

Homework for  
**Komplexitätstheorie**  
A. Y. 16/17  
Assignment 6

**Exercise 6.1**

In the lecture, we introduced the MST-based PTAA  $A$  for TSP with a performance ratio of at most 2. Show that there are instances for which this bound is actually tight.

**Exercise 6.2**

BIN PACKING as an optimization problem asks for packing items of sizes  $s_1, \dots, s_n$  into as few as possible bins, taken from a possibly infinite sequence  $B_1, B_2, \dots$ , where each bin  $B_j$  has a capacity of  $C$  and is empty initially. There is a PTAA named FIRST FIT (FF) for BP which works as follows: for  $i = 1, \dots, n$ , put item  $i$  into the lowest-indexed bin in which it fits (without exceeding the capacity). Show that the performance ratio of FF is at most 2.

**Exercise 6.3**

In the lecture, Sahni's PTA-scheme  $(A_k)_{k \geq 0}$  for the KNAPSACK problem has been presented. Run  $A_0$  and  $A_1$  on the following instance of KNAPSACK and determine the packings they deliver.

- $U = \{u_1, u_2, u_3, u_4, u_5\}$  is the set of items.
- $W = 19$  is the knapsack capacity.
- The weight function  $w: U \rightarrow \mathbb{Z}^+$  is defined as follows:  $w(u_1) = 4$ ,  $w(u_2) = 11$ ,  $w(u_3) = 8$ ,  $w(u_4) = 12$ ,  $w(u_5) = 10$ .
- The profit function  $p: U \rightarrow \mathbb{Z}^+$  is defined as follows:  $p(u_1) = 5$ ,  $p(u_2) = 24$ ,  $p(u_3) = 18$ ,  $p(u_4) = 30$ ,  $p(u_5) = 16$ .

**Exercise 6.4**

Show that Sahni's algorithm  $A_0$  for KNAPSACK has performance ratio  $\infty$ .