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ACTIVE CONTROL OF SMART STRUCTURES – AN OVERALL APPROACH

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Abstract. *The paper presents active control of smart structures within a focused frame of piezoelectric applications in active vibration and noise attenuation with potentials for the use in mechanical and civil engineering. An overall approach to active control of piezoelectric structures involves subsequent steps of modeling, control, simulation, experimental verification and implementation. Each of these steps is regarded in details. Different application examples showing the feasibility of the active structural control will be presented.*

Key words: *smart structures, modeling, control, simulation*

1. INTRODUCTION

Active structures have been intensively investigated in the recent years. Due to their complexity, demands on performances and realisation forms this topic represents an important interdisciplinary field of research with great application potentials. Smart structures and systems in general comprise integrity (structural and functional) of a structure or a system, multifunctional materials, actuators/sensors and appropriate control in order to achieve desired performances under varying environmental conditions. The interdisciplinary field encompasses material science, applied mechanics (vibration, acoustics, fracture mechanics, elasticity), electronics (actuators, sensors, control), manufacturing, biomechanics etc.

In this paper an overall approach to active control of piezoelectric structures, which involves subsequent steps of modeling, control, simulation, experimental verification and implementation, will be presented and documented by illustrative examples.

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2. Finite Element Based Modeling of Smart Structures

The finite element (FE) based modeling of piezoelectric smart structures represents a good basis for the overall simulation and design procedure. The FE analysis is based on the finite element semi-discrete form of the equations of motion of a piezoelectric smart system describing its electro-mechanical behavior. The set of electro-mechanical equations can be augmented by appropriate equations describing the acoustic fluid and in that way the modeling can be performed for electro-mechanical vibro-acoustic problems. These equations can be derived using the established approximation methods for displacements, electric potential and velocity potential of an acoustic fluid as degrees of freedom and the standard finite element procedure [3]. In this way the finite element semi-discrete form of equations of motion is obtained:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}_d\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \overline{\mathbf{E}}\mathbf{f}(t) + \overline{\mathbf{B}}\mathbf{u}(t) \tag{1}$$

where vector $\mathbf{q}$ represents the vector of generalized displacements including mechanical displacements, electric potential and velocity potential of an acoustic fluid and contains all considered degrees of freedom. Matrices $\mathbf{M}$, $\mathbf{D}_d$ and $\mathbf{K}$ are the mass matrix, the damping matrix and the stiffness matrix, respectively. The total load vector is divided into the vector of the external forces $\mathbf{F}_E = \overline{\mathbf{E}}\mathbf{f}(t)$ and the vector of the control forces $\mathbf{F}_C = \overline{\mathbf{B}}\mathbf{u}(t)$, where the forces are generalized quantities including also electric charges. Vector $\mathbf{f}(t)$ represents the vector of external disturbances, and $\mathbf{u}(t)$ is the vector of the controller influence on the structure. Matrices $\overline{\mathbf{E}}$ and $\overline{\mathbf{B}}$ describe the positions of the forces and the control parameters in the finite element structure, respectively.

The tools for modal reduction are also included in the FE modeling procedure, which enable development of appropriate models with reduced orders for the controller design. Based on the modal truncation, which was adopted as a suitable technique for the reduction of the number of equations in the FE models, a state space model of an actively controlled structure can be obtained in the form convenient for the controller design. A limited number of eigenmodes of interest is taken into account, while the remaining modes are truncated. Introducing the modal coordinates $\mathbf{z}$

$$\mathbf{q}(t) = \mathbf{\Phi}_{\mathrm{m}}\mathbf{z}(t) \tag{2}$$

into equation (1), where $\mathbf{\Phi}_{\mathrm{m}}$ represents the modal matrix, and applying the ortho-normalization with $\mathbf{\Phi}_{\mathrm{m}}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_{\mathrm{m}} = \mathbf{I}$, $\mathbf{\Phi}_{\mathrm{m}}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}_{\mathrm{m}} = \mathbf{\Omega}$, $\mathbf{\Delta} = \mathbf{\Phi}_{\mathrm{m}}^{\mathrm{T}}\mathbf{D}_{\mathrm{d}}\mathbf{\Phi}_{\mathrm{m}}$, where $\mathbf{\Omega}$ represents the spectral matrix and $\mathbf{\Delta}$ the modal damping matrix, the state space model of the modally reduced system can be obtained in the form:

