed, therefore we use the index set

$$\mathcal{F} = \{27, 32, 61, 110, 145\}\,. \tag{20}$$

The other thirteen of those eighteen departments can be optimized which means that they can be closed or relocated. This stations are sited at

$$\mathcal{E} = \{13, 21, 26, 28, 41, 71, 91, 100, 115, 125, 146, 148, 155\}\,. \tag{21}$$

In figure 1 on page 5 the light gray sites represent the location of the thirteen fire departments belonging to the volunteer fire brigade which could be relocated and the dark gray sites represent the fixed fire stations. It has to be remarked that there is a very low coverage in the north-eastern part of the city, especially before the station at site 110 was newly built. However, for practical and economic reasons it is assumed that only the fire departments of the volunteer fire brigade can be repositioned. The main departments can be seen as fixed due to earlier planning. Another reposition of the main fire departments is entailed with unacceptable costs. For sites that are unsuitable for economic or geographic reasons the index set

$$\mathcal{U} = \{11, 22, 37, 60, 62, 73, 80, 93, 98, 112, 116, 118, 128, 137, 150, 153, 158\} \tag{22}$$

is introduced. The eighteen fire departments are not equally distributed over the entire city. This is on the one hand due to the fact that areas with a high population density lead to a higher number of operations and on the other hand due to historical placing. Historical placing implies for example incorporations of outskirts to Bochum with low population density and therefore lower number of operations but nevertheless located close to several fire departments. For our investigation we consider a time horizon with $1,710$ fire emergency calls from the 166 demand points. The assumed demand distribution for the 166 demand nodes can be seen in figure 1 on page 5 as the number below the demand point index and is typical for an urban region. The demand distribution varies significantly from 0 up to 85 calls in some regions. Before turning to the optimization of the current situation we consider how many fire stations are needed at least to ensure total coverage. Therefore, a simple set covering model is used.

4.1 Covering aspects and lower bound

In order to calculate the minimum number of facilities we use the set covering location problem model (SCLP). The SCLP was introduced by Toregas et al. [20]. With $t_{ij} = d_{ij}/v$ for all $i, j \in I$ and

$$N_i := \{j \in J \mid t_{ij} \leq T\} \tag{23}$$

the SCLP model can be formulated as follows:

$$\min \quad \sum_{j \in J} y_j \tag{24}$$

$$\text{s.t.} \quad \sum_{j \in N_i} y_j \geq 1 \qquad \forall i \in I \tag{25}$$

$$y_j \in \{0, 1\} \qquad \forall j \in J \tag{26}$$

The objective function (24) minimizes the total number of facilities required. Constraint (25) states that all demand points must be covered at least once. $y_j = 1$ if a fire station is located at site $j \in J$ and $y_j = 0$ otherwise (for all $j \in J$). The value T is set to $T := 10.8$ minutes to consider a dispatch delay. Without any additional constraints we calculate an optimal solution for the SCLP with four fire stations at the sites $\{18, 58, 91, 152\}$ (with $v = 25$ km/h, $T = 10.8$ minutes). In this solution only the fire station at site 91 already exists. With respect to the existing facilities there is a large number of optimal solutions, yet none of them uses more of the existing locations. With the additional constraint $y_j = 0$ for all $j \in \mathcal{U}$ (sites in $\mathcal{U}$ could be excluded because of too high setup costs) but without the restriction on existing facilities we calculate the minimum number of facilities $z^\star = 4$ with the same solution as before (see the left figure 2 on

