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Homeworks for

Komplexitätstheorie

A. Y. 13/14

Sheet 11

Hinweis: Am Mittwoch, den 12.02., und am Mittwoch, den 02.04., werden Termine für die mündliche Prüfung angeboten. Bitte vor der Anmeldung zur Prüfung sich eine Uhrzeit von Frau Weissmann (Sekretariat in NA 1/72) geben lassen.

Exercise 11.1 For every $d \in \mathbb{N}_{>0}$, a language L is in NC^d if L can be decided by a family of logspace-uniform circuits $\{C_n\}$, where C_n has poly(n) size and depth[1] $O(log^d(n))$. Show that $\mathrm{NC}^1 \subseteq \mathcal{L}$.

Exercise 11.2 Show that there is a PPTM that, given $N \in \mathbb{N}$, achieves the following:

a) It returns as output an element of the set $\{1, \ldots, N\} \cup \{?\}$.

b) The question mark is returned with probability at most 1/2.

c) Conditioned to the event of not returning "?", the output is uniformly distributed in $\{1, \ldots, N\}$.

Note that the PPTM must be poly($\log N$) time-bounded.

Exercise 11.3 Show that a coin with $\Pr[\text{Head}] = \rho$ can be simulated by a PPTM in expected time $O(1)$ provided that the i-th bit of ρ is computable in poly(i) time.

Exercise 11.4 Describe a real number $0 < \rho < 1$ such that a Turing machine equipped with a coin that throws "Head" with probability ρ can decide an undecidable language in polynomial time.

[1] The depth of a circuit is the length of the longest path from an input node to the output node.