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega} & -\mathbf{\Delta} \end{bmatrix}\mathbf{x} + \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}^{\mathrm{T}}\overline{\mathbf{B}} \end{bmatrix}\mathbf{u}(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}^{\mathrm{T}}\overline{\mathbf{E}} \end{bmatrix}\mathbf{f}(t) \tag{3}$$

where $\mathbf{x}(t) = \begin{bmatrix} \mathbf{z} & \dot{\mathbf{z}} \end{bmatrix}^{\mathrm{T}}$ represents a state-space vector. With the state and the output equations, the state space model is represented in the form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}\mathbf{f}(t)\,,\ \ \mathbf{y} = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) + \mathbf{F}\mathbf{f}(t) \tag{4}$$

which is convenient for the controller design.

As an alternative method for modeling identification techniques can also be used, if a real structure or a prototype is available. The method suggested by the authors for this purposes is the subspace identification method (for more details see e.g. [4], [5]), which also results in a state space model representation and offers thus a good basis for the model comparison, verification and improvement.

3. Optimal Controller with Additional Dynamics

An optimal LQ controller design with additional dynamics results in a successful vibration and noise reduction [6], [8].

Controller design includes available a priori knowledge about occurring disturbance type contained in the additional dynamics. Such an a priori knowledge is available in terms of the type of the disturbance function which has to be rejected or whose influence should be suppressed by the controller. Periodic disturbances with frequencies corresponding to the eigenfrequencies of the smart structure can cause resonance and their suppression is therefore important. They are taken into account via the additional dynamics.

Discrete-time state space equivalent (5) of the state space model (4) developed through the FE procedure and modal reduction is used for the controller design.

$$\mathbf{x}[k+1] = \mathbf{\Phi}\mathbf{x}[k] + \mathbf{\Gamma}\mathbf{u}[k] + \boldsymbol{\varepsilon}\mathbf{w}[k], \qquad \mathbf{y}[k] = \mathbf{C}\mathbf{x}[k] + \mathbf{D}\mathbf{u}[k] + \mathbf{F}\mathbf{w}[k] \tag{5}$$

Using the a priori knowledge about the disturbance class, which has to be suppressed, the model of the disturbance is represented in an appropriate state space form, where the disturbance is assumed to be the output of the state space representation. The poles λ_i of the disturbance transfer function are used to define the additional dynamics using the coefficients of the polynomial:

$$\delta(z) = \prod_i (z - \mathrm{e}^{\lambda_i T})^{m_i} = z^s + \delta_1 z^{s-1} + \ldots + \delta_s \tag{6}$$

where m_i represents the multiplicity of the pole λ_i. Additional dynamics is expressed in the following state space form:

$$\mathbf{x}_a[k+1] = \mathbf{\Phi}_a \mathbf{x}_a[k] + \mathbf{\Gamma}_a \mathbf{e}[k]; \tag{7}$$

where $\mathbf{x}_a$ represents the vector of the state variables for the additional dynamics, $\mathbf{e}$ is the error signal and:

$$\mathbf{\Phi}_a = \begin{bmatrix} -\delta_1 & 1 & 0 & \cdots & 0 \\ -\delta_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\delta_{s-1} & 0 & 0 & \cdots & 1 \\ -\delta_s & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \mathbf{\Gamma}_a = \begin{bmatrix} -\delta_1 \\ -\delta_2 \\ \vdots \\ -\delta_{s-1} \\ -\delta_s \end{bmatrix}. \tag{8}$$

For multiple-input multiple-output (MIMO) systems additional dynamics is replicated q times (once per each output). Replicated additional dynamics is described by:

$$\overline{\mathbf{\Phi}} \overset{def}{=} diag(\underbrace{\mathbf{\Phi}_a,...,\mathbf{\Phi}_a}_{q\,\text{times}}), \quad \overline{\mathbf{\Gamma}} \overset{def}{=} diag(\underbrace{\mathbf{\Gamma}_a,...,\mathbf{\Gamma}_a}_{q\,\text{times}}) \tag{9}$$