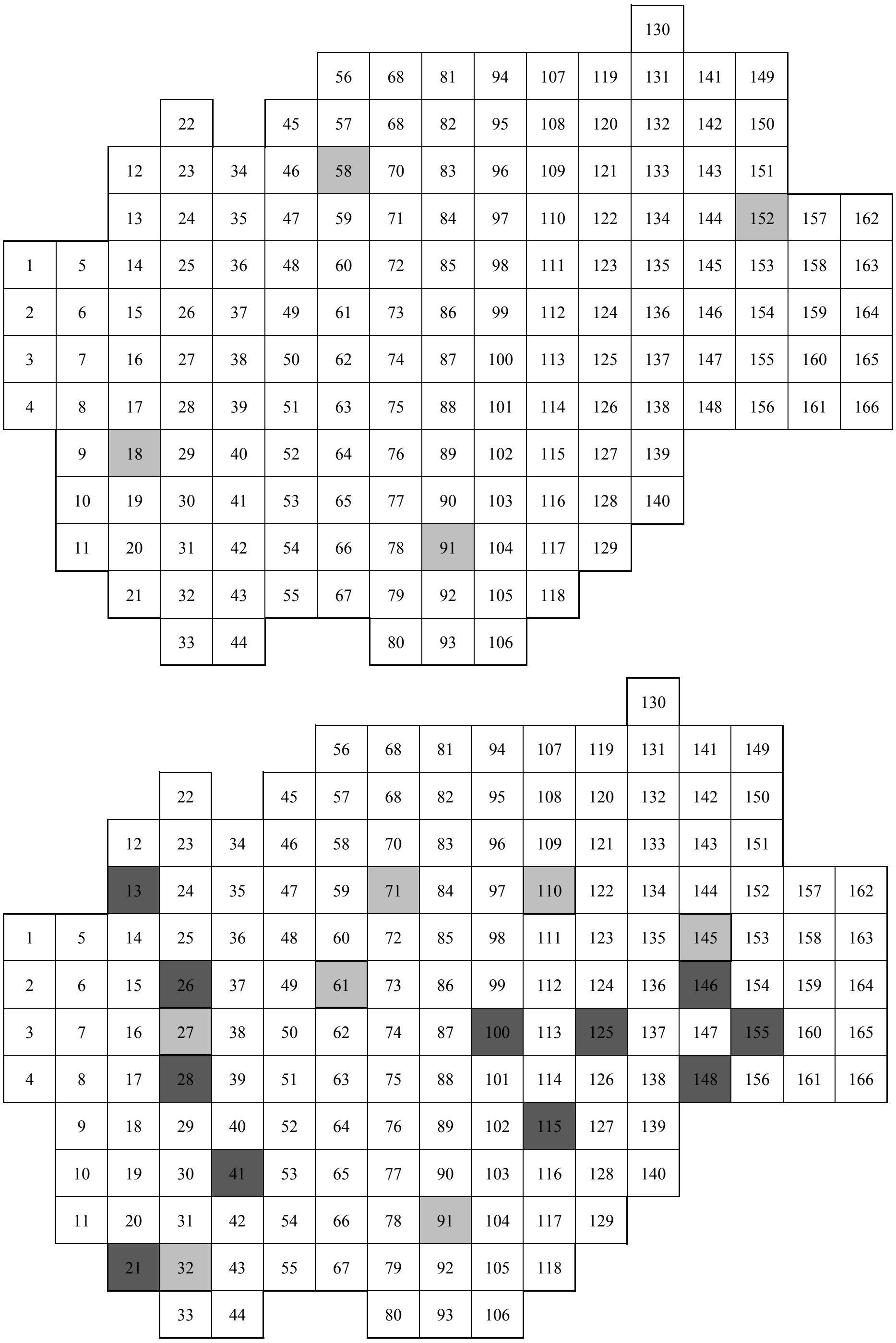


Abbildung 2: On the top: Solution of the SCLP without restriction to existing facilities, on the buttom: Solution of the SCLP with restriction to existing facilities

criterium	total number of stations	number of (new | existing)	solution $y_j = 1$ for j in
SCLP	4	(3 | 1)	$\{18, 58, 91, 152\}$
SCLP+$\mathcal{U}$	4	(3 | 1)	$\{18, 58, 91, 152\}$
SCLP+$(\mathcal{E} \cup \mathcal{F})$	6	(0 | 6)	$\{27, 41, 71, 91, 110, 145\}$ or
SCLP+$\mathcal{E} + \mathcal{F}$	7	(0 | 7)	$\{27, 32, 61, 71, 91, 110, 145\}$
			$\{27, 32, 61, 71, 110, 115, 145\}$
status quo	18	(0 | 18)	$\mathcal{E} \cup \mathcal{F}$

