# Analysis of Material Instabilities of Inelastic Solids based on Incremental Variational and Relaxation Methods

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We propose a new approach to the treatment of material instabilities and shear band localizations in strain–softening elastic–plastic solids based on energy minimization principles associated with micro–structure developments. The point of departure is a general internal variable formulation that determines the inelastic response as a generalized standard medium. Consistent with this type of inelasticity we develop an incremental variational formulation of the local constitutive response where a quasi–hyperelastic stress potential is obtained from a local constitutive minimization problem with respect to the internal variables [2]. The existence of this variational formulation allows to define the material stability of an inelastic solid based on weak convexity conditions of the incremental stress potential in analogy to the treatments of finite elasticity. Furthermore, localization phenomena are interpreted as micro–structure developments on multiple scales related to non–convex incremental stress potentials in analogy to elastic phase decomposition problems. These micro–structures can be resolved by the relaxation of non–convex energy functions based on a convexification of the stress potential [1], [4]. The relaxed problem provides a well–posed formulation for a mesh–objective analysis of localizations as close as possible to the non–convex problem. We develop based on an approximated rank–one–convexification of the incremental stress potential a computational two–scale procedure for a mesh–objective treatment of localization problems. It constitutes a local minimization problem for a relaxed incremental stress potential with just one scalar variable representing the intensity of the micro–shearing of a rank–one laminate aligned to the shear band [3]. This problem is sufficiently robust with regard to applications to large–scale inhomogeneous deformation processes of elastic–plastic solids. The performance of the proposed energy relaxation method is demonstrated for a representative set of numerical simulations of straight and curved shear bands which report on the mesh–independence of the result.

## REFERENCES

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