The discrete-time design model ($\mathbf{\Phi}_d$, $\mathbf{\Gamma}_d$) is formed as a cascade combination of the additional dynamics ($\mathbf{\Phi}_a$, $\mathbf{\Gamma}_a$) or $(\overline{\mathbf{\Phi}},\overline{\mathbf{\Gamma}})$ and the discrete-time plant model ($\mathbf{\Phi}$, $\mathbf{\Gamma}$):

$$\mathbf{x}_d[k+1] = \mathbf{\Phi}_d \mathbf{x}_d[k] + \mathbf{\Gamma}_d \mathbf{u}[k]\,; \tag{10}$$

$$\mathbf{\Phi}_d = \begin{bmatrix} \mathbf{\Phi} & 0 \\ \mathbf{\Gamma}^*\mathbf{C} & \mathbf{\Phi}^* \end{bmatrix}, \mathbf{\Gamma}_d = \begin{bmatrix} \mathbf{\Gamma} \\ \mathbf{0} \end{bmatrix}, \ \mathbf{x}_d = \begin{bmatrix} \mathbf{x}[k] \\ \mathbf{x}_a[k] \end{bmatrix} \tag{11}$$

where $\mathbf{\Phi}^*$ and $\mathbf{\Gamma}^*$ denote respectively $\mathbf{\Phi}_a$ and $\mathbf{\Gamma}_a$ in the case of single-input single-output systems or $\overline{\mathbf{\Phi}}$ and $\overline{\mathbf{\Gamma}}$ for MIMO systems. For the design model (11) the feedback gain matrix **L** of the optimal LQ regulator is calculated in such a way that the feedback control law $\mathbf{u}[k] = -\mathbf{L}x_d[k]$ minimizes the performance index (12) subject to the constraint (11), where **Q** and **R** are symmetric, positive-definite matrices.

$$J = \frac{1}{2}\sum_{k=0}^{\infty}(\mathbf{x}_d[k]^T \mathbf{Q}\mathbf{x}_d[k] + \mathbf{u}[k]^T \mathbf{R}\mathbf{u}[k]) \tag{12}$$

The feedback gain matrix **L** is then partitioned into

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_1 & \mathbf{L}_2 \end{bmatrix} \tag{13}$$

so that $\mathbf{L}_1$ corresponds to the state space model of the structure, and $\mathbf{L}_2$ to the modeled additional dynamics.

4. Model Reference Adaptive Control

Direct adaptive control technique, model reference adaptive control (MRAC) is an alternative method for the controller design for vibration suppression and noise attenuation of smart structures. This control technique comprises several advantages for large flexible structures. In the case of piezoelectric smart structures the term large can regard high number of degrees of freedom of the finite element model used for the modeling of the structure behavior. With the model truncation, which is a necessary step to adapt the structure model to the controller design purpose, resulting state space model does not exactly reflect the real behavior of the structure. Inaccuracies introduced in this way can be viewed as a source of the parameter variation with respect to the modeled case. The presence of disturbances in real environment also introduces variation of the parameters in comparison with the modeled case. This causes the need for the adaptive control algorithm, which can successfully face the insufficient prior knowledge or the unknown changes of the system parameters.

The advantage of a robust adaptive controller over a fixed-gain controller can be viewed through the fact that in design of large flexible smart structures a large degree of the model uncertainty is allowable in the sense of the possible parameter variation as well as with respect to the order of the controlled structure. The robustness assumes stability in the presence of disturbances and unmodeled dynamics.

The idea of the model reference adaptive control is based upon the existence of the reference model, specified by the designer, which reflects the desired behavior of the controlled structure. The output of the controlled structure should track the output of the reference model (Fig. 1).