Tabelle 1: Number of stations to ensure full coverage ($v = 25$ km/h, $T = 10.8$ min)

page 10). In order to consider only sites where fire stations currently exist we add the constraint $y_j = 0$ for all $j \in J \setminus (\mathcal{E} \cup \mathcal{F})$ to the model and calculate the minimal number of facilities as $z^\star = 6$ with the optimal solution $y_j = 1$ for $j \in \{27, 41, 71, 91, 110, 145\}$. In this solution one of the fixed stations ($j = 32$) is not considered. With respect to the fact that all fixed stations must be part of a solution, the constraint $y_i = 1$ for all $j \in \mathcal{F}$ is added to the model and we finally calculate the minimal number of facilities as $z^\star = 7$ with two optimal solutions $y_j = 1$ for $j \in \{27, 32, 61, 71, 91, 110, 145\}$ or $j \in \{27, 32, 61, 71, 110, 115, 145\}$. In this solution all fixed stations are included ($y_j = 1$ for all $j \in \mathcal{F}$). In this calculation the demand at each site $i \in I$ is not considered. The following table 1 compares the different solutions by successively reducing the feasible region. The upper picture of figure 2 shows an optimal solution for the SCLP without any additional restrictions. As typical for SCLP-model solutions the facilities tend to be at the periphery of the city, which means far away from demand nodes with high demand. The second image of figure 2 shows the minimum number and location of facilities out of the existing facilities that are required to cover all demand points $i \in I = \{1, \ldots, 166\}$. Only the light gray colored facilities $\{27, 32, 61, 71, 91, 110, 145\}$ are needed to cover all demand points. The dark gray colored facilities could be relocated. This optimal value of $z^\star = 7$ is a lower bound for the parameter p in the multi-criteria model.

4.2 Demand covering aspects and reduction of facilities

In this section we take the demand into account and consider two different interests in a multi-criteria approach. Minimization of the average travel time as the main interest of the firefighting management and minimization of the maximum travel time as the main interest of the residents. In a weighted sum approach the vector of weights $\lambda = (\lambda_1, \ldots, \lambda_K)$ is set a priori by the decision maker. Typically, the weights are set such that $\sum_{k=1}^{K} \lambda_k = 1$ and $\lambda_k > 0$ for all $k = 1, \ldots, K$. The relative value of the weights reflects the relative importance of the objectives. Using the weights $\lambda = (0.5, 0.5)$ means that both criteria are treated equally. Due to the discrete structure of the decision situation small changes of the parameter λ_k do not have a major impact on the solutions. Annual operating costs for the facilities and setup costs for new facilities will be implicitly considered by varying the values of the parameters p and q. In order to evaluate the current situation we set the number of facilities that could be established to $p = 18$ and the number of existing facilities that have to be preserved to $q = 13$ (status quo). Figure 2 on page 13 shows the results for all parameter combinations (p, q) with $p = 7, \ldots, 18$ and $q = 0, \ldots, p - |\mathcal{F}|$. Simultaneous reduction of p and q means closing the facility with the smallest contribution to the objective functions step by step (moving along the principle diagonals from the down-right up to the upper-left corner). Fixing p and decreasing q of one means replacing the worst facility (moving in a fixed row from right to left). All in all 102 instances of the multi-criteria optimization models have been solved. For each setting (p, q) the model is solved using both individual objective functions and the weighted sum approach ($\lambda = (0.5, 0.5)$). One element in figure 2 shows the results for one setting (p, q)

$$\begin{bmatrix} z_1^\star & z_2(\arg\min(z_1^\star)) \\ z_1(\arg\min(z_2^\star)) & z_2^\star \\ z_1(\arg\min(\lambda_1 z_1 + \lambda_2 z_2)) & z_2(\arg\min(\lambda_1 z_1 + \lambda_2 z_2)) \end{bmatrix} \tag{27}$$

