Hans U. Simon
Francesco Aldà

Bochum, November 3rd 2016
Deadline on November 10th 2016

Homework for

Komplexitätstheorie

A. Y. 16/17

Assignment 3

Exercise 3.1

Prove the claims in Remark 2.8 of the lecture notes.

Exercise 3.2

Consider the following CNF-formulas.

$$\varphi_1(x_1, x_2, x_3) = \overline{x_1} \wedge (x_1 \vee x_2 \vee x_3) \wedge (\overline{x_2} \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3})$$
$$\varphi_2(x_1, x_2, x_3) = \overline{x_1} \wedge (x_1 \vee x_2) \wedge (x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3})$$
$$\varphi_3(x_1, x_2, x_3) = (x_1 \vee x_2 \vee \overline{x_3}) \wedge (x_1 \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$$

For each, specify a satisfying assignment or prove its unsatisfiability.

Exercise 3.3

Let RECTANGLE–PACKING be the following problem: Given a set of rectangles $R_1, \ldots, R_n$ and a target rectangle R of area $A(R) = \sum_{i=1}^{n} A(R_i)$, can $R_1, \ldots, R_n$ be packed without any overlap into R?

Show PARTITION $\leq_{pol}$ RECTANGLE–PACKING.

Exercise 3.4

Consider the SUBSET–SUM problem.

Decision problem (EP). Given a set of natural numbers $A = \{a_1, \ldots, a_n\}$ and a target value $T \in \mathbb{N}$, is there $M \subseteq A$ such that $\sum_{a \in M} a = T$?

Search problem (KP). Given a set of natural numbers $A = \{a_1, \ldots, a_n\}$ and a target value $T \in \mathbb{N}$, find, if there is any, $M \subseteq A$ such that $\sum_{a \in M} a = T$.

Show that KP $\to$ EP holds for SUBSET–SUM, i.e., the search problem can be polynomially reduced to the corresponding decision problem.