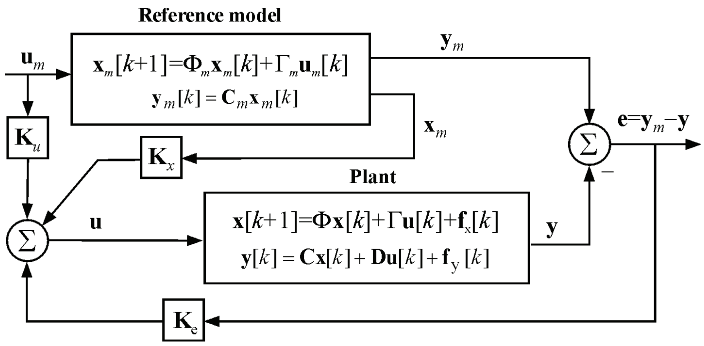

Fig. 1 General form of a discrete-time MRAC

A general form of a discrete-time model reference adaptive system is represented in Fig. 1. Plant representation is a discrete-time state space realization, which corresponds to the plant model (5), whereas the reference model is represented by equations in Fig. 1, where $\mathbf{f}_x$ and $\mathbf{f}_y$ represent bounded unmeasurable plant and output disturbances in a general case and $\mathbf{e}=\mathbf{e}_y$ is the output error, i.e. the difference between the desired output of the reference model and the real plant output.

Discrete-time direct model reference adaptive control law is expressed in the following form:

$$\mathbf{u}[k]=\mathbf{K}_r[k]\mathbf{r}[k]=\mathbf{K}_e[k]\mathbf{e}_y[k]+\mathbf{K}_x[k]\mathbf{x}_m[k]+\mathbf{K}_u[k]\mathbf{u}_m[k]\,. \tag{14}$$

The adaptive gain $\mathbf{K}_r[k]$ in is determined as a sum of proportional $\mathbf{K}_p$ and integral part $\mathbf{K}_I$:

$$\mathbf{K}_r[k]=\mathbf{K}_p[k]+\mathbf{K}_I[k] \tag{15}$$

According to the basic model reference adaptive algorithm the proportional and integral gains are adapted in the following way:

$$\mathbf{K}_p[k]=\mathbf{e}_y\mathbf{r}^{\mathrm{T}}(t)\overline{\mathbf{T}},\quad \mathbf{K}_I[k+1]=\mathbf{e}_y\mathbf{r}^{\mathrm{T}}[k]\mathbf{T},\quad \mathbf{K}_I(0)=\mathbf{K}_{I0} \tag{16}$$

where $\mathbf{T}$ and $\overline{\mathbf{T}}$ are $n_r \times n_r$ time-invariant weighting matrices and $\mathbf{K}_{I0}$ is the initial integral gain, selected by the designer. In the robust model reference adaptive control approach the integral gain is determined in the form (17). This modification of the integral gain in (16) by adding a σ-term is introduced to provide the convergence of the integral gain [6] since in realistic environment due to disturbances the error **e** does not reach the zero value and the integral gain would thus never stop increasing without its limiting by the σ-term.

$$\mathbf{K}_I[k+1] = \mathbf{e}_y[k]\mathbf{r}^T[k]\mathbf{T} - \sigma\mathbf{K}_I[k] \tag{17}$$

The control law for a general plant in Fig. 1 including disturbances or excitations is globally stable with respect to boundedness if the disturbances are bounded and the plant is almost strictly positive real. The proof of the condition is based on the selection of the Lyapunov candidate positive definite function and on analyzing the sign of its derivative. In order to guarantee for the robust stability, perfect tracking is not obtained in general, but the adaptive controller maintains a small tracking error over large ranges of non-ideal conditions and uncertainties.

5. Application of Active Structural Control

The feasibility of the suggested overall approach to design and control of smart structures will be illustrated by experimental examples.

5.1 Vibration Control Of A Clamped Cantilever Beam

Investigated clamped beam, 300×30×2 [mm], is considered through an overall development procedure including modeling (numerical and experimental), controller design, simulation and experimental study. The beam is considered as an active structure controlled by four piezoelectric patch actuators attached to the beam, two on the top and two on the bottom of the beam (Fig. 2a). Numerical modeling is performed using the FE procedure with appropriate piezoelectric finite elements used to model the actuator behavior. Besides the numerical modal analysis with the standard FE software, the experimental modal analysis was performed in order to verify the results for the obtained eigenfrequencies. An experimental setup for the modal analysis with shaker an laser vibrometer is represented in Fig. 2b.

