Hans U. Simon
Francesco Aldà

Bochum, November 10^{th} 2016
Deadline on November 17^{th} 2016

Homework for

Komplexitätstheorie

A. Y. 16/17

Assignment 4

Exercise 4.1

In the lecture, we have shown that an efficient procedure for deciding TSP can be transformed into an efficient procedure for the corresponding optimization problem. This actually applies to any search problem $\mathcal{S}_R$ provided that R is polynomially verifiable and self-reducible and provided that the target function used in the optimization version of the problem can be evaluated in polynomial time. Sketch a proof of this claim.

Exercise 4.2

Consider the following NP-complete problems.
CLIQUE: Given a graph $G = (V, E)$ and a positive integer $k \leq |V|$, does G contain a clique of size k or more?
BP: Given a finite set U of items, a size $s(u) \in \mathbb{N}_0$ for each $u \in U$, a positive integer bin capacity B, and a positive integer k, is there a partition of U into disjoint sets $U_1, \ldots, U_k$ such that, for every $1 \leq i \leq k$, $\sum_{u \in U_i} s(u) \leq B$?

a) Define the corresponding search problems in terms of the relations R_{CLIQUE} and R_{BP}.

b) Show that R_{CLIQUE} and R_{BP} are self-reducible (without making use of Theorem 4.2).

Exercise 4.3

Let INDEPENDENT–SET (IS) be the following problem. Given a graph $G = (V, E)$, and a positive integer $k \leq |V|$, does G contain a subset $V' \subseteq V$ such that $|V'| \geq k$ and no two vertices in V' are joined by an edge in E?

Show $R_{3\text{-SAT}} \leq_L R_{\text{IS}}$. *Hint:* It might be helpful to map clauses to triangles and add some cleverly chosen edges.

Exercise 4.4

Consider the following NP-complete problem.
NM: Given disjoints sets X and Y, each containing m elements, a size $s(a) \in \mathbb{N}_0$ for each element $a \in X \cup Y$, and a target vector $(B_1, \ldots, B_m)$ of positive integers, can $X \cup Y$ be partitioned into m disjoints sets $A_1, \ldots, A_m$, each containing exactly one element from each of X and Y, such that, for $1 \leq i \leq m$, $\sum_{a \in A_i} s(a) = B_i$?

Show that NM is solvable in polynomial time if $B_1 = B_2 = \ldots = B_m$.