Abstract

In this work we prove Anderson localisation for a randomly perturbed, onedimensional, periodic Schrödinger operator

$$H^\omega := -\Delta + V_0 + V^\omega$$

The potential V_0 is periodic and the random perturbation is of alloy type:

$$V^\omega := \sum_{i\in\mathbb{Z}} t(\omega, i)\chi(\cdot - i) ,$$

with χ continuous, nonnegative and of compact support. Here $t(\omega, i)$, $i \in \mathbb{Z}$ are independent, identically distributed random variables on a probability space $(\Omega, \mathcal{A}, \mathbb{P})$ with common distribution μ. The measure μ has a bounded density g of compact support.

The first part of this work contains a multiscale analysis very much like the one for the discretised Schrödinger operator in the paper of von Dreifus and Klein, but adapted to the continuous case. This method is an induction argument over increasing length scales Λ_k. On each scale one considers the differential operator H^ω restricted to $L^2(\Lambda_k)$ with Dirichlet boundary conditions and ensures exponential decay of the corresponding Green's function with good probability. Here the qualification 'good' is understood in terms of the length scale we are dealing with.

To start the induction we use the so-called Lifschitz Tails of the integrated density of states $N(E)$ of H^ω. As Mezincescu proved, they appear at the band edges of the spectrum of the periodic operator $-\Delta+V_0$. Assuming that V^ω is sufficiently small, we use a transformation introduced by Combes and Thomas and thus obtain the exponential decay of the Green's function on the initial length scale Λ_0.

Afterwards we prove a Wegner-type estimate, which is needed for the induction step of the multiscale analysis.

On the one hand, the multiscale analysis is in itself of interest, since it is a tool applicable to the case of multidimensional Schrödinger operators. In that setting there are much more open problems than in the onedimensional case. On the other hand, we achieve results which are not contained in previous works, e.g. in the one by Barbaroux, Combes and Hislop. This article comes closest to the problem we consider. For example, we are able to treat a potential V^ω with random coupling constants $t(\omega, i)$ of uniform density, i.e. g with constant, a case not covered in the last mentioned paper.

For the references look at the end of the work.