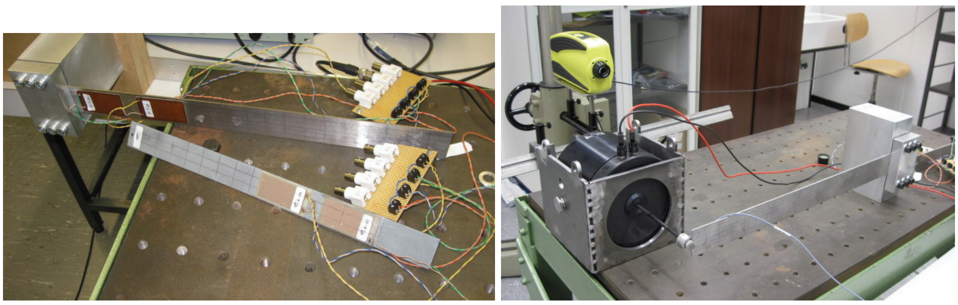

Fig. 2 a) Active cantilever beam with piezoelectric actuators;
b) experimental setup for the modal analysis with excitation shaker and laser vibrometer

Performing the modal analysis, the eigenforms and eigenfrequencies of interest for the controller design are determined. These bending modes are represented in Fig. 3. Corresponding bending eigenfrequencies of interest are: f_1=18.8Hz, f_2=113.1Hz, f_3=314.4Hz, f_6=619.2Hz (the eigenmodes 4 and 5 are torsion modes, and they are not relevant for the bending vibration suppression).

As a starting point for the controller design an appropriate state space model was developed as described in Section 2, based on the modal truncation of the finite element model of a much higher order, in such a way that the modally reduced state space model contains important information on the eigenmodes in the frequency range of interest. For the solution of the control task an optimal LQ controller is designed (based on the procedure in Section 3) in such a way that the vibration amplitudes due to periodic excitation forces with frequencies corresponding to the eigenfrequencies of the clamped beam, are significantly suppressed in comparison with uncontrolled case.

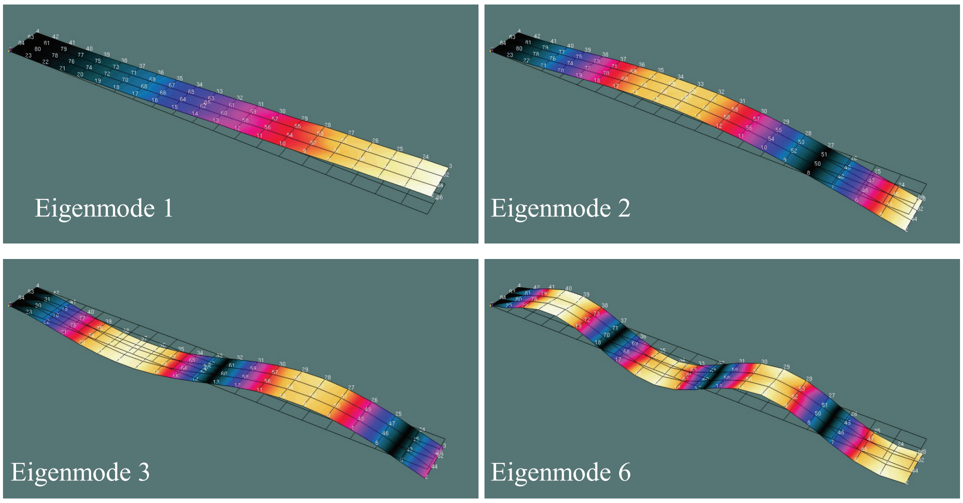

Fig. 3 Experimentally determined bending eingemodes of interest used for the controller design

Scheme of a discrete-time closed-loop control system for the vibration suppression of the clamped beam is represented in Fig. 4. Exciting forces F(*t*)=Asin(ω*t*) exerted on the corner points at the tip of the beam are chosen with regard to the eigenfrequencies ω of interest.

