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Deadline on November 24^{th} 2016

Homework for

Komplexitätstheorie

A. Y. 16/17

Assignment 5

Wichtige Anzeige: Die Vorlesung am Donnerstag, den 24.11.16, wird ausnahmsweise im Raum 02/99 stattfinden. Die Übung wird stattdessen im Raum 2/24 stattfinden.

Exercise 5.1

Prove Lemma 6.1 of the lecture notes.

Exercise 5.2

Consider the polynomial reduction of 3–SAT to 3–COLORABILITY and prove the following properties of the clause-component (Skript, p. 35) and the crossover-gadget (Handzettel, p. 4):

a) If A, B, C have the color 0 then Z must have the color 0, too.

b) If one of the nodes A, B, C has the color 1 then Z can also be colored with 1.

c) For every $i, j \in \{0, 1, 2\}$ there exists a feasible 3-coloring f such that $f(x) = f(x') = i$ and $f(y) = f(y') = j$.

d) Any feasible 3-coloring f satisfies $f(x) = f(x')$ and $f(y) = f(y')$.

Exercise 5.3

Consider the following variant of PARTITION:

Instance: $a_1, \ldots, a_{2n} \in \mathbb{N}$ such that $\sum_{i=1}^{2n} a_i = S$

Question: Is there a subset $I \subseteq [2n]$ of size $|I| = n$ such that $\sum_{i \in I} a_i = S/2$?

Show that this variant of PARTITION can be solved in pseudo-polynomial time.

Exercise 5.4

In assignment 3, we saw PARTITION $\leq_{pol}$ RECTANGLE–PACKING. This only shows that RECTANGLE–PACKING is weakly NP-hard. Prove that this problem is actually strongly NP-hard.

Hint: You may show that 4–PARTITION is pseudo-polynomially Karp-reducible to RECTANGLE–PACKING.