```
    begin
1.      for i ← 0 until n do
            begin
2.              w_ii ← q_i;
3.              c_ii ← 0
            end;
4.      for l ← 1 until n do
5.          for i ← 0 until n − l do
                begin
6.                  j ← i + l;
7.                  w_ij ← w_{i,j−1} + p_j + q_j;
8.                  let m be a value of k, i < k ≤ j, for which c_{i,k−1} + c_kj
                      is minimum;
9.                  c_ij ← w_ij + c_{i,m−1} + c_mj;
10.                 r_ij ← a_m
                end
    end
```

**Fig. 4.9.** Algorithm to compute roots of optimal subtrees.

```
procedure BUILDTREE(i, j):
begin
    create vertex v_ij, the root of T_ij;
    label v_ij by r_ij;
    let m be the subscript of r_ij (i.e., r_ij = a_m);
    if i < m − 1 then make BUILDTREE(i, m − 1) the left subtree of v_ij;
    if m < j then make BUILDTREE(m, j) the right subtree of v_ij
end
```

**Fig. 4.10.** Procedure to construct optimal binary search tree.

**Example 4.5.** Consider the four elements $a_1 < a_2 < a_3 < a_4$ with $q_0 = \frac{1}{8}$, $q_1 = \frac{3}{16}$, $q_2 = q_3 = q_4 = \frac{1}{16}$, and $p_1 = \frac{1}{4}$, $p_2 = \frac{1}{8}$, $p_3 = p_4 = \frac{1}{16}$. Figure 4.11 shows the values of $w_{ij}$, $r_{ij}$, and $c_{ij}$ computed by the algorithm given in Fig. 4.9. For notational convenience, the values of $w_{ij}$ and $c_{ij}$ in this table have all been multiplied by 16.

| $l = j - i$ ↓ | $i \rightarrow$ 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | $w_{00} = 2$<br>$c_{00} = 0$ | $w_{11} = 3$<br>$c_{11} = 0$ | $w_{22} = 1$<br>$c_{22} = 0$ | $w_{33} = 1$<br>$c_{33} = 0$ | $w_{44} = 1$<br>$c_{44} = 0$ |
| 1 | $w_{01} = 9$<br>$c_{01} = 9$<br>$r_{01} = a_1$ | $w_{12} = 6$<br>$c_{12} = 6$<br>$r_{12} = a_2$ | $w_{23} = 3$<br>$c_{23} = 3$<br>$r_{23} = a_3$ | $w_{34} = 3$<br>$c_{34} = 3$<br>$r_{34} = a_4$ | |
| 2 | $w_{02} = 12$<br>$c_{02} = 18$<br>$r_{02} = a_1$ | $w_{13} = 8$<br>$c_{13} = 11$<br>$r_{13} = a_2$ | $w_{24} = 5$<br>$c_{24} = 8$<br>$r_{24} = a_4$ | | |
| 3 | $w_{03} = 14$<br>$c_{03} = 25$<br>$r_{03} = a_1$ | $w_{14} = 10$<br>$c_{14} = 18$<br>$r_{14} = a_2$ | | | |
| 4 | $w_{04} = 16$<br>$c_{04} = 33$<br>$r_{04} = a_2$ | | | | |

**Fig. 4.11.** Values of $w_{ij}$, $c_{ij}$, and $r_{ij}$.