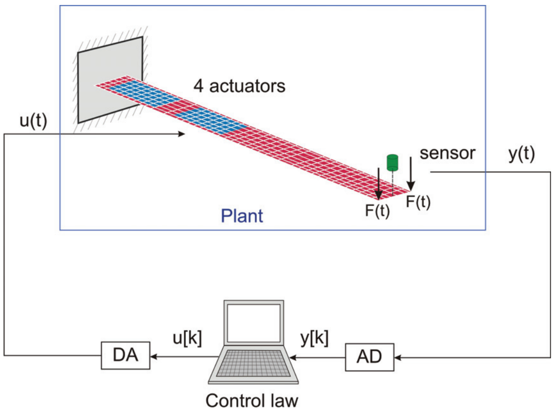

Fig. 4 Closed-loop system control system

Results of the controller design, with periodical excitation with the frequency corresponding to the first eigenfrequency of the beam, are represented in Fig. 5. Diagram 5*a*) represents the uncontrolled and controlled (after 0.5 s) output – displacement at the tip of the beam, while diagram 5*b*) represents the control signals (actuating voltages on piezo patches) without and with control. A significant reduction of the vibration magnitudes can be observed in the presence of the controller.

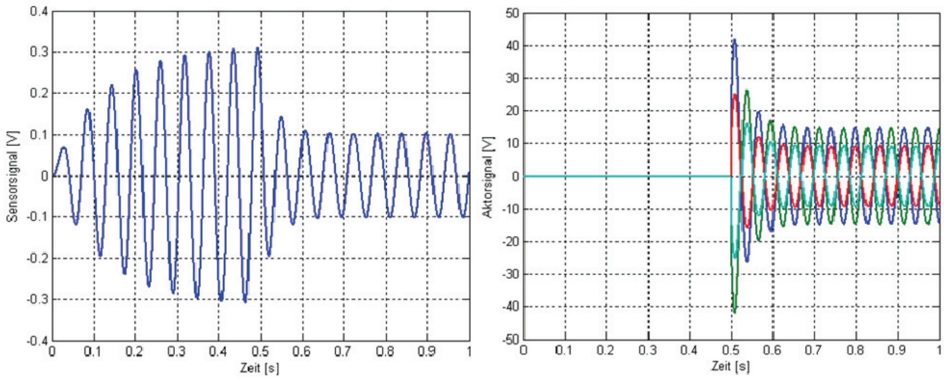

Fig. 5 a) Controlled output; b) actuating inputs – without and with control (after 0.5s)

5.2 Noise Control Of A Smart Acoustic Box

An actively controlled smart acoustic box consisting of the clamped plate with attached piezoelectirc patches used as actuators and of the wooden box surrounding the clamped plate is designed and investigated in order to reduce the plate vibrations and the air pressure at selected points inside the box (Fig. 6).

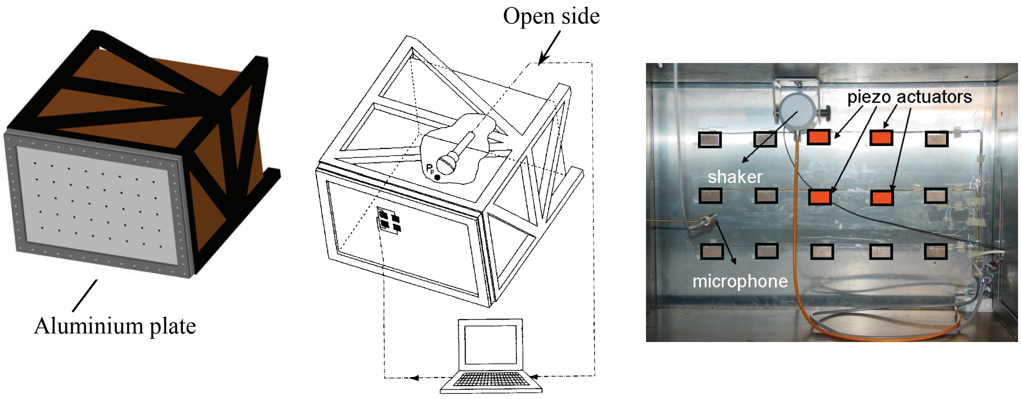

Fig. 6 Scheme of the acoustic box and photo of the clamped plate with piezo-actuators

The plate is excited by a shaker and the plate and the acoustic fluid vibrations are measured using the laser scanning vibrometer (for the velocity and displacement measurements at selected points on the plate surface) and the microphone (for the air pressure measurement) respectively. The piezopatches and the microphone are located inside the acoustic box.

