

Homeworks for  
**Komplexitätstheorie**  
A. Y. 13/14  
Sheet 9

**Exercise 9.1** Show that the problem LINEAR SPACE ACCEPTANCE is PSpace-hard.

**Hint:** It is easy to see that every language that can be recognized by a DTM with space bound  $O(n)$  can be polynomially reduced to LINEAR SPACE ACCEPTANCE. Use a “padding argument” (padding = making an input string longer by adding redundant symbols) to show that this generalizes to languages that can be recognized within polynomial space.

**Exercise 9.2** Show that the language GM – related to the game “Go-Moku” – belongs to PSpace. See the end of Section 17.3 of the Lecture Notes for a definition of Go-Moku and GM.

**Exercise 9.3** Find a class  $\mathcal{C}$  of languages for which the statement  $\mathcal{C} \subseteq (\exists)_{pol}[\mathcal{C}]$  is false (and argue why).

**Hint:** You can find such a class exploiting the natural encoding function introduced in the lecture.

**Exercise 9.4** During the lecture, we discussed the closure property

$$L \in \mathcal{C} \implies L_\varepsilon \in \mathcal{C}.$$

Show that the complexity classes  $\Sigma_k, \Pi_k$  actually have this closure property for each choice of  $k \geq 0$ .