were $z_k^\star$ is the optimal objective value and $\arg\min(z_k^\star)$ is the optimal solution, if only objective function $k \in \{1, 2\}$ is considered. $\arg\min(\lambda_1 z_1 + \lambda_2 z_2)$ indicates the optimal solution for the weighted sum approach. $z_k(\arg\min(\lambda_1 z_1 + \lambda_2 z_2))$ evaluates objective function z_k with this solution. Obviously, for settings with $p < q + |\mathcal{F}|$ no feasible solution exists. In consequence of the rough discrete structure of the values of the maximum travel time, the solutions of both criteria are similar. The multi-criteria approach determines a solution which is also optimal for the second individual goal (minimization of the maximum travel time) in the majority of cases. These solutions are Pareto optimal and guarantee 'good' individual values for the first goal. In the current situation every demand point could be reached within 10.73 minutes at most and the average travel time is 2.92 minutes. The maximum travel time without building a new facility is constant because it is determined by the distance $d_{110,130} \approx 10.73$ minutes. But it could be improved by relocating a single station from site 148 to site 131 (solving the problem instance with $p = 18, q = 12$) to ≈ 7.59 minutes ($\approx 30\,\%$) with a simultaneous reduction in the average travel time from 0.22 minutes. The reduction of fire stations results in increasing the average travel time from 2.92 minutes (18 fire stations) to 3.93 minutes (7 fire stations). Table 3 shows the average travel time and the optimal solution if we reduce the number of stations from $p = 18, \ldots, 7$ and $q = p - |\mathcal{F}|$ step by step. Table 3 is organized as follows. The first part includes stations that could be closed. The given order represents the sequence in which they will be closed step by step. For example the station at site 148 is the first station to be closed and the station at site 21 is the second one. In the second part of the table there are the two existing stations at sites 71 and 115 that are needed to ensure total coverage (together with the fixed stations). The third part represents the fixed stations. The resulting closing sequence is

$$(148, 21, 28, 146, 125, 41, 26, 91, 100, 13, 155)\,.$$

Figure 3 visualizes the step by step reduction of the existing fire stations from the current situation to the optimal solution. The dark gray colored sites illustrate remaining fire stations, the light gray colored site those that will be closed. The number below the site index is the closing sequence. Figure 4 on page 15 shows the increase in average travel time when reducing the number of existing fire stations from 18 (status quo) down to 7. It turns out that the average travel time increases with the reduction of stations. In the current situation with 18 station the average travel time is 2.92 minutes. A reduction to 7 fire stations leads to an average travel time of 3.93 minutes which means an increase of 34.6 %.

4.3 Sensitivity of the solution according to partial relocation

Different consequences arise due to partial relocation of a subset of facilities regarding a fixed overall number of facilities. The values of p determine the costs for operating facilities whereas by variations of q the setup costs for new facilities can be observed. Implicitly impacts of changes of the available budget on the average and maximum travel time can be measured. Evaluating a single row of figure 2 from the right to the left the effective degrees of freedom increase for a given number p of facilities. Fixing $p = 18$ (♦ line) and relocating $r = 0, 1, 2, \ldots$ facilities (which means decreasing $q = p - |\mathcal{F}|, \ldots, 0$), the average and the maximum travel time improve by trend. Correspondingly, the same effects occur for values $p = 14$ (■ line) and $p = 10$ (▲ line). The following diagrams in figure 5 show relative changes of average and maximum travel time corresponding to the maximal allowed number of facilities that can be relocated. With $p = 14$ and $q = 7$ four of the existing eighteen facilities have to be closed and two $(7-|\mathcal{F}|)$ facilities can be relocated. This leads to an improvement of the average travel time of 10.02 % (0.31 min) and an improvement of the maximum travel time of 29.29 % (3.14 min) in contrast to reducing only the number of facilities to 14. Figure 5 also shows that the average travel time increases while the degrees of freedom rise. This is caused by the rough discrete structure of

