Continuous symmetries of dissipative constitutive laws: Master curves and Noether's analysis

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An analysis of the continuous symmetries of the constitutive laws of inelastic materials written within a thermodynamical framework of relaxation is performed. This framework relies on the generalization of Gibb's relationship outside the equilibrium of a uniform system, and the use of the fluctuation theory to model the material dissipation due to its internal reorganization. The constitutive laws split into the state laws relating intensive variables (thermodynamics forces) to extensive-like variables, and the complementary evolution laws of the internal variables associated to the dissipative mechanisms. An interpretation of a non-equilibrium thermodynamic approach of irreversible processes in terms of an extremum principle is proposed, associated to a Lagrangian functional. It is shown that one possible choice for the Lagrangian kernel is the material derivative of the internal energy density, augmented by a complementary term that accounts for the evolution laws of the internal variables. Interpreting the material behavior during the non equilibrium evolution as the Euler-Lagrange equations of the resulting action integral, a differential condition expressing both the local and variational symmetries encapsulated into the Lagrangian formulation is formulated. The predictive nature of the symmetry analysis is highlighted, as a systematic tool for the exploitation of the constitutive response. Its performance and utility are exemplified by the construction of a time-temperature equivalence principle for a dry viscous polymer (PA66); the calculated shift factor is shown to well agree with the empirical shift factor given by WLF expression.
Adopting the point of view of differential geometry, an interpretation of the least action principle in terms of the invariance of the action integral under suitable Lie groups then leads to the elaboration of conservation laws involving appropriate Eshelby tensors. On the basis of this Noetherian analysis, new possible directions for Configurational Mechanics of dissipative media are envisaged.