The aluminium plate of the acoustic box and the acoustic fluid inside it are modeled using the FE approach, taking into consideration the acoustic behavior via the appropriate acoustic finite elements. Based on the modally reduced state space model obtained through the modal truncation (Section 2), the simulation and subsequently the experimental control of the plate vibration and of the fluid pressure were performed using the optimal LQ controller with additional dynamics (Section 3) and the model reference adaptive control (Section 4).

The optimal LQ controller was tested with different excitation signals: periodic as well as random. For periodic excitations the sinusoidal signals with frequencies corresponding to the eigenfrequencies of interest of the plate (f_{w1}=66.7Hz, f_{w2}=106.2Hz, f_{w2}=163.8Hz) and their combinations were used.

The results of the adaptive MRAC controller testing are shown in Fig. 7. The adaptive controller is compared with the optimal LQ controller in the presence of the periodic excitation with the frequency f_{w2}. Uncontrolled and controlled signals are represented in Fig. 7*a*), and a zoomed portion in Fig. 7*b*). Both controllers perform the air pressure reduction at the microphone point. In this case adaptive controller has performed slightly better.

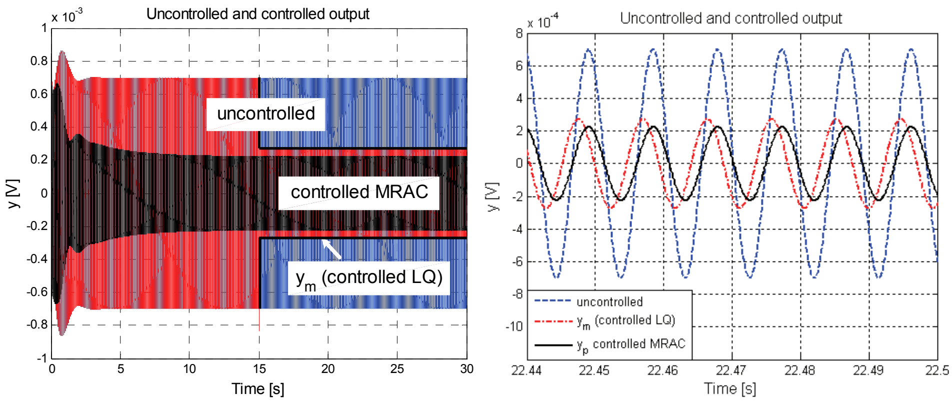

Fig. 7 a) Comparison of the MRAC and optimal LQ controller; b) zoomed portion

6 CONCLUSION

Design of actively controlled smart structures is addressed in this paper considering several phases in the overall design approach, with the focus on structural control of lightweight structures which use piezoelectric materials as active elements. Main development phases, such as modeling, controller design, simulation and testing are presented. The main control objective is vibration suppression and the noise attenuation. The feasibility of the approach is demonstrated by application examples.

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AKTIVNO UPRAVLJANJE KONSTRUKCIJAMA – SVEOBUHVATNI PRISTUP

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U radu se razmatraju aktivne konstrukcije i sistemi sa akcentom na piezoelektričnim strukturama za aktivnu redukciju vibracija i buke sa potencijalnim primenama u aktivnim mašinskim ili građevinskim konstrukcijama. Sveobuhvatni pristup aktivnom upravljanju strukturama sa integrisanim piezoelektričnim aktuatorima i senzorima, obuhvata sledeće značajne faze: modeliranje, upravljanje, simulaciju, eksperimentalnu verifikaciju i implementaciju. Svaka od ovih faza razmatra se detaljno. Realizacija aktivnog upravljanja konstrukcijama prikazana je na primerima

Ključne reči: *aktivne konstrukcije, modeliranje, upravljanje, verifikacija*