	q=0		q=1		q=2		q=3		q=4		q=5		q=6		q=7		q=8		q=9		q=10		q=11		q=12		q=13	
p=7	3.93	10.73	3.93	10.73	3.93	10.73																						
	5.02	10.73	5.06	10.73	4.40	10.73																						
	3.93	10.73	3.93	10.73	3.93	10.73																						
p=8	3.66	10.73	3.66	10.73	3.66	10.73	3.72	10.73																				
	4.78	10.18	4.63	10.18	4.78	10.18	4.80	10.73																				
	3.76	10.18	4.01	10.18	3.66	10.73	3.72	10.73																				
p=9	3.40	10.73	3.40	10.73	3.42	10.73	3.44	10.73	3.52	10.73																		
	4.53	8.65	4.35	8.65	4.38	8.65	4.64	9.60	4.80	10.73																		
	3.71	8.65	3.71	8.65	3.97	8.65	4.20	9.60	3.52	10.73																		
p=10	3.16	10.73	3.16	10.73	3.18	10.73	3.20	10.73	3.29	10.73	3.38	10.73																
	4.19	7.59	4.12	7.59	4.26	7.59	4.16	7.59	4.41	7.59	4.79	10.73																
	3.51	7.59	3.53	7.59	3.55	7.59	3.61	7.59	3.75	7.59	3.38	10.73																
p=11	2.94	10.73	2.94	10.73	2.94	10.73	2.98	10.73	3.05	10.73	3.14	10.73	3.23	10.73														
	4.06	7.59	4.15	7.59	4.22	7.59	4.07	7.59	3.73	7.59	3.89	7.59	4.82	10.73														
	3.03	7.59	3.03	7.59	3.03	7.59	3.05	7.59	3.07	7.59	3.14	7.59	3.23	10.73														
p=12	2.72	9.90	2.72	9.90	2.72	9.90	2.79	7.59	2.85	10.73	2.90	10.73	2.99	7.59	3.16	10.73												
	3.33	6.79	3.26	6.79	3.65	6.79	3.42	6.79	3.57	7.20	3.79	7.59	4.11	7.59	4.82	10.73												
	2.95	6.79	2.95	6.79	2.95	6.79	2.99	6.79	3.14	7.20	2.92	7.59	2.99	7.59	3.16	10.73												
p=13	2.59	8.65	2.59	8.65	2.59	8.65	2.61	8.65	2.66	7.59	2.72	7.59	2.79	7.59	2.92	7.59	3.09	10.73										
	3.28	5.37	3.20	5.37	3.43	6.79	3.28	6.79	3.41	6.79	3.51	7.20	3.98	7.59	3.91	7.59	4.82	10.73										
	3.17	5.37	3.17	5.37	2.71	6.79	2.76	6.79	2.85	6.79	2.98	7.20	2.80	7.59	2.92	7.59	3.09	10.73										
p=14	2.46	8.65	2.46	8.65	2.46	8.65	2.48	8.65	2.52	8.65	2.57	7.59	2.64	7.59	2.72	7.59	2.85	7.59	3.03	10.73								
	3.54	5.37	3.20	5.37	3.40	5.37	3.22	5.37	3.19	5.37	3.19	5.37	3.57	7.20	3.91	7.59	3.98	7.59	4.74	10.73								
	2.87	5.37	2.87	5.37	2.87	5.37	2.91	5.37	2.93	5.37	2.99	5.37	2.83	7.20	2.72	7.59	2.85	7.59	3.03	10.73								
p=15	2.37	8.65	2.37	8.65	2.37	8.65	2.38	8.65	2.40	8.65	2.44	7.59	2.50	7.59	2.57	7.59	2.67	7.59	2.79	7.59	2.99	10.73						
	3.42	5.37	3.31	5.37	3.15	5.37	3.25	5.37	3.39	5.37	3.27	5.37	3.09	5.37	3.93	7.20	3.95	7.59	3.73	7.59	4.74	10.73						
	2.60	5.37	2.60	5.37	2.63	5.37	2.63	5.37	2.63	5.37	2.70	5.37	2.79	5.37	2.75	7.20	2.67	7.59	2.79	7.59	2.99	10.73						
p=16	2.27	8.65	2.27	8.65	2.27	8.65	2.28	8.65	2.30	8.65	2.33	7.59	2.38	7.59	2.44	7.59	2.52	7.59	2.62	7.59	2.75	7.59	2.95	10.73				
	3.17	5.37	3.42	5.37	3.38	5.37	3.21	5.37	3.22	5.37	3.10	5.37	3.19	5.37	3.04	5.37	3.78	7.20	3.18	7.59	3.70	7.59	4.70	10.73				
	2.40	5.37	2.40	5.37	2.40	5.37	2.41	5.37	2.45	5.37	2.53	5.37	2.58	5.37	2.66	5.37	2.68	7.20	2.62	7.59	2.75	7.59	2.95	10.73				
p=17	2.20	8.65	2.20	8.65	2.20	8.65	2.20	8.65	2.21	8.65	2.24	8.65	2.27	7.59	2.33	7.59	2.39	7.59	2.47	7.59	2.58	7.59	2.71	7.59	2.93	10.73		
	3.25	5.37	3.02	5.37	2.97	5.37	3.14	5.37	3.11	5.37	3.02	5.37	3.34	5.37	3.04	5.37	3.03	5.37	3.73	7.20	3.38	7.59	3.75	7.59	4.90	10.73		
	2.29	5.37	2.29	5.37	2.29	5.37	2.30	5.37	2.31	5.37	2.37	5.37	2.44	5.37	2.49	5.37	2.57	5.37	2.62	7.20	2.58	7.59	2.71	7.59	2.93	10.73		
p=18	2.12	8.65	2.12	8.65	2.12	8.65	2.13	8.65	2.14	8.65	2.16	8.65	2.19	8.65	2.21	7.59	2.27	7.59	2.34	7.59	2.43	7.59	2.55	7.59	2.69	7.59	2.92	10.73
	3.21	5.37	3.03	5.37	3.09	5.37	3.08	5.37	2.72	5.37	3.10	5.37	3.18	5.37	3.14	5.37	3.03	5.37	3.03	5.37	3.47	7.20	3.42	7.59	3.66	7.59	2.98	10.73
	2.23	5.37	2.23	5.37	2.23	5.37	2.23	5.37	2.23	5.37	2.23	5.37	2.30	5.37	2.36	5.37	2.42	5.37	2.50	5.37	2.56	7.20	2.55	7.59	2.69	7.59	2.92	10.73

Tabelle 2: Average and maximum travel time for all settings (p, q)

	number of stations											
solution	18	17	16	15	14	13	12	11	10	9	8	7
y_{148}	1											
y_{21}	1	1										
y_{28}	1	1	1									
y_{146}	1	1	1	1								
y_{125}	1	1	1	1	1							
y_{41}	1	1	1	1	1	1						
y_{26}	1	1	1	1	1	1	1					
y_{91}	1	1	1	1	1	1	1	1				
y_{100}	1	1	1	1	1	1	1	1	1			
y_{13}	1	1	1	1	1	1	1	1	1	1		
y_{155}	1	1	1	1	1	1	1	1	1	1	1	
y_{71}	1	1	1	1	1	1	1	1	1	1	1	1
y_{115}	1	1	1	1	1	1	1	1	1	1	1	1
y_{27}	1	1	1	1	1	1	1	1	1	1	1	1
y_{32}	1	1	1	1	1	1	1	1	1	1	1	1
y_{61}	1	1	1	1	1	1	1	1	1	1	1	1
y_{110}	1	1	1	1	1	1	1	1	1	1	1	1
y_{145}	1	1	1	1	1	1	1	1	1	1	1	1
average travel time	2.92	2.93	2.95	2.99	3.03	3.09	3.16	3.23	3.38	3.52	3.72	3.93

Tabelle 3: Optimal solution $y^\star$ and average travel time $z_1(y^\star)$ according to the number of facilities

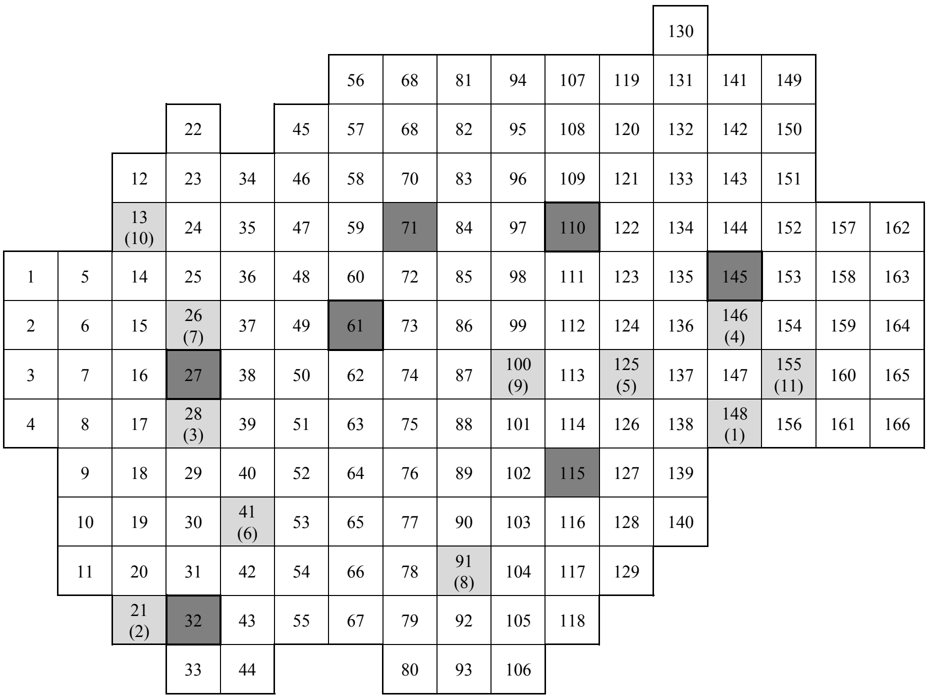


Abbildung 3: Light and dark gray sites represent the current situation, the dark gray facilities will be closed (12 instances of the problem have been solved).

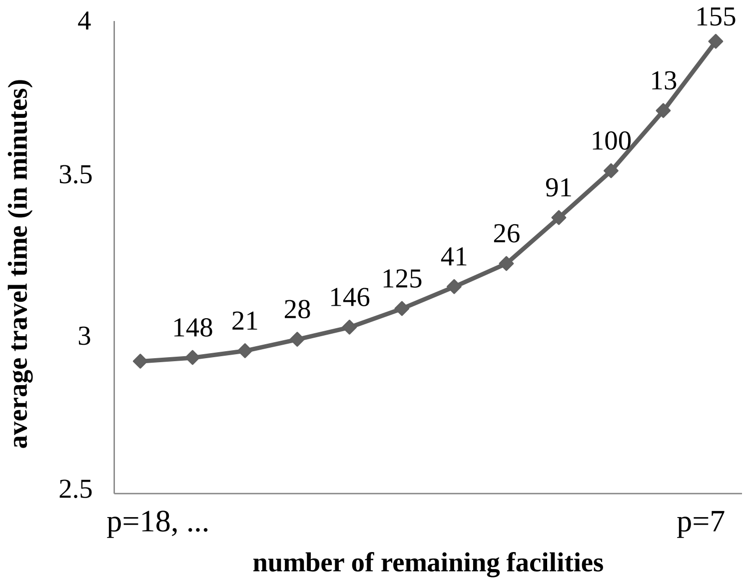


Abbildung 4: Average travel time depending on the number of fire stations (Numbers at nodes indicate closing sequence)

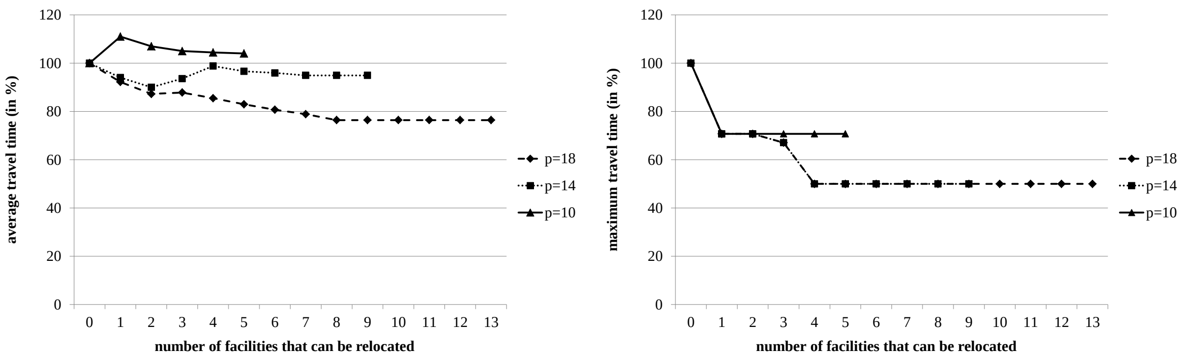


Abbildung 5: On the left: Percentage decreasing of the average travel time corresponding to the number of relocated facilities. On the right: Percentage decreasing of the maximum travel time corresponding to the number of relocated facilities

	total number of stations			
multiple coverage	$p = 18$	$p = 14$	$p = 10$	$p = 7$
1	166	166	166	166
2	163	163	150	122
3	151	139	102	74
4	139	114	71	29
5	116	83	41	8
6	91	57	14	
7	60	29	3	
8	42	14		
9	20	3		
10	7	1		
11	2			

Tabelle 4: Multiple coverage depending on the total number of stations

the values of the maximum travel time and the disproportional decrease in maximum travel time. A small increase in the average travel time corresponds to a broad improvement in the maximum travel time. For example for $p = 10$ and the possibility to relocate one facility causes an increase in the average travel time of $\approx 10\,\%$ and an improvement in the maximum travel time of $\approx 30\,\%$.

4.4 Multiple coverage aspects

In this section we investigate another very important aspect of volunteer fire departments. In case of large-scale disasters the professional fire departments request support of volunteer fire departments. In this case a large number of fire fighters and equipment is needed and a multiple coverage of the disaster region gets into the focus. We investigate how often different demand regions could be covered depending on the total number of fire stations ($p = 18, 14, 10, 7$). The following table 4 indicates the number of demand sites that are covered by more than one station. In the current situation with 18 stations nearly each demand point is covered three times and can be reached from three different stations. In a setting with only 7 stations still nearly half of the demand points are covered three times. It is remarkable that with 7 stations a full coverage can be ensured while still 122 sites get a double coverage.

5 Conclusions

In this paper a multi-criteria optimization model to decide on locations and relocations of volunteer fire stations is presented and analyzed. The optimization model is a mixed-integer linear program which provides an optimal solution for each instance (p, q) within a reasonable time by using Xpress for real size problems. As a case study the multi-criteria model is applied to the VFD of Bochum. The model and the analysis help the planners of the fire department to evaluate different strategic decisions and show the consequences of the alternatives. The two main interests (average travel time as the interest of the fire department and maximum travel time as the interest of the population) are considered explicitly in terms of objective functions (a priori multi-criteria approach). Cost reduction is considered implicitly by varying the parameters (p, q) (a posteriori approach). We showed that the current situation guarantees full multiple-coverage and an average travel time of only 2.92 minutes. Two general statements about the model can be made. (1) In general a broad reduction of facilities is possible without loosing the total coverage, but clearly results in increasing the average travel time and decreasing the multiple-coverage. Closing only some facilities saves operating costs and could also ensure a high (first-coverage) service level. (2) The relocation of a few stations improves the performance of the system considerably. The average travel time as well as the maximum travel time can be clearly improved by replacing only some facilities. Particularly decisions regarding the location of VFD are influenced by many additional qualitative factors including social, regulatory and political considerations that are beyond the scope of this research. A high availability of well qualified volunteer fire fighters improves the security of